

Influence of structural parameter included in nonlocal rock mass model on stress concentration around circular tunnel

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Abstract. A model of elastic body, including local curvature of elementary volume, is matched with a nonlocal model with a linear structural parameter in the differential approximation. The problem on deformation of rock mass around a circular cross section tunnel is solved numerically. The contours of the calculated stresses are plotted. It is shown that inclusion of local bends in the model results in expansion of influence zone of the tunnel and reduces stress concentration factor at the tunnel boundary.

1. Introduction

The problem on deformation of rock mass surrounding a tunnel is one of the basic problems in geomechanics. Many formulations of the problem have been analyzed [1–5]. This problem is very attractive for the use in the theoretical research as there are known exact solutions in some limiting cases. Such solutions can be test solutions for more complex models, for instance, taking into account internal structure of rock mass [6–8]. Internal structure is one of the fundamental properties of rocks and is a source of numerous equilibrium states and capacity of rock mass to accumulate some energy of external impact in the form of internal self-balancing stresses [9–11].

There many ways of taking into account internal structure of rock mass. The mathematical models with the internal variables in [12–14] describe behavior of the medium in terms of its structural elements. The non-Archimedean analysis methods are used in [15, 16] to describe multi-scale rock mass and its deformation and failure.

It is noteworthy that models of media having internal structure are used in the other research areas, such as mechanic of composites and nanomaterials, dynamics of crystal lattices [17, 18].

In this paper, in the framework of the linear elasticity model with a structural parameter [19, 20] and taking into account local curvature of a medium, the authors solve the problem on deformation of rock mass surrounding a mined-out tunnel.

2. Mathematical model

Let us use the approach from [19]. Let a discrete number of elastic particles seat at the points of a square lattice (plain-strain deformation). Let L_1, L_2 —linear dimensions of the deformation domain; m, n —particles that fall on L_1, L_2 , i.e. $L_1/m = L_2/n$ —linear dimension of particle (diameter); $N = m \cdot n$ —total number of particles. In a general case, at the particle contacts, it is possible to introduce sliding by plasticity laws, and dry or viscous friction. In case under analysis, we discuss an elastic model and assume that sliding is absent at the particle contacts. It is also assumed that the point



moments are not transferred through the contacts. Then, for each particle (with a number i), four force vectors \bar{T} and four displacement vectors \bar{u} will be determined at the contact points A_i, B_i, C_i, D_i with the neighbor articles (Figure 1). One vector conforms with two scalar components. Temperature and other parameters can be included additionally. The total number of the scalar unknowns is: $4N - 2m - 2n$ —displacements at the contacts; $4N - 2m - 2n$ —forces at the contacts; $4m + 4n$ —displacements at the boundaries; $4m + 4n$ —forces at the boundaries. All in all, there are $8N + 4m + 4n$ unknowns.

For each out of N particles, there are two equations of equilibrium and one equation of moment of forces, which makes total $3N$ equations. Then, at each of $2m + 2n$ boundary contacts, two conditions should be set (either in terms of forces or displacements, or their combination); i.e. there are $4m + 4n$ conditions. Regarding constitutive equations, the four points A_i, B_i, C_i, D_i agree with the four vectors of displacements—8 degrees of freedom. Constitutive equations can only include such combinations that are independent of translation and rotation of particles, which means that three degrees of freedom should be withdrawn. Accordingly, we have five invariant combinations of displacement. There are also 8 degrees of freedom for the forces. The vector of sum of forces and the moment should equal zero. So, we have five degrees of freedom as a result. Consequently, the constitutive equations should relate five invariant combinations of displacements with five force characteristics. There should be five equations for a particle and $5N$ equations for all particles. We have $8N + 4m + 4n$ equations all in all. The system is closed and reduced to $8N + 4m + 4n$ algebraic equations for $8N + 4m + 4n$ unknowns.

In this manner, we need five constitutive equations for elementary volume. On the other hand, there are three constitutive equations for an elastic body. This means that the classical theory contains assumptions that have equal strength of two equations. Moreover, these equations are of equal significance as in the Hook law. Let us formulate such equations in explicit form.

