

# Time dependent variation of carrying capacity of prestressed precast beam

Tuan D Le<sup>1,2,\*</sup>, Petr Konečný<sup>2,\*</sup> and Pavlína Matečková<sup>2,\*</sup>

<sup>1</sup> Faculty of Civil Engineering, Saigon Technology University, 180 Cao Lo Str., Ward 4, Dist. 8, HCMC 700000, Vietnam

<sup>2</sup>VSB – Technical University of Ostrava, Faculty of Civil Engineering, L. Podeste 1875, 708 33 Ostrava-Poruba, Czech Republic

\*Corresponding authors: tuan.leduc@stu.edu.vn, petr.konecny@vsb.cz, pavlina.mateckova@vsb.cz

**Abstract.** The article deals with the evaluation of the precast concrete element time dependent carrying capacity. The variation of the resistance is inherited property of laboratory as well as in-situ members. Thus the specification of highest, yet possible, laboratory sample resistance is important with respect to evaluation of laboratory experiments based on the test machine loading capabilities. The ultimate capacity is evaluated through the bending moment resistance of a simply supported prestressed concrete beam. The probabilistic assessment is applied. Scatter of random variables of compressive strength of concrete and effective height of the cross section is considered. Monte Carlo simulation technique is used to investigate the performance of the cross section of the beam with changes of tendons' positions and compressive strength of concrete.

## 1. Introduction

Prestressed concrete has been a widely used material in construction due to its conveniences in comparison with reinforced concrete such as less deep and larger in span if exposed in similar conditions, thus reducing the carbon dioxide footprint. Moreover, progressive high performance materials are being developed in order to further improve concrete performance and reduce the cement usage [1], [2], [3], [4], [5]. Industrial by-products such as slag and fly ash are used. However, it needs to be noted that prestressed concrete structures require higher quality materials in prestressing and more complicated formwork [6]. Consequently, attention should be paid to the design of these structures to get better understanding of their behavior and performance [7].

When new material and structural precast elements are being developed, experimental investigations help to verify the numerical modelling. The numerical models applicable for the design purposes should be calibrated with experiments.

When the test of the samples is prepared, the question might arise how large and how reinforced might the samples be. One wants to prepare samples large enough while address the limit of testing machines with respect to dimensions and maximum loading capacity. So, when designing the experiments, the ultimate capacity of the sample is of great importance.

There is also factor of concrete maturing because the hydration process is time dependent and concrete strength is being developed over the time. Therefore, it is necessary to address the increase of concrete resistance over the time. The EN 1992-1-1 Eurocode 2 [8] gives estimated compressive strength of concrete at various ages:

$$f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm} \quad (1)$$



where:

$f_{cm}(t)$ : average strength (MPa) of concrete in compression at age  $t$  (days);

$\beta_{cc}(t)$ : coefficient depends on the age of the concrete.

If measured data are available and approximation given in equation (1) does not fit measured data, other approximations are available. See e.g. [9] and [10] for applications in design of underground shotcreting works such as [11].

Design methods for reinforced and prestressed concrete structures have been developed during past decades [6] and are still under development with respect to the evaluation of advanced non-linear modelling capabilities [12], [13]. Current codes [8], [14] are based on both deterministic and probabilistic theories. However, actual application is rather deterministic from the engineer standpoint. Since the input parameters governing the resistances of reinforced as well as prestressed concrete structures show high variability, probability-based approaches [15], [16], [17], employed e.g. in [18], [19] in case of reinforced concrete, are robust tools in order to address the upper percentile of the prestressed concrete beams carrying capacity distribution. Probabilistic simulation allows addressing the question of ultimate force of foreseen experimental bending test.

In case of prestressed concrete, it is important to take of influence of rheology into account. This includes creep and shrinkage of concrete as well as relaxation loss of tendons. However, time dependent change of the prestressing force does not affect the level of ultimate bending resistance [20] in contrast to the change of concrete strength.

