

Tensile fracture of rocks under uniaxial compression

VP Efimov

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences,
Novosibirsk, Russia

E-mail: *efimov-pedan@mail.ru

Abstract. The uniaxial compressive tests of brittle rock sample show that the samples fail in the form of columnar fracturing along the axis of the force application. The researcher attempts to relate the test data with the characteristic values of tensile strength.

The uniaxial compression often induces cleaving of rock samples along a plane coinciding with the direction of the compressive force application. Such fracture is typical for brittle rocks but is sometimes observed in plastic rocks when loading pates have sizes equal to the dimension of cross section of a sample, or when some measures are undertaken to reduce constraint on the sample faces [1, 2]. The term ‘cleaving’ highlight the difference of this type of fracture from shearing, which also takes place under uniaxial compression. The experiments conducted by the present paper author and by other researches [3–5] aimed to determine physical parameters of rocks that govern their failure show that the initial energy of fracture activation in hard rocks is independent of the type of applied fracturing load. This parameter is the same for the compression and tension of such rocks. This fact suggests that the compression fracture of such rocks is based on the mechanism of fracture of strained bonds by tension. In passing from the breakage of the strained bonds to the increase in the existing defects in the framework of a model of thermal fluctuations in failure, it is possible to suggest development of failure under the action of tension in the conditions of total compression. The relationship between the compression and tension strengths in Figure 1 indirectly proves the said model of failure.

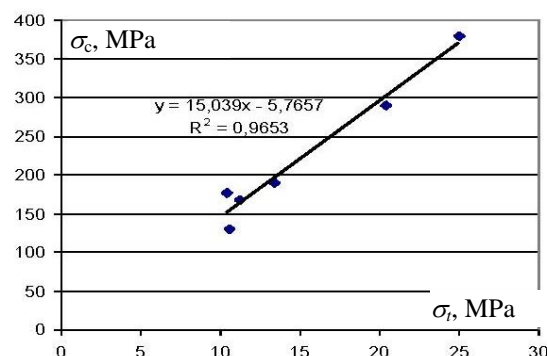


Figure 1. Compression strength versus tension strength of tested rocks.

Griffiths [6] showed that tensile stresses appeared at the tips of cracks oriented at an angle to the compression axis. Furthermore, the conclusion drawn on the basis that a solid is a medium containing



differently oriented defects in the form of cracks is well known: ratio of compression and tension strengths for brittle media is equal to 8. It is worthy of mentioning that the criterion of failure was chosen the tension strength of molecular bond.

Presentation of a sample as a medium with cracks is not a single possible model to transform compressive stresses to tensile stresses. The well known solution of an elastic problem on stress distribution around a spherical pore in an infinite body subjected to compression at infinity [7] yields tensile stress at the poles of the pore, and this stress also can be a center of initiation of tensile micro-cracks leading to macro-failure. The values of the initiated tensile stresses are close to the values of the compressive stresses but are extremely localized in space. The use of the local strength criteria in the determination of tensile stresses in such geometry should result in a small difference in the values of the compression and tension strengths in samples containing such defects, which contradicts the reality. For the description of failure in nonuniform stress fields, the models involve nonlocal strength criteria [8–14] which is effective in this case. In such models, failure is not analyzed at a hazardous point of material but in some vicinity the linear dimension of which is a characteristic of the material. It is assumed that failure takes place when stresses, averaged over a certain length (or equivalent) reach the value equal to tension strength measured by standard in a uniform field. With such approach, the ratio of the compression and tension strengths depends on the characteristics of material. Let us analyze this approach in terms of compression of a sample with a single spherical pore.