We select five invariant combinations of displacements capable to be included in the constitute equations. The number of choices is unlimited. In our case, we take the closest variant to the linear elasticity:

$$\begin{aligned} \frac{E}{2l} [u_1(A_i) - u_1(C_i)] &= \frac{t_{11}(A_i) + t_{11}(C_i)}{2} - \nu \frac{t_{22}(B_i) + t_{22}(D_i)}{2}, \\ \frac{E}{2l} [u_2(B_i) - u_2(D_i)] &= \frac{t_{22}(B_i) + t_{22}(D_i)}{2} - \nu \frac{t_{11}(A_i) + t_{11}(C_i)}{2}, \\ \frac{E}{2(1+\nu)} \frac{1}{2l} [u_2(A_i) - u_2(C_i) + u_1(B_i) - u_1(D_i)] &= \frac{1}{4} [t_{12}(A_i) + t_{12}(C_i) + t_{21}(B_i) + t_{21}(D_i)], \\ \frac{1}{2l} \left[\frac{u_1(A_i) + u_1(C_i)}{2} - \frac{u_1(B_i) + u_1(D_i)}{2} \right] &= \xi \frac{1}{2} [t_{11}(A_i) - t_{11}(C_i) - t_{21}(B_i) + t_{21}(D_i)], \\ \frac{1}{2l} \left[\frac{u_2(B_i) + u_2(D_i)}{2} - \frac{u_2(A_i) + u_2(C_i)}{2} \right] &= \xi \frac{1}{2} [t_{22}(B_i) - t_{22}(D_i) - t_{12}(A_i) + t_{12}(C_i)], \end{aligned} \quad i = \overline{1, N}, \quad (1)$$

where E, ν —Young's modulus and Poisson's ratio; $2l$ —linear dimension (diameter) of particle; ξ —structural parameter dimension of which is inversely proportional to dimension of stresses. The first three equations are the discrete analog of the Hook law. The last two equations are not formulated explicitly in the classical theory. Though, there is information about them. The classical theory involves (as a matter of course) the postulate on diffeomorphism [21]. More simply, it is assumed that all functions are sufficiently smooth. This means that any functions can locally be presented as a linear function, for instance: $u_1 = a_{11}x_1 + a_{12}x_2$, $u_2 = a_{12}x_1 + a_{22}x_2$, where a_{11}, \dots, a_{22} —const. Placing this representation in (1) yields $\xi = 0$. The contrary is valid, too. Consequently, the two equations from

(1) are “hidden” in the postulate of diffeomorphism. When $\xi \neq 0$ we obtain a linear elasticity theory with a structural parameters, and Eq. (1) describe local bends.

The system of equations of equilibrium is given by:

$$\begin{aligned} t_{11}(A_i) - t_{11}(C_i) + t_{21}(B_i) - t_{21}(D_i) + X_1(O_i) &= 0, \\ t_{12}(A_i) - t_{12}(C_i) + t_{22}(B_i) - t_{22}(D_i) + X_2(O_i) &= 0, \quad i = \overline{1, N}. \\ t_{12}(A_i) + t_{12}(C_i) - t_{21}(B_i) - t_{21}(D_i) &= 0, \end{aligned} \tag{2}$$

Here, X_1, X_2 —mass forces in the center of particle. The first two equations describe the zero main vector of forces and the third equation—zero main moment of forces.

The conditions of interaction of grains (total adherence) take on form (Figure 1):

$$\begin{aligned} t_{11}(C_j) = t_{11}(A_i); \quad t_{12}(C_j) = t_{12}(A_i); \\ u_1(C_j) = u_1(A_i); \quad u_2(C_j) = u_2(A_i); \end{aligned} \quad i, j = \overline{1, n(m-1)}, \tag{3}$$

$$\begin{aligned} t_{22}(D_k) = t_{22}(B_i); \quad t_{21}(D_k) = t_{21}(B_i); \\ u_1(D_k) = u_1(B_i); \quad u_2(D_k) = u_2(B_i); \end{aligned} \quad i, k = \overline{1, m(n-1)}. \tag{4}$$

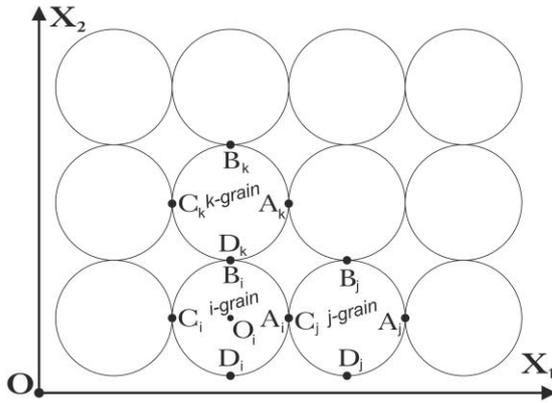


Figure 1. Internal structure of the medium: arrangement of contacts of particles.