In this paper, the preparation of simple model for the probability-based bending resistance computation of a simply supported prestressed concrete beam is shown. The goal of the development is to prepare the procedure for the estimation of time dependent experimental simply supported beam ultimate resistance in three-point bending test (figure 1) and the newly prepared concrete mixtures. The availability of time dependent behavior of basic properties is considered. The bending moment resistance of the beam with variations of cylinder compressive strength of concrete and position of prestressing tendons in the section is evaluated. The Monte Carlo simulation based approach will be adopted in order to include scatter of input parameters. Random variations of compressive strength of concrete and effective height of the cross section are built up based on normal distributions. Procedures are composed using Matlab compatible environment [21], [22] to facilitate the simulations.



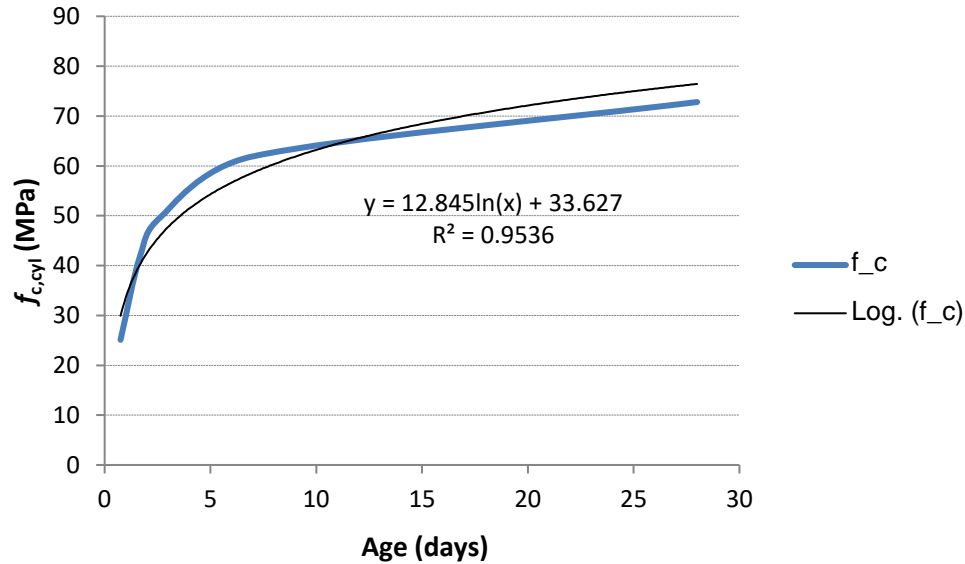
**Figure 1.** Preparation of laboratory evaluation of carrying capacity: (a) Samples of prestressed concrete beams and (b) Test machine pressing head.

## 2. Material properties

In order to address necessary time dependent material behavior especially in case of precast prestressed girders, the time dependent concrete strength and modulus of elasticity is crucial of interest.

The approximation of sample concrete cylindrical strength may be based on codes [8]. If the laboratory data of strength are obtained during the concrete aging, the data may be approximated by suitable curve. Such sample case is given below in figure 2 on which the strength is approximated in a

time by regression function, equation (2). The regression function is obtained using the spreadsheet calculation. It has good coefficient of determination  $R^2=0.9536$ . Thus, the approximation in equation (2) has a good fit.



**Figure 2.** Approximation of considered cylinder strength time dependent behavior.

$$f_{c,cyl}(t) = 12.845\ln(t) + 33.627 \quad (2)$$

where:

$f_{c,cyl}(t)$ : cylindrical strength (MPa) of a reinforced concrete tested sample at age  $t$  (days).

If the variation of concrete strength is available as given in table 1 on which available data are based on the evaluation of cubic strength, the standard deviation can be recomputed to dimensionless coefficient of variation. Thus, assumption is made here that the relative variation of cubic strength and cylinder-based strength is the same. With such assumption, the confidence bounds for the simulated cylinder-based concrete strength may be evaluated with respect to given selected percentiles (5.th and 95.th) related to its lower and upper bounds.

