Study on Wind-induced Vibration and Fatigue Life of Cable-stayed Flexible Antenna

Kongde He[1, a], Xuehui He[2, b], Zifan Fang[2, c], Xiaowei Zheng[2, d], Hongchang Yu[2, e]

[1]Hubei Key Laboratory of Hydroelectric Machinery Design & Maintenance, China Three Gorges University, Yichang 443002, China

[2]College of Mechanical & Power Engineering, China Three Gorges University., Yichang 443002, China

*Corresponding Author

[a]E-mail: hekongde@ctgu.edu.cn   [b]E-mail: 1038575452@qq.com   [c]E-mail: fzf@ctgu.edu.cn   [d]E-mail: 603215450@qq.com   [e]E-mail: 1933763048@qq.com

Abstract. The cable-stayed flexible antenna is a large-span space structure composed of flexible multibody, with low frequency of vibration, vortex-induced resonance can occur under the action of Stochastic wind, and a larger amplitude is generated when resonance occurs. To solve this problem, based on the theory of vortex-induced vibration, this paper analyzes the vortex-induced vibration of a cable-stayed flexible antenna under the action of Wind. Based on the sinusoidal force model and Autoregressive Model (AR) method, the vortex-induced force is simulated, then the fatigue analysis of the structure is based on the linear fatigue cumulative damage principle and the rain-flow method. The minimum fatigue life of the structure is calculated to verify the vibration fatigue performance of the structure.

1. Introduction

The cable-stayed flexible antenna is a kind of signal transmitting antenna. when it under the action of wind load, the cable-stayed flexible antenna can produce a vortex-induced resonance with a transverse direction, which produces large amplitude when resonance occurs, and the structure is easily destroyed.

The structure of cable-stayed flexible antenna is similar to the cable-stayed bridge and the spatial latticed structure, so the analysis method of these structures is used for reference. Qingkuan Liu introduced wind load calculating methods, types of wind induced vibration and mechanisms of cables and summarized the research achievements of vibration control methods designed from aerodynamic perspective [1]. Tingting Liu studied on the wind-induced vibration response and wind-induced fatigue damage of two typical flexible structures of super high-rise buildings and Long-span bridges under vortex-induced vibration and buffeting-induced vibrations [2]. Based on the results of the above references, the vortex-induced vibration response and fatigue life of the cable-stayed flexible antenna are studied on the basis of the theory of vortex-induced vibration, the sinusoidal force model, the fatigue cumulative damage theory and the rain-flow method.

2. Wind - induced Vibration Analysis of Cable - stayed Flexible Antenna

2.1 Vortex-induced Vibration Principle

When the wind passes through the structural surface at a certain speed, the vortices are periodically alternately formed on both sides of the back surface and shedding from the structural surface at a definite frequency, producing a pair of opposing vortices in the left and right sides of the tail flow that are paired, alternately arranged, and opposite to the rotation direction, that is, Karman vortex. When the vortices alternately appear on both sides of the structure, a across-wind load with the same frequency as the vortex is formed, resulting in the lateral vibration of the structure perpendicular to the wind direction, namely vortex-induced vibration.

2.1.1 Requirements for Vortex-induced Vibration Generation

Vortex-induced resonance occurs when the vortex shedding frequency and the natural frequencies of the structure occur simultaneously. When resonance occurs, the vortex shedding frequency is controlled by the vibration frequency of the structure, even if the wind speed is increased, the vortex shedding frequency remains unchanged, this particular phenomenon is called "lock" [2]. In this locked area, the vortex shedding frequency is a constant rather than a linear function. From the above phenomenon, it is known that resonance occurs in a certain range of wind speed when the induced resonance begins, and the initial wind speed of the resonance is called the critical wind speed. Its equation is:

$$\upsilon_{cr,i} = \frac{L}{T_i St} \tag{2-1}$$

In the formula: $\upsilon_{cr,i}$ is the critical wind speed corresponding to the ith order vibration mode of the structure, St is the Strouhal number, for the round section to take 0.2, the rest of the section takes 0.15, T_i is the oscillation period of the ith order vibration mode of the structure, L is the diameter of structure section.

The condition for the structure to produce the Carmen Vortex is that the value of the resonant critical wind speed is within the range of the possible wind speed of the structure, that is $0 \le \upsilon_{cr} \le \upsilon_H$.