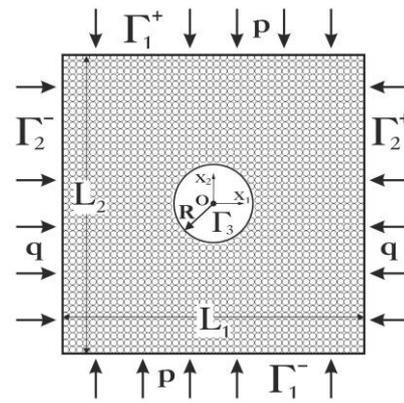


Figure 2. Problem on deformation of rock mass around a tunnel.

3. Problem formulation and calculation results

Regarding the formulation of the boundary value problem, let the calculation domain with the dimensions L_1, L_2 surround a tunnel with a radius R (Figure 2). We present the calculation domain as a set of discrete particles with a radius l . The calculation domain is subjected to gravitational and tectonic compression, and the tunnel boundary is free from stress. The boundary conditions at the boundary of the calculation domain are given by:

$$\begin{aligned} t_{22}(B)|_{\Gamma_1^+} = t_{22}(D)|_{\Gamma_1^-} = -p; \quad t_{21}(B)|_{\Gamma_1^+} = t_{21}(D)|_{\Gamma_1^-} = 0; \\ t_{11}(C)|_{\Gamma_2^-} = t_{11}(A)|_{\Gamma_2^+} = -q; \quad t_{12}(A)|_{\Gamma_1^+} = t_{12}(C)|_{\Gamma_1^-} = 0; \end{aligned} \tag{5}$$

Similarly, at the boundary of the tunnel:

$$t_{nm}|_{\Gamma_3} = t_{nt}|_{\Gamma_3} = 0; \tag{6}$$

where t_{nm}, t_{nt} denote, respectively, the normal and shear forces at one of the points A, B, C, D .

The system (1)–(6) is a closed system of algebraic equations to be solved numerically using the Gauss method.

We select the dimensionless parameters of the problem (the values having dimension of length are related to L_1 , the values having dimension of stress—to p):

$$p = 1, \quad q = 0,4, \quad E = 2000, \quad \nu = 0,25, \quad L_1 = 1, \quad L_2 = 1, \quad R = 0,26. \quad (7)$$

In the analysis of the influence exerted by the structural parameter ξ , it is first assumed that $\xi = 0$, which is case of the classical linear elasticity theory. The resultant contours of the vertical t_{22} and maximal shear $\tau_{\max} = 0.5\sqrt{(t_{11} - t_{22})^2 + 4t_{12}^2}$ stresses calculated from (1)–(6) are shown in Figures 3a and 4a, respectively. The structure has no influence in this case, the local bends are absent. The highest concentration of the stresses, both t_{22} and τ_{\max} is observed in comparatively small zones at the side boundaries of the tunnel.