**Table 1.** Statistical parameters variation of concrete strength for 28 days.

t (days)	No. of samples	$f_{c,cube}$ (MPa)				
		$f_{c,mean}$	$f_{c,std}$	$f_{c,cov}$	$f_{c,05}$	$f_{c,95}$
28	13	87.8	3.4	0.0388	82.2	93.4

Since the probabilistic modelling is of question, the statistic characteristic of considered probability density function shall be considered as indicated on the equation (3).

$$f_{c,cyl}(t) = \mu(f_{c,cyl}(t)) + cov(f_{c,cube}(28)) \cdot \mu(f_{c,cyl}(t)) \quad (3)$$

where:

$f_{c,cyl}(t)$ : cylindrical strength (MPa) of sample at age  $t$  (days);

$\mu(f_{c,cyl}(t))$ : mean value of cylindrical strength of sample at age  $t$ ;

$cov(f_{c,cube}(28))$ : coefficient of variation of cubic strength of sample,  $t = 28$  days.

### 3. Probabilistic modelling of beam resistance

The ultimate resistance model is specified first while the description of sample input data follows.

### 3.1. Computational model

Considering three-point bending test loading scheme, the ultimate force  $F_r$  for bending is:

$$F_r = \frac{4M_r}{l} \quad (4)$$

where:

$M_r$ : ultimate bending moment resistance (kNm) given in equation (5);

$l$ : loading span (m) of the beam.

Ultimate bending moment resistance of a critical rectangular cross section of an underreinforced beam (effect of reinforcement is ignored) is depicted as:

$$M_r = F_c(d - 0.4x) \quad (5)$$

where:

$d$ : effective height (m) of considering cross section of the beam, the distance from extreme compression fiber to centroid of prestressed steel;

$x$ : height of compression zone (m), calculated as:

$$x = \frac{A_p f_{p01}}{0.8 f_{c,cyl} b} \quad (6)$$

$F_c$ : compressive force (kN) in concrete and is determined as:

$$F_c = 0.8 f_{c,cyl} b x \quad (7)$$

$A_p$ : cross-sectional area (m<sup>2</sup>) of prestressing tendons;

$f_{p01}$ : 0.1% proof-stress (kPa) of prestressing steel;

$f_{c,cyl}$ : cylinder compressive strength (kPa) of concrete;

$b$ : width (m) of cross section of the considered beam.

### 3.2. Monte Carlo simulation technique

Monte Carlo method is one of the widely used simulation techniques based on a process of repeated random sampling to obtain numerical results. It was first systematically used very early and can be considered as the art of approximating an expectation by the sample mean of a function of simulated random variables [23]. Steps of the approach include: creating the sample, running the model and analyzing the data.

### 3.3. Normal distribution generation of random variables

According to [24], a desired distribution can be modeled using the following transformation formula:

$$N(\mu, \sigma) = \mu + \sigma \times N(0, 1) \quad (8)$$

where:

$N(0, 1)$ : represents random numbers from the normalized normal distribution;

$\mu$ : specified mean value;

$\sigma$ : specified standard deviation;

$N(\mu, \sigma)$ : represents random numbers from the generated normal distribution.

If the parameter of interest is time dependent, then:

$$N(\mu, \sigma, t) = \mu(t) + \sigma(t) \times N(0, 1) \quad (9)$$

where:

$\mu(t)$ : specified time dependent mean value at age  $t$  (days);

$\sigma(t)$ : specified time dependent standard deviation at age  $t$ ;

$N(\mu, \sigma, t)$ : represents time dependent normal distribution from the normalized one at age  $t$ .

## 4. Sample numerical example

A simply supported prestressed concrete beam with dimensions of cross section of 0.9 m wide and 0.56 m high as depicted in figure 1 is considered. The length of the beam is 7 m. There are 2 layers of

bottom tendons with 8 wires in lower layer and 4 wires (their redundant segments were amputated at the face of the beam) in the upper one. The top layer of tendons was not considered in the calculation because it locates close to the neutral axis of the beam. Area of one-wire prestressing tendon is  $A_p = 150 \times 10^{-6} \text{ m}^2$ . Distance between the two layers of tendons is 0.05 m. A 0.08 m concrete cover is assumed and effect of reinforcement is ignored. The cross-sectional bending resistance at midspan of the beam will be realized through Monte Carlo simulation technique.