The stresses $\sigma_{\theta\theta}$ and $\sigma_{\varphi\varphi}$ under uniaxial compression or tension are determined from the solution to the problem of Leon [7]:

$$\begin{aligned}\sigma_{\theta\theta} &= \sigma \left[1 + \frac{4-5\nu}{14-10\nu} \frac{a^3}{r^3} + \frac{9}{14-10\nu} \frac{a^5}{r^5} + \left(-1 + 5 \frac{2-5\nu}{14-10\nu} \frac{a^3}{r^3} - \frac{21}{14-10\nu} \frac{a^5}{r^5} \right) \cos^2 \theta \right], \\ \sigma_{\varphi\varphi} &= \sigma \left[-3 \frac{2-5\nu}{14-10\nu} \frac{a^3}{r^3} + \frac{3}{14-10\nu} \frac{a^5}{r^5} + \left(15 \frac{1-2\nu}{14-10\nu} \frac{a^3}{r^3} - \frac{15}{14-10\nu} \frac{a^5}{r^5} \right) \cos^2 \theta \right],\end{aligned}\quad (1)$$

where a —radius of the pore; r —distance from the center of the pore; θ —polar angle counted from the axis of force application; φ —azimuth direction angle; ν —Poisson's ratio. The distribution of stresses in the vicinity of hazardous points:

—at the poles along the force application line:

$$\sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \frac{3\sigma}{14-10\nu} \left((3-5\nu) \frac{a^3}{r^3} - 4 \frac{a^5}{r^5} \right); \quad (2)$$

—at the equator of the pore:

$$\sigma_{\theta\theta} = \sigma \left(1 + \frac{4-5\nu}{14-10\nu} \frac{a^3}{r^3} + \frac{9}{14-10\nu} \frac{a^5}{r^5} \right). \quad (3)$$

It is evident from (2) that the stresses at the poles have opposite sign to the applied force while there is the concentration of stresses at the equator. Stress distribution decreases with the growth of r .

We take the failure criterion as the integral criterion by Novozhilov:

$$\sigma_t = \frac{1}{\delta} \int_0^\delta \sigma_n(r) dr, \quad (4)$$

where σ_t —tension strength measured in uniform field; δ —characteristic structural constant of a material. Integrating (2) and (3) produces the ratio of the compressive and tensile strengths:

$$\frac{\sigma_c}{\sigma_t} = \frac{1}{6} \left(\frac{4(14-10\nu)(1+b)^4 + 2(4-5\nu)(2+b)(1+b)^2 + 9(4+6b+4b^2+b^3)}{(3-5\nu)(2+b)(1+b)^2 - 2(4+6b+4b^2+b^3)} \right), \quad (5)$$

where $b=\delta/a$ —dimensionless parameter.

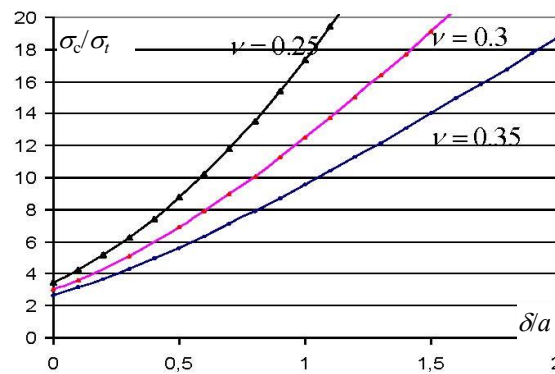


Figure 2. The ratio of compressive and tensile strength versus the ratio of structural parameter and pore radius at different values of Poisson's ratio.

Figure 2 shows the curves of this ratio and the ratio of structural parameter to pore radius. Evidently, the compression/tension strength ratio depends on the structural characteristics of materials.

In passing to a real rock, it is expected to obtain qualitatively the same results, namely, the influence of the structural features of rocks on the ratio of compression and tension strength. The works [15, 16] analyzed different models of failure under compression in materials having ordered structure some elements of which can transform compression to tension. Considering intact rock mass not as a solid medium with averaged parameter but as an aggregation of grains with different mechanical characteristics, we obtain a heterogeneous medium and transformation of the compressive stresses to tensile stresses is possible at the boundaries of these grains. In hard rocks, such granite, porosity is low, and the discussed effect of transformation of the compressive stresses to tensile stresses should not be connected with the size of pores but with the size of grains, and the hardest grains should be assumed inclusions and soft grains—filled pores. The relationship between the strength and average size of grains in such rocks is depicted in Figure 3.

