

The Mode Analysis of Sandwich Beam Subjected Under Winkler Elastic Foundation

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Abstract: On the basis of accurately considering axial extension and the first-order transverse shearing, geometrically nonlinear governing equations for composite sandwich beams with elastic support, subjected to thermal loads, are formulated. By using a shooting method, the two-point boundary value problem for nonlinear ordinary differential equations are solved numerically and the numerical solutions to static thermal post-buckling and thermal bending deformation of the sandwich beams with pinned-fixed ends under transverse uniform thermal loads are obtained. And critical thermal buckling modes transition feature are given. With the increasing foundation stiffness parameters, the corresponding physical meaning and the curves of relationship between temperature parameters and the boundary force and moment is given.

1. Introduction

Thermal expansion along axial direction under temperature load leads to thermal post-buckling deformation of beams and rods, which is required to be studied with axial extension. Measurement of stability of axially extensible beams has been one basic calculation problem in physics, and one subject of interest to researchers^[1-6]. Many studies have been done of elastic beams and panels. Li^[7] explored displacement and angular rotation of beams on a Winkler foundation under randomly distributed mechanical load. Zhang^[8] studied the elastic beams on a non-linear foundation subjected to a longitudinal force, and different mode transition caused by different mechanical load. Pate^[9] numerically analyzed the effect of nonlinear free bending vibration and transverse shearing deformations on beams on a two-parameter elastic foundation. Shen^[10] examined the effect of post-buckling on composite panels under mechanical and thermal load by using Galerkin method. niu^[11] studied, based on Timoshenko theory, mode transition of elastic beam on a Winkler foundation subjected to uniform temperature rise.

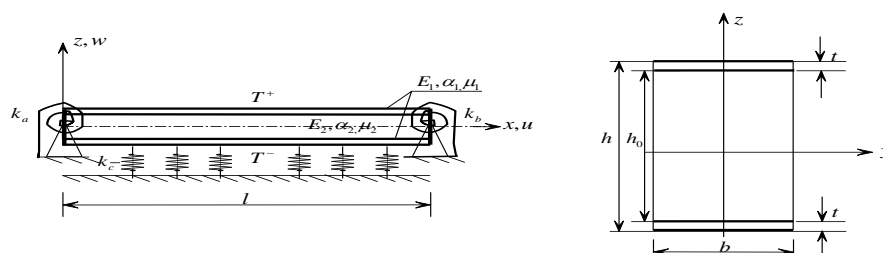


Figure 1. Geometry of sandwich beam under elastic foundation and rotational springs

Literature survey shows that there are still few researches into the effect of stiffness change on thermal buckling and thermal post-buckling behaviors of composite sandwich beams on an elastic



foundation under temperature load. In this paper, based on axial extension and first-order shear deformation theory^[3,11], governing equations for nonlinear thermal buckling of composite sandwich beams subjected to temperature load on an elastic foundation are derived. With the shooting method, numerical solutions for thermal bending and buckling of a sandwich beam under uniform transverse temperature rise are obtained. This paper highlights mode transition caused by foundation stiffness change, to which the corresponding stiffness and critical temperature are presented.

2. Governing Equation

Supposing there is a composite sandwich straight beam with rectangular section, its length, width and height are l , b and h_0 , respectively (see Figure 1). Modulus of elasticity, coefficient of expansion, and Poisson's ratio of the panel and the core are given in Figure 1a. Made up of same material, both the upper and lower panels are t in thickness; therefore, the core is $h = h_0 - 2t$ in thickness. The beam has a rectangular section, as shows in Figure 1b. Assuming that the beam is subjected to a temperature rise which changes with thickness, the temperature rises, a transverse linear rise, in the upper and lower panels are T^+ and T^- , respectively. It is assumed that physical parameters of different layers of the beam do not change with the rise of the temperature. The beam, at the bottom, is supported by an elastic foundation, which is K_c in stiffness, and in both ends, is restrained by two movable springs, which are k_a and k_b in stiffness, respectively. The static response of thermal buckling of the beam was analyzed, with axial extension and transverse shear deformation taken into consideration. Its corresponding dimensionless governing equation is given below^[3]:

$$\frac{dS}{d\xi} = \Lambda_0, \quad \frac{dU}{d\xi} = \Lambda_0 \cos \theta - 1, \quad \frac{dW}{d\xi} = \Lambda_0 \sin \theta, \quad \frac{d\varphi}{d\xi} = m + m_T \quad (1)$$