#### 4.1. Input parameters

The variables include the cylinder compressive strength of concrete depicted in table 1 and the distance from extreme compression fiber to centroid of prestressed steel given in table 2.

**Table 2.** Information on input parameters employed in the study.

Parameter	Notation	Mean	Coefficient of variation	Transformation
Concrete strength (kPa)	$f_{c,cyl}(t)$	Equation (1)	0.0388	$\mu(f_{c,cyl}(t)) + 0.0388 \times N(0,1)$
Distance from extreme compression fiber to centroid of prestressed steel (m)	$d$	0.52	0.0096	$d = 0.52 + 0.005 \times N(0,1)$
Loading span (m)	$l$	6.85	-	-
Width of cross section of the considered beam (m)	$b$	0.9	-	-
Height of cross section of the beam (m)	$h$	0.56	-	-
Cross-sectional area of prestressing tendons ( $\text{m}^2$ )	$A_p$	$150 \times 10^{-6}$	-	-
0.1% proof-stress of prestressing steel (kPa)	$f_{p01}$	$1687 \times 10^3$	-	-
Thickness of concrete covered layer (m)	$c$	0.08	-	-

#### 4.2. Distributions of random variables

Histograms of cylinder compressive strength of concrete and effective height of considering cross section of the beam are built up through using equation (8).

#### 4.3. Results

Equation (5) is used to compute ultimate bending moment resistance of the critical cross section. It needs to be noted that the resistance is provided by concrete and prestressed tendons. The conventional reinforcing steel is neglected herein. Distribution of bending moment resistance of the cross section of the beams at different ages of concrete are displayed in figure 3.

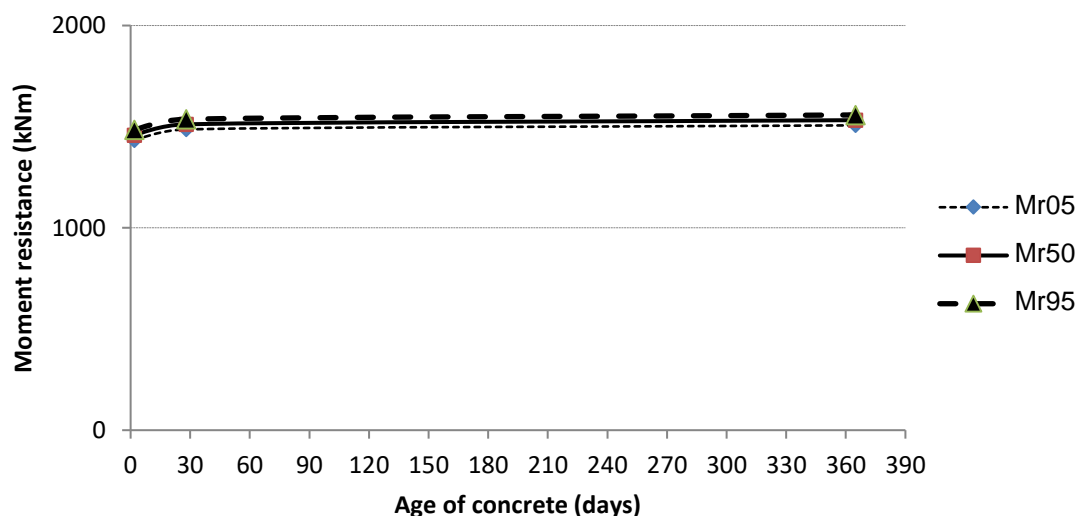
It is likely that bending moment resistance of the cross section at different ages of concrete have normal distributions and these distributions are different with the progress of maturing of concrete.



**Figure 3.** Distribution of bending moment resistance of the cross section of the beam with: (a)  $t = 2$  days, (b)  $t = 28$  days and (c)  $t = 365$  days.

**Table 3.** Simulation results on bending moment resistance and ultimate force of cross section of the beam and their confidence bounds.