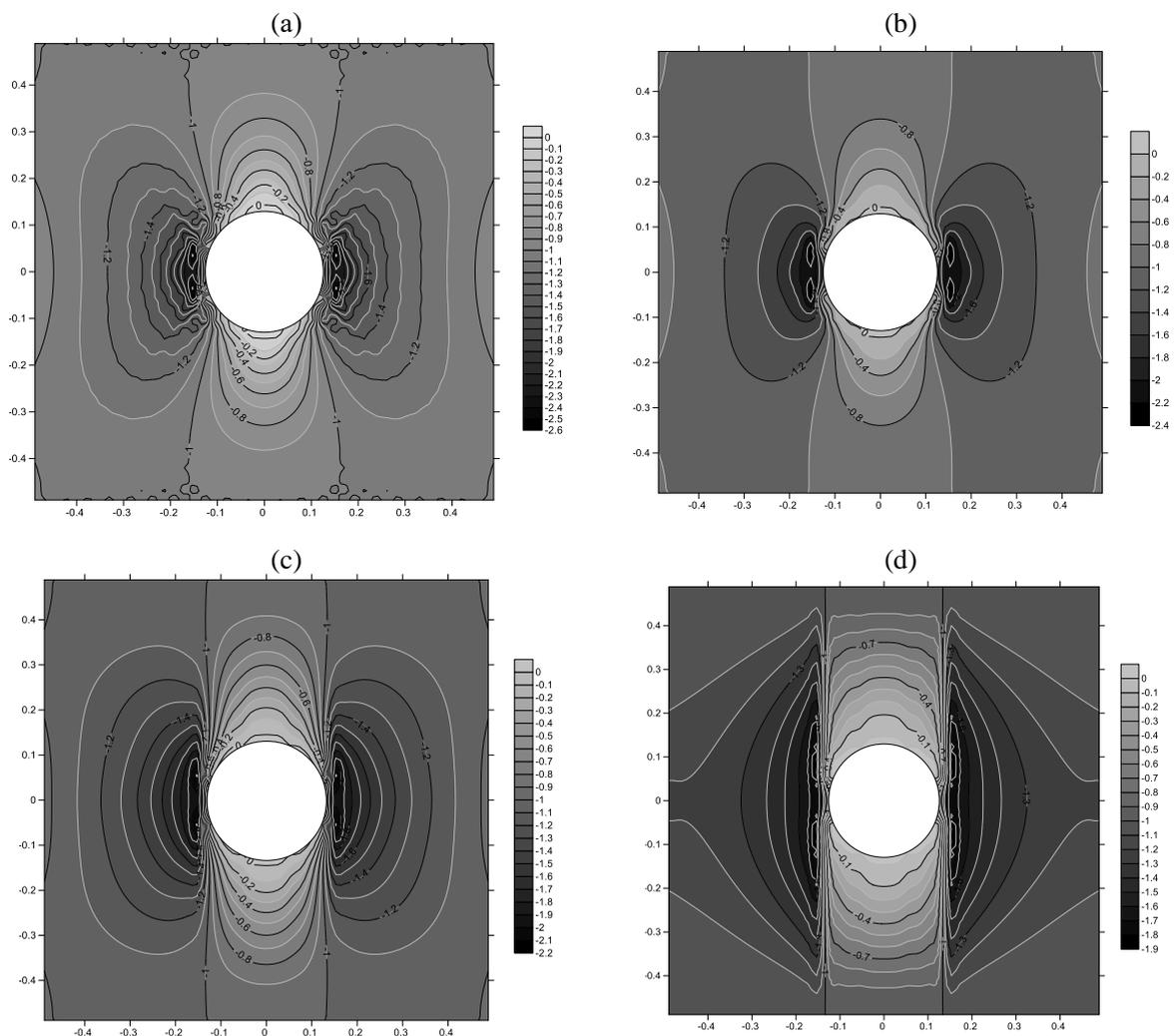


Figure 3. Contour lines of the stress t_{22} depending on the structural parameter: (a) $\xi = 0$; (b) $\xi = 5 \cdot 10^{-4}$; (c) $\xi = 5 \cdot 10^{-3}$; (d) $\xi = 5 \cdot 10^{-2}$.

Gradual increase in the value of the structural parameter due to local curvature results in considerable redistribution of stresses in the adjacent rock mass. It is seen that the stresses t_{22} , as ξ is increasing, concentrate in the consistently expanding rock mass zones in the horizontal direction (Figures 3b–3d). The maximal concentration of t_{22} changes from $t_{22} = -2.8$ (Figure 3a) to

$t_{22} = -1.9$ (Figure 3d). The maximum shear stress τ_{\max} behaves similarly: it concentrates in the successively expanding zones leftward and rightward the tunnel in the vertical direction (Figures 4b–4d). The maximum concentration of τ_{\max} changes from $\tau_{\max} = 1.5$ (Figure 4a) to $\tau_{\max} = 1.0$ (Figure 4d). In this manner, with the local curvature taken into account in modeling, the maximum concentration of stresses is “smeared”, literally, in rock mass surrounding a tunnel.