$$\frac{dP_H}{d\xi} = 0, \quad \frac{dP_V}{d\xi} = \Lambda_0 W K_c, \quad \frac{dm}{d\xi} = \Lambda_0 (P_H \sin \theta - P_V \cos \theta) \quad (2)$$

$$\Lambda_0 \cos \theta = \Lambda_0 \cos(\varphi - \gamma) = D_1 \cos \varphi + D_2 \sin \varphi \quad (3)$$

$$\Lambda_0 \sin \theta = \Lambda_0 \sin(\varphi - \gamma) = D_1 \sin \varphi - D_2 \cos \varphi \quad (4)$$

$$D_1 = \frac{(P_H \cos \varphi + P_V \sin \varphi + P_T)}{F_1} + 1 \quad (5)$$

$$D_2 = \frac{\beta(P_H \sin \varphi - P_V \cos \varphi)\phi_3}{[12\delta^2(c + (1-c)K_1)]} \quad (6)$$

$$\Lambda_0 = \sqrt{D_1^2 + D_2^2}, \quad \tan \gamma = D_2 / D_1 \quad (7)$$

$$P_T = \tau_m \eta_1 / \phi_3, \quad m_T = \tau_d \eta_2 / (12\delta\phi_3) \quad (8)$$

$$F_1 = 12\delta^2 \phi_1 / \phi_3, \quad F_4 = 12\delta^2 \eta_3 / \phi_3 \quad (9)$$

The dimensionless boundary conditions are as follows:

$$S(0) = 0, \quad U(0) = 0, \quad W(0) = 0 \quad (10a)$$

$$\varphi(0) = \beta_0, \quad m(0) = K_a \varphi(0) \quad (10b)$$

$$U(1) = 0, \quad W(1) = 0, \quad m(1) = -K_b \varphi(1) \quad (11)$$

According to Eqs. (6) and (7), dimensionless parameter β is a determinant of shear deformation [5].

$$\beta = \frac{k\bar{E}}{\bar{G}} = \frac{2k(c + (1-c)K_1)}{(c/(1+\nu_1) + (1-c)K_1/(1+\nu_2))} \quad (12)$$

$$\bar{E} = cE_1 + (1-c)E_2, \quad \bar{G} = cG_1 + (1-c)G_2 \quad (13)$$

If $\beta = 0$, the dimensionless governing equation can be transformed into a governing equation for Euler-Bernoulli beam without transverse shear deformation, where $\gamma = 0$, and $\varphi = \theta$. c represents

the uniformity of transverse material and is a constant. When $0 < c < 1$, beams are non-uniform in material; $c = 0$ represents that beams are made up of only core material while $c = 1$ represents that of only panel material.

3. Numerical Calculation and Results Analysis

Eqs.(1) and (2) are nonlinear ordinary equations with boundary value problems to be solved; therefore, they are hard to be solved with the analytical method. In this paper, with the shooting method [3-6], numerical solutions are obtained. When calculating, assume that the upper and lower layer are made up of metal while the middle of core.

As an example, a beam constrained by an elastic foundation but not by rotational springs is considered. Assuming that the end constraints of the beam are not symmetrical, i.e. there is unmovable hinge supports on the left side while a fixed pin on the right. The effects of other forms of elastic restraint on the stability of beams are further discussed in the subsequent paper. The characteristics of buckling and post-buckling deformation of composite sandwich beams subjected to uniform temperature rise are analyzed when $c = 0.1$, and $\delta = 30$.

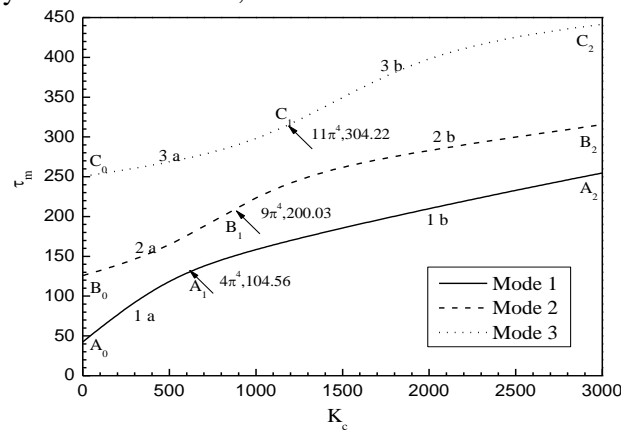


Figure 2. Characteristic curves of the τ_{cr} versus K_c for the first three buckling modes