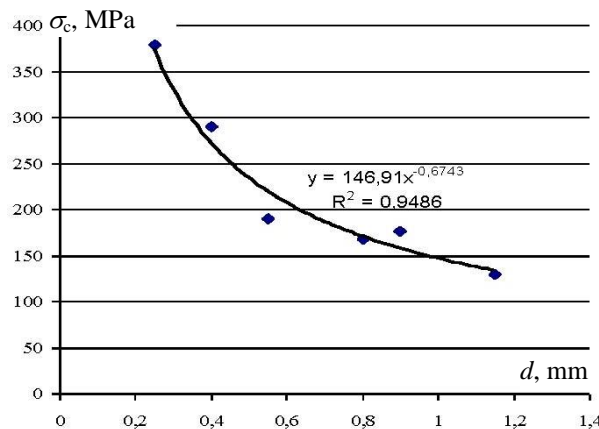


Figure 3. Compression strength versus grain size of rocks.

The curve shows the power dependence of strength on grain size in rocks. This curve is plotted for the test rocks the mechanical characteristics of which are described in the table: d —average grain size; K_{lc} —critical SIF of tensile cracks; σ_c —compression strength; σ_t —Brazilian test tension strength; σ_b^4 —4-point bending strength; δ —structural parameter having dimension of length, determined as the ratio of critical SIF to tension strength [10]:

$$\delta = \frac{2}{\pi} \left(\frac{K_{lc}}{\sigma_t} \right)^2. \quad (6)$$

Mechanical characteristics of tested rocks.

Rock	d , mm	K_{Ic} , $\text{MPa}\cdot\text{m}^{1/2}$	σ_c , MPa	σ_t , MPa	σ_b^4 , MPa	$\frac{\sigma_c}{\sigma_e}$	$\frac{\sigma_b^4}{\sigma_t}$	δ , m m	δ/d
Dolerite	0.25	1.90	379	25.0	33.4	15.20	1.34	4.3	17.20
Gabbroide	0.40	2.00	290	20.4	33.6	14.20	1.65	5.9	14.75
Gabbro-diorite	0.45	1.10	190	13.4	19.4	14.20	1.45	5.6	12.40
Granite	0.80	1.14	168	11.2	16.6	15.00	1.38	6.6	8.25
Biotite granite	0.90	0.70	177	10.4	11.2	17.00	1.08	2.9	3.20
Leucocratic granite	1.15	0.80	130	10.6	13.4	12.30	1.26	3.6	3.10

The ratio of the structural parameter to the grain size, as follows from the table, takes on value from 3 to 20 for the tested rocks, i.e. the structural parameter covers a few grains at the least. The dependence of the compressive stresses on the dimensionless parameter δ/d is shown in Figure 4. The conclusion is that the more grains are covered by the structural parameter, the stronger is rock. This conclusion is valid both in compression and tension.

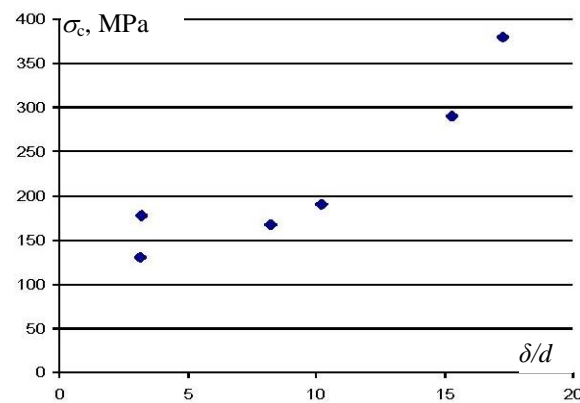


Figure 4. Compression strength versus dimensionless parameter δ/d .