Age of concrete $t$ (days)	Bending moment resistance of the cross section of the beam (kNm)			Ultimate force for bending of the cross section of the beam (kN)		
	$M_{r05}$ (5%)	$M_{r50}$ (50%)	$M_{r95}$ (95%)	$F_{r05}$ (5%)	$F_{r50}$ (50%)	$F_{r95}$ (95%)
2	1432.901	1458.424	1483.947	836.731	851.634	866.538
28	1486.796	1511.803	1536.810	868.202	882.805	897.408
365	1506.599	1532.259	1557.918	879.766	894.750	909.733



**Figure 4.** Bending moment resistance of the cross section of the beam and its confidence bounds.

It can be seen from figure 4 that moment resistance of the cross section increases over the time. The rise took place at concrete age of several days to one week. This trend almost slowed down after the 1<sup>st</sup> week of age and there was only a gradually light growth from 28 days onwards. The reason is the rectangular concrete beam cross section that cannot fully utilize the potential of high quality concrete. It can be also seen that the variation is almost negligible. It is within the range of  $\pm 2$  percent. If the more sophisticated shape of the beam, such as T-section or I-section, would be employed, the increase of bending resistance and its variation might be more significant.



According to three-point bending moment formula equation (4), the loading force  $F_r$  is computed in order to find the most likely ultimate value for the laboratory experiment considering the scatter of input parameters. The resulting loading forces are given in table 3.

The resulting necessary loading forces given in table 3 are below 1000 kN. If, for example, the limit for the loading unit would be 1000 kN, the samples would break under bending considering assumed simplifications.

## 5. Conclusions

The presented numerical procedure allows for the preliminary evaluation of the bending resistance of prestressed concrete beams. The bending resistance allows for the calculation of ultimate force necessary to break the laboratory sample valuable when considering limit of the loading unit.

The variation and time-dependency of concrete strength is considered. The variation of concrete cover is also considered. Both random parameters were modeled as normal distributions.

The results showed that bending moment resistance variation and its increase with concrete maturing was not significant. The carrying capacity growth was limited by the rectangular shape of the beam.

In addition, the random variation of bending resistance and ultimate loading force was not significant herein. Thus for further experimental preparations with rectangular shape prestressed beams, the average values for the problem specification are enough.

The current model deals only with rather simple case of carrying capacity of prestressing tendons from bottom part because tendons at top part were very near to the beam's neutral axis. In addition, the longitudinal reinforcement was not considered due to their small influence to carrying capacity of the beam. Factor of shear resistance was not also been studied yet.

In future development of this research, therefore, computation using method of limit strain on which the carrying capacity of both parts of prestressing tendons included should be exploited. Furthermore, influence of the longitudinal reinforcement to the carrying capacity should be taken into account. Expansion of this topic with inclusion of shear resistance would be a next step. Deflection, cracking, stiffness change and shape optimization of the beam are other directions of development.

## References

- [1] Aitcin P C 1998 High performance concrete (Taylor & Francis) ISBN-10: 0419192700, ISBN-13: 978-0419192701
- [2] Konecny P, Lehner P, Ghosh P and Tran Q 2017 Variation of diffusion coefficient for selected binary and ternary concrete mixtures considering concrete aging effect *Proc. 6th Int. Conf. Non-traditional Cement & Concrete* June 19-22-2017 Brno (Preprint gr-qc/0303038)
- [3] Ghosh P, Konecny P, Lehner P and Tikalsky P J 2017 Probabilistic time-dependent sensitivity analysis of HPC bridge deck exposed to chlorides *J. Computers and Concrete* vol **19** No. 3 pp 305-313 DOI: <https://doi.org/10.12989/cac.2017.19.3.305>
- [4] Cajka R, Vaskova J and Smirakova M Stress and strain of fiber reinforced concrete composites *Proc. 6th Int. Conf. Non-traditional Cement & Concrete* June 19-22-2017 Brno (Preprint gr-qc/0303038)
- [5] Keulenl A, Kakebeeke P J T, Grunewald S and Brouwers H J H Durability evaluation of aam using a plasticizing additive, enhancing fresh concrete rheology *Proc. 6th Int. Conf. Non-traditional Cement & Concrete* June 19-22-2017 Brno (Preprint gr-qc/0303038)
- [6] Nawy E G 2009 *Prestressed concrete - a fundamental approach* 5th edition update (NJ Upper Saddle River: Prentice Hall)
- [7] Biondini F, Frangopol D M, Bontempi F and Malerba P G 2004 Reliability of material and geometrically non-linear reinforced and prestressed concrete structures *J. Computer & Structures*
- [8] EN 1992-1-1 2004 Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings *The European Union Per Regulation* 305/2011