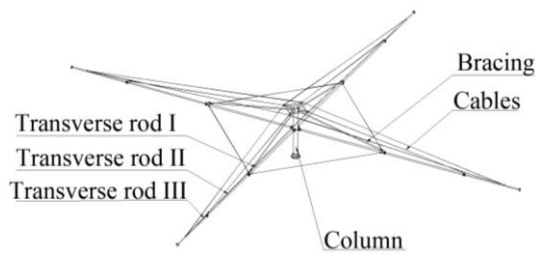$$\upsilon_H = \sqrt{\frac{2000\mu_H w_0}{\rho}} \tag{2-2}$$

$$\mu_H = 1.379 \left(\frac{z}{10}\right)^{0.24} \tag{2-3}$$

In the formula : υ_H is the wind speed at the top of the structure, μ_H is the height of the height of the wind pressure variation coefficient, w_0 is the area of the basic wind pressure ,and its value is $0.8^{KN/m^2}$, ρ is the density of air, and its value is $1.25^{kg/m^3}$.
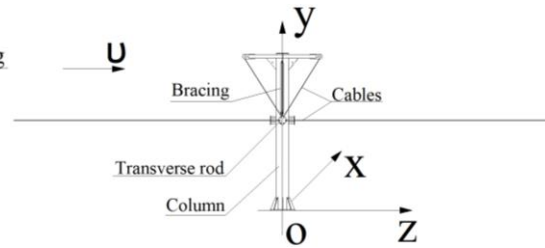
2.2 Empirical Model of Vortex-induced Vibration

2.2.1 Dynamic Equation of Vortex-induced Vibration of Flexible Antennas

The direction of the wind is relative to the horizontal plane, but the angle is usually less than 5 °, so the horizontal wind is usually considered. The simplified model diagram and the main coordinate system of the cable-stayed flexible antenna are shown inFiguer2.1and Figuer2.2. The midpoint of the bottom of the main pipe is the coordinate origin, the axis of the base is the x axis, the y-axis is perpendicular to the head axis of the base, and the positive direction of the wind speed is the z-axis positive direction.

ICEESE 2017

IOP Conf. Series: Earth and Environmental Science **128** (2018) 012119

IOP Publishing

doi:10.1088/1755-1315/128/1/012119

Figure 2.1 Schematic diagram of cable-stayed flexible antenna structure

Figure 2.2: Schematic diagram of simplified cable-stayed flexible antenna

Table 2.1: Structural parameters

Parameter's name	Value (m)	Parameter's name	Value (m)
Maximum height of the structure	2.528	The diameter of the transverse rod III	0.0045
The height of the axis of the transverse rod	1.45	Length of the transverse rod I	4.4
The diameter of the transverse rod I	0.006	Length of the transverse rod II	4.0
The diameter of the transverse rod II	0.006	Length of the transverse rod III	2.8

The vortex-induced vibration of flexible structures is more complex, vortex shedding is not only affected by the structure shape, but also considering the coupling of wind and structure, the simplified empirical mathematical model is often used in engineering, the parameters are determined by experiments, the vortex-induced force is obtained, and the vortex-induced force is added to the motion equation to solve the structural vibration [3].

The basic form of the sinusoidal force model is:

$$[M]\left(\ddot{y} + 2\xi\omega_n \dot{y} + \omega_n^2 y\right) = \frac{1}{2}\rho\upsilon^2 B\mu_L \sin\left(\omega_s t + \varphi\right) \tag{2-4}$$

In the formula: υ is the mean wind speed, B is the structure reference width, ξ is the damping ratio, ω_n is the structure vibration angular frequency, ω_s is the vortex shedding angular frequency, φ is the initial phase angle, μ_L is the lift coefficient of across-wind .

The vortex-induced force formula at height z is:

$$F(z,t) = \frac{1}{2}\rho\upsilon^2(z)B(z)\cdot \mu_L \sin\omega_s t \tag{2-5}$$

2.3 Dimensionless Parameters Related to Vortex-induced Vibrations

The Navier-stokes equation of one-dimensional flow of air is:

$$\rho\left(\frac{\partial\upsilon}{\partial t} + \upsilon\frac{\partial\upsilon}{\partial x}\right) = \rho F - \frac{\partial\rho}{\partial x} + \mu\frac{\partial^2\upsilon}{\partial x^2} \tag{2-6}$$

According to the literature [2], the dimensionless relation of force F is:

$$\frac{F}{\rho\upsilon^2 L^2} = \left(\frac{fL}{\upsilon}\right)^{x_4}\left(\frac{\mu}{\rho L\upsilon}\right)^{x_5}\left(\frac{gL}{\upsilon^2}\right)^{x_6} = (St)^{x_4}\left(\frac{1}{\mathrm{Re}}\right)^{x_5}(Fr)^{x_6} \tag{2-7}$$

In the formula, x_4, x_5, x_6 is the undetermined coefficient.

Therefore, there are three dimensionless parameters Reynolds number *Re*, Strouhal number *St* and Froude number *Fr* related to the vortex-induced vibration force, where the Froude number is the dimensionless correlation dimensionless parameter, which is only used Gravity and inertia forces play an important role in the flow, such as surface ship movement and open channel flow, so this paper is not considered. The value of the Strouhal number is generally given by experiments or specifications, with a general value of 0.2 for circular sections.