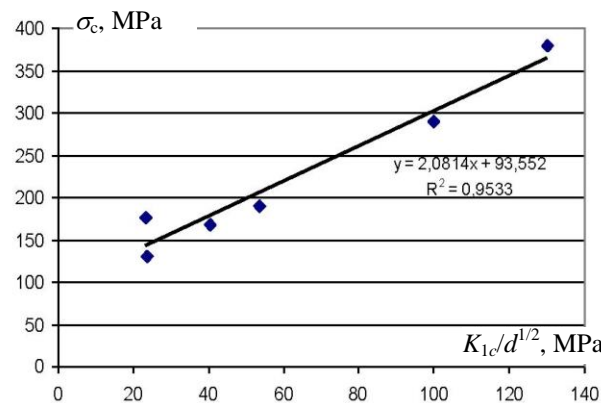


Figure 5. Curve of compression strength of rocks.

Figure 5 demonstrates, in reduced coordinates, the curve of compression strength in the tested rocks on the value which has dimension of stresses and is composed of the product of SIF for tensile cracks and inverse value of average grain size to the $1/2$ power.

Conclusion

Fracture of rocks by compression greatly depends on the surface conditions of compressing plates. There can be three scenarios: failure along a conical or pyramidal surface, along the diagonal or cleaving along the force application line. Fracture with the formation of cones or pyramids at the contact surface due to strong constraint of materials initiates in the central part. When the constraint is weak, fracture fragments can have a clearly columnar structure. The correlation relations between compression strength, tension strength and tensile crack resistance in hard rocks are reflective of the close connection of fracture processes in these tests in case when compression initiates formation of the columnar structure. The average ratio of compression and tension strengths for the test rocks is equal to 15, and it should be adjusted according to the mechanical characteristics of grains composing rocks.

References

- [1] Fairhurst C and Cook NGW 1966 The phenomenon of rock splitting parallel to the direction of maximum compression in the neighbourhood of a surface *Proc. I ISRM Congress*, Lisbon, pp 687–692
- [2] Bock H (Ed) 1978 *An Introduction to Rock Mechanics* Townsville: James Cook University of North Queensland
- [3] Efimov VP 2007 Investigation into the long term strength of rock under loading with a constant rate *J. Min. Sci.* Vol 43 No 6 pp 600–606
- [4] Kucheryavy FI, Mikhalyuk AV and Demchenko LA 1980 Energy of activation and energy intensity of rock fracture *Izv. Vuzov Gorny. Zh.* No 5 pp 57–63
- [5] Wexler YuA 1979 Durability of rocks under compression *J. Min. Sci.* No 3 pp 71–76
- [6] Griffith AA 1924 Theory of Rupture *Proc. I Int. Congr. On Applied Mechanics* Delft pp 55–63
- [7] Hahn H 1985 *Elastizitätstheorie* Stuttgart (in German)
- [8] Novozhilov VV 1969 To the basics of the theory of equilibrium cracks *J. Appl. Math. Mech.* Vol 33 No 5 pp 797–812
- [9] Lajtai EZ 1972 Effect of tensile stress gradient on brittle fracture initiation *Int. J. Rock Mech. Min. Sci.* 9 pp 569–578
- [10] Suknev SV and Novopashin MD 2000 Determination of local mechanical properties of materials *Dokl. RAN* Vol 373 No 1 pp 487–500
- [11] Legan MA 1993 Correlation of local strength gradient criteria in a stress concentration zone with linear fracture mechanics *J. Appl. Mech. Tech. Phys.* Vol 34 No 5 pp 585–592
- [12] Kharlab VD and Minin VA 1989 Strength criterion considering effect of stress gradient *Research on Mechanics of Engineering Structures and Materials: Collected Papers* Leningrad: LISI pp 53–57 (in Russian)
- [13] Kornev VM 2002 Generalized sufficient strength criterion. Description of the pre-fracture zone *J. Appl. Mech. Tech. Phys.* Vol 43 No 5 pp 763–769
- [14] Efimov VP 2013 Rock tests in nonuniform fields of tensile strength *J. Appl. Mech. Tech. Phys.* Vol 54 No 5 pp 857–865
- [15] Goldstein RV 2003 Fracture under compression *Usp. Mekhaniki* Vol 2 pp 3–20
- [16] Goldstein RV and Osipenko NM 1999 Structures in failure processes of fracture *Izv. RAN Mekh. Tverd. Tela* No 5 pp 49–71