- [9] Oreste P P 2003 A procedure for determining the reaction curve of shotcrete lining considering transient conditions *Rock Mechanics and Rock Engineering* vol **36** (3) pp 209-236 DOI: 10.1007/s00603-002-0043-z
- [10] Klepsatel F, Kusý P and Mařík L 2003 Construction of tunnels in rock massive (Výstavba tunelů ve skalních horninách) (Bratislava: Jaga group) ISBN 80-88905-43-5 (in Czech)
- [11] Vojtasik K, Mohyla M and Hrubesova E 2017 Static design of steel-concrete lining for traffic tunnels *IOP Conference Series: Materials Science and Engineering*, vol **236** (1) Art. no. 012083
- [12] Králík J and Klabník M 2016 Nonlinear analysis of the failure of nuclear hermetic reinforced concrete structure due to extreme pressure and temperature, *Transactions of the VŠB – Technical University of Ostrava Civil Engineering Series* No. 2 vol **16** paper#17
- [13] Sucharda O, Bílek V, Šmiráková, Martina K J and Čajka R 2017 *Periodica Polytechnica Civil Engineering* vol **61** No. 4 pp 972-986
- [14] Matthews S L, Bigaj-Van Vliet A J and Dieteren G G A 2016 Fib model code 2020 – A new development in structural codes: Towards a general code for both new and existing concrete structures ed Beushausen H *Proc. fib Symp. 2016 Performance-based approaches for concrete structures* (Cape Town) pp 22-31
- [15] Marek P et al 2003 *Probabilistic Assessment of Structures using Monte Carlo Simulation - Background, Exercises and Software* TeReCo 2<sup>nd</sup> edition (Praha: Institute of Theoretical and Applied Mechanics, Academy of Sciences of Czech Republic)
- [16] Melchers R 1999 *Structural reliability analysis and prediction* (Civil Engineering) (West Sussex: Wiley)
- [17] Stewart M G and Rosowsky D V 1998 Time-dependent reliability of deteriorating reinforced concrete bridge decks *Structural Safety* vol **20** (1) pp 91-109
- [18] Janas P, Krejsa M, Šejnoha J and Krejsa V 2017 DOPRO-based reliability analysis of structures *Structural Engineering and Mechanics* vol **64** No. 4 pp 413-426
- [19] Teplý B and Vorechovská D 2012 Reinforcement corrosion: Limit states, reliability and modelling *J. Advanced Concrete Technology* vol **10** pp 353-362
- [20] Navrátil J 2014 *Prestressed concrete structures* 2<sup>nd</sup> edition (Ostrava: VSB – Technical University of Ostrava, Faculty of Civil Engineering) ISBN: 978-80-248-3625-6
- [21] MatLab: The language of technical computing *MathWorks*- on-line: [www.mathworks.com](http://www.mathworks.com)
- [22] GNU Octave Scientific programming language on-line: [www.octave.org](http://www.octave.org)
- [23] Anderson E C 1999 *Monte Carlo methods and importance sampling* - Lecture notes for Stat 578C, Statistical Genetics
- [24] Fegan G and Gustar M 2003 *Chapter 2 – Monte Carlo Simulation* in: Marek P et al *Probabilistic Assessment of Structures using Monte Carlo Simulation - Background, Exercises and Software* TeReCo 2<sup>nd</sup> edition (Praha: Institute of Theoretical and Applied Mechanics, Academy of Sciences of Czech Republic)

### Acknowledgments

The article preparation was supported from sources for conceptual development of research, development and innovations at VŠB-Technical University of Ostrava for year 2017, which were granted by the Ministry of Education, Youth and Sports of the Czech Republic.