2.3.1 Reynolds Number

In the experiment, the karman-dunn of smooth cylindrical body in viscous fluid is studied, and the effect of Reynolds number on the flow around the smooth cylinder is pointed out. The characteristics of vortex shedding can be divided into three stages according to Reynolds number: When the

Reynolds number is within the range of $300 \le \mathrm{Re} \le 3\times10^5$, which is the subcritical range, the vortex is formed with regular and periodic shedding; When Reynolds number is within the range of $3\times10^5 \le \mathrm{Re} \le 3\times10^6$, which is the supercritical range, the vortex is irregular motion; When the Reynolds number is within the range of $\mathrm{Re} > 3\times10^6$, which is the critical range, the vortex gradually becomes a regular movement, with periodic deterministic vibrations.

The expression of Reynolds number in the air is：

$$Re = 69000 \cdot \upsilon \cdot L \tag{2-8}$$

In the formula: υ is the wind speed used for calculation, and its value is critical wind speed υ_{cr}.

2.4 Numerical Analysis of Vortex - induced Vibration of Cable - stayed Flexible Antenna

2.4.1 Calculation of Natural Frequency and Reynolds Number of Cable - stayed Flexible Antenna
The free vibration equation of a cable-stayed flexible antenna is：

$$[M]\ddot{x} + [K]x = \{0\} \tag{2-9}$$

In the formula: $[M]$, $[K]$ are the mass matrix and stiffness matrix of flexible antennas.

Therefore, the natural frequency values of the cable-stayed flexible antennas are shown in Table 2.3.

Table 2.3: Natural frequency values for all levels of cable-stayed flexible antenna

Modal number	1	2	3	4	5	6	7	8	9	10
Natural frequency （Hz）	0.421	0.428	0.429	0.429	1.015	1.015	1.052	1.053	2.434	2.473

From the equations (2-1) and (2-9), the critical wind speed and Reynolds Number corresponding to the natural frequencies of the cable-type flexible antenna are shown in Table 2.4.

Table2.4: Critical wind speed and Reynolds number

Modal number	1	2	3	4	5	6	7	8	9	10
Critical wind speed （m/s）	0.126	0.128	0.129	0.129	0.305	0.305	0.316	0.316	0.730	0.742
Reynolds number (s^{-1})	521.6	529.9	534.1	534.1	1262.7	1262.7	1380.2	1380.2	3022.2	3071.9

2.4.2 Wind-induced Structural Vibration Frequency
The wind-induced vibration frequency is the structure vibration frequency of the structure under the wind action, and the vibration of the structure is caused by the vortex shedding. Therefore, by the definition of the Strouhal Number, the expression of wind-induced vibration frequency n_s is:

$$n_s = f = \frac{St \cdot \upsilon_z}{L} \tag{2-10}$$

$$\upsilon_z/\upsilon_1 = (z/z_1)^{\alpha} \tag{2-11}$$

In the formula: υ_z is the average wind speed of the structure at a certain time (generally taking 10min) in height z, L is the diameter of the flexible antenna transverse rod, z_1 is the standard height (10m), υ_1 is the average wind speed under the standard height, α is the index related to the roughness of the ground, its values are 0.12, 0.16, 0.22, 0.3.

The cable diameter of cable-stayed flexible antenna is 8mm, The shortest length of cable is 10.59m, according to the fundamental frequency formula of cable [4]：

$$f_1 = \frac{1}{2l}\sqrt{\frac{F_1}{\rho}} \tag{2-12}$$

F_1 is the tensile force of the cable, the fundamental frequency of the cable is 0.047Hz, According to the formula (2-1), the critical wind speed of the vortex-induced resonance of the cable is 0.002m/s, and the Vortex force produced by the low wind speed will be difficult to provide the energy to excite the large amplitude vibration of the cables, so the vortex-induced vibration of the cable is not considered.

In summary, the wind-induced structural vibration frequencies of transverse rod at wind levels of 2 to 9 are shown in Table 2.5.

Table2.5: Wind - induced frequency of flexible antenna at transverse rod under each wind level

Wind level	2	3	4	5	6	7	8	9
frequency (Hz)	6.33	11.63	17.70	24.73	32.53	40.96	50.09	59.76

2.4.3 Aggregate analysis

By formula (2-2), it is known that the wind speed of the top of the cable-stayed flexible antenna is υ_H =35.62m/s greater than $\upsilon_{cr,4}$ =0.129m/s, so the structure can form the Karman Vortex. Composite tables (2.3), (2.4) and (2.5), the Reynolds number of flexible antennas Re less than 3×10^5 , it's in the subcritical range and the natural frequencies of flexible antennas and wind-induced vibration frequencies are not equal, so the flexible antenna does not occur in subcritical range of the wind resonance.