In Fig 2, the relationship curve between buckling temperature and foundation stiffness of composite sandwich beams is given when there is unmovable hinge supports on the left side while a fixed-pin on the right. Three relationship curves without cross-over points are given when $K_c \leq 3000$. Three buckling modes corresponding to three buckling temperature points are presented when $K_c = 0$. There is one special point in each curve, i.e. the stiffness point for mode transition, and it divides each curve into two parts. In the figure, three stiffness points, A_1, B_1, C_1 , are showed, and their positions are $(K_c, \tau_m)_1 = (4\pi^4, 105.5610)$, $(K_c, \tau_m)_2 = (9\pi^4, 200.03)$ and $(K_c, \tau_m)_3 = (11\pi^4, 304.22)$, respectively. Near these stiffness points, mode transition takes place in sandwich beams. Whether there is a change of polarity of moment of flexure, or whether the curvature is zero is also our basis for the determination of the occurrence of mode transition.

Now comes the analysis of the load conditions of beams. With the rise of foundation stiffness, the relationship curve between dimensionless temperature and longitudinal tensile force in the unmovable hinge support is presented in Fig 3 and moment of flexure in the fixed end in Fig 4. As shows in Fig 3, with the rise of temperature, the longitudinal tensile force monotonically increases which is imposed on beams on a foundation of same stiffness. At the same temperature, the larger the foundation stiffness is, the steeper the curve, i.e. the larger the longitudinal tensile force. However, once a certain high temperature is reached, the longitudinal tensile force decreases, which can be explained by the occurrence of mode transition.

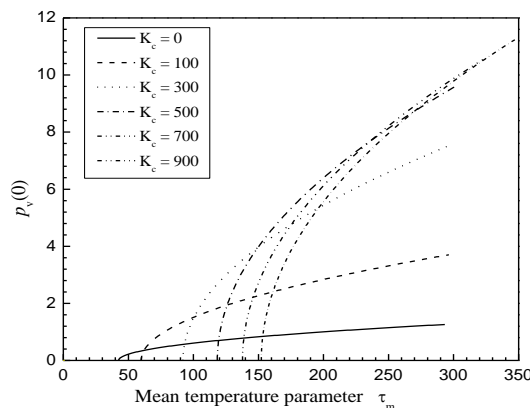


Figure 3 The curves of relationship between τ_m and $P_v(0)$ for the different values of K_c

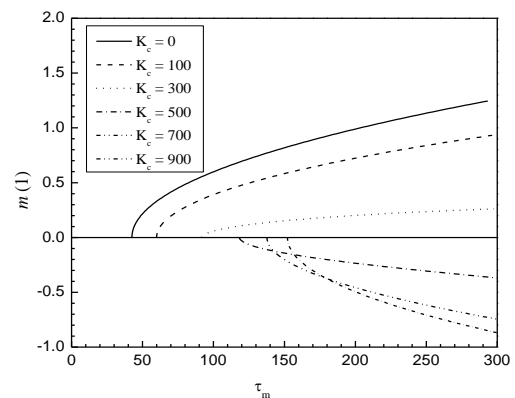


Figure 4 The curves of relationship between τ_m and $m(1)$ for the different values of K_c

As can be seen in Fig 4, change of the direction of moment of flexure also represents the occurrence of mode transition. When $K_c < 4\pi^4$, moment of flexure in the end of beams is positive. This is corresponding to the first-order buckling state where mode transition does not occur. When $K_c = 4\pi^4$, the direction of moment of flexure in the end of beams changes. When $K_c > 4\pi^4$, moment of flexure in the end of beams is negative. This post-buckling equilibrium configuration is developed from the second-order critical buckling mode. The higher the temperature is, the greater impact foundation stiffness has on post-buckling deformation of and force on composite sandwich beams.

4. Conclusion

Based on axial extension and first-order shear deformation theory, the numerical solutions of boundary value problems of ordinary equations are obtained with the shooting method. The buckling and post-buckling characteristics of sandwich beams on an elastic foundation are also discussed with emphasis on the effect of foundation stiffness on critical buckling state. The major findings are as follows:

- (1) For the first three kinds of critical buckling modes, critical temperature monotonically rises with the increase of foundation stiffness. The three curves have no cross-over points, but each has a mode transition point. The transition stiffness and critical temperature corresponding to these three points. Different transition stiffness values represent the transition characteristics of different modes.
- (2) With the rise of temperature, the longitudinal tensile force monotonically increases which is imposed on beams on a foundation of same stiffness. At the same temperature, the larger the foundation stiffness is, the steeper the curve, i.e. the larger the longitudinal tensile force. However, once a certain high temperature is reached, the longitudinal tensile force decreases, which can be explained by the occurrence of mode transition.

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