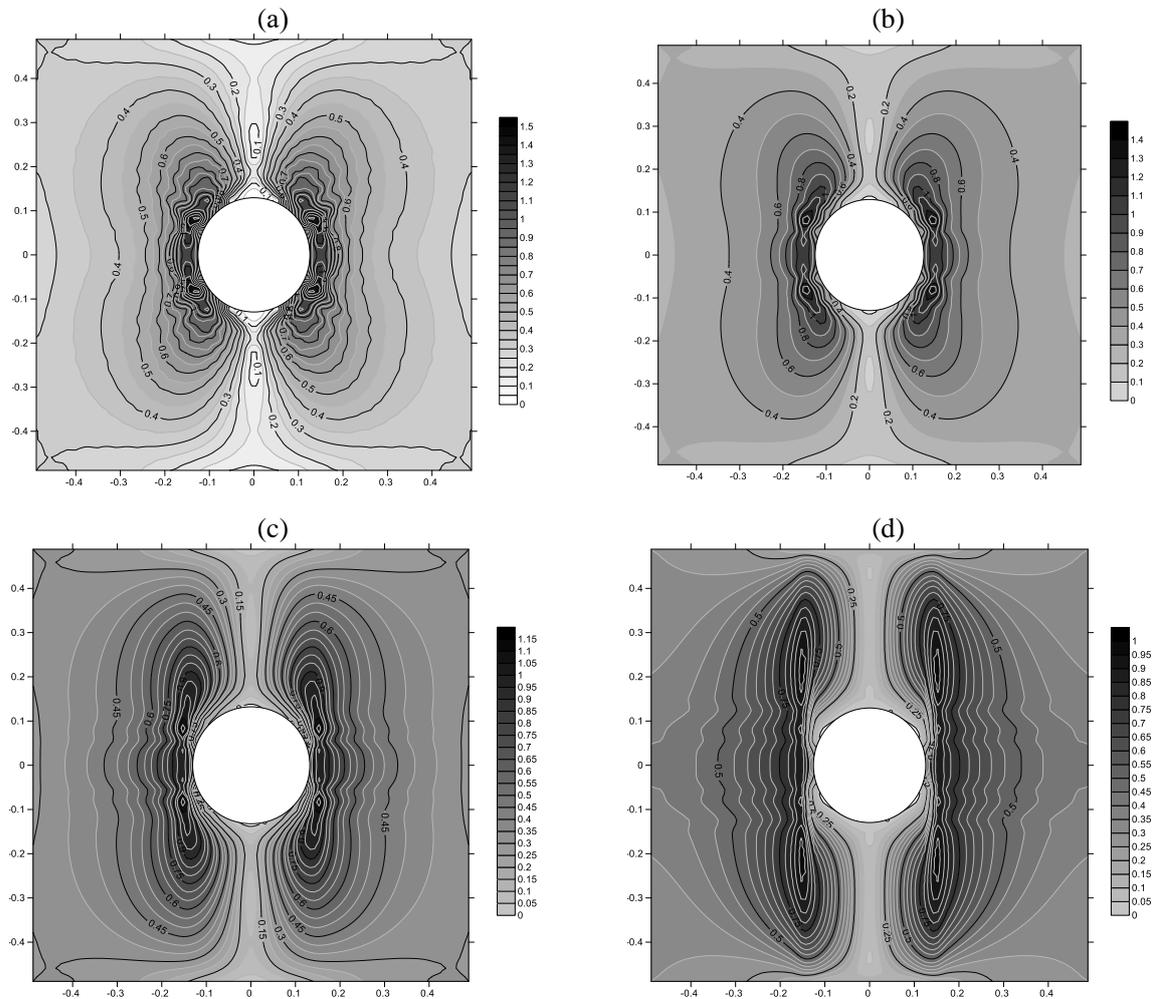


Figure 4. Contour lines of the stress τ_{\max} depending on the structural parameter: (a) $\xi = 0$; (b) $\xi = 5 \cdot 10^{-4}$; (c) $\xi = 5 \cdot 10^{-3}$; (d) $\xi = 5 \cdot 10^{-2}$.

4. Conclusion

In the models of elastic deformation of rock mass surrounding a tunnel with taking into account local bends, the influence zones of the tunnel is expanded and the stress concentration factor at the tunnel boundary is lower.

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