3.Analysis of wind-induced fatigue life of cable-stayed flexible antenna

3.1 Wind speed and vortex simulation based on auto regressive model

The autoregressive model of the column vectors of the space related time series of fluctuating wind speed $U(X,Y,Z,t)$ can be expressed as:

$$U(X,Y,Z,t) = \sum_{k=1}^{p} \psi_k U(X,Y,Z,t-k\Delta t) + N(t) \tag{3-1}$$

In the formula: X,Y,Z is the matrix of the coordinate vector, (x_i, y_i, z_i) is the coordinate of the nth point in the space, $i=1,2,\cdots,M$. p is the order of AR model, ψ_k is the coefficient matrix of AR model autoregressive, $k=1,\cdots,p$. Δt is the time step of simulating the wind speed time. $N(t)=Ln(t)$, $n(t)$ is the random process quantity of the independent normal distribution within the interval (0, 1). L is the Mth order lower triangular matrix.

Therefore, M-related stochastic wind processes is：

$$\begin{bmatrix} \upsilon^1(j\Delta t) \\ \vdots \\ \upsilon^M(j\Delta t) \end{bmatrix} = \sum_{k=1}^{p} [\psi_k] \begin{bmatrix} \upsilon^1[(j-k)\Delta t] \\ \vdots \\ \upsilon^M[(j-k)\Delta t] \end{bmatrix} + \begin{bmatrix} N^1(j\Delta t) \\ \vdots \\ N^M(j\Delta t) \end{bmatrix} \tag{3-2}$$

Thus we can get M discrete pulsating wind speed process vectors with time and space correlation and time intervals of Δt [5].

Davenport spectrum is simple and easy to use and does not change with height, therefore, this paper uses autoregressive model (AR) into wind speed simulation and confirms the rationality of wind speed simulation method.

According to China's norms on the definition of basic wind pressure, the length of simulation time is 600s, the wind power spectral density curve and the Davenport Power spectral density curve in the center of the transverse rod under the action of the 9-level wind and
the time series of vortex-induced force are shown from Figure 3.1 to 3.3.
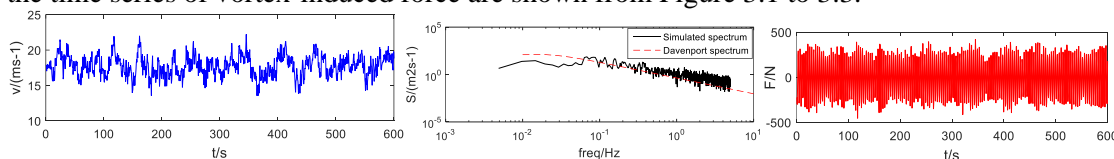
Figure 3.1 The time series   **Figure 3.2** Simulated and Davenport   **Figure 3.3** The vortex-induced
   of transverse rod   power spectral density curve   force at the transverse rod

3.2 Fatigue life analysis of cable-stayed flexible antennas

The linear fatigue Cumulative damage theory is that under the action of the regular variable amplitude cyclic stress, the damage caused by each stress to the structure is independent and the damage linearly accumulates into total damage, and when the life damage rate of each stress is equal to 1, the structure breaks down.

The expression of the total damage D is:

$$D = \sum_{i=1}^{k} D_i = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{3-3}$$

Under the action of stochastic wind load, the stress response of cable-stayed flexible antenna belongs to stochastic stress time series, so the stress is calculated by rain-flow method. In this paper, the finite Element software (Workbench) is used to analyze the fatigue life of flexible antennas, the shortest fatigue life of structure is 2 years, the overall stress response and fatigue life isoline of flexible antenna is shown in Figure 3.4 and 3.5.
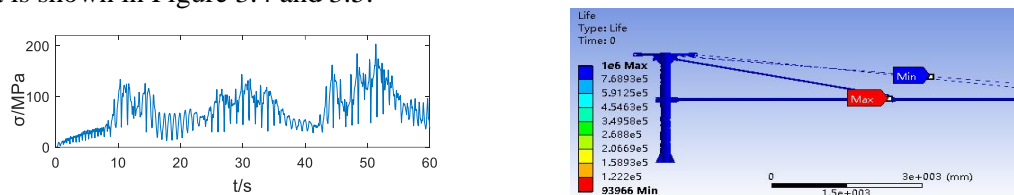
Figure3.4 Stress response diagram of flexible antenna **Figure3.5** Contour Diagram of fatigue life (cycle)

4.Conclusion

Based on the theory of vortex-induced vibration, the analysis process of vortex-induced vibration of a cable-stayed flexible antenna under low speed wind is presented. The base frequency of the cable of the flexible antenna is small, so the critical wind speed of the resonance is small, the vortex force under the wind speed is not enough to provide the large vibration energy of the cable, so the vortex-induced vibration of the cable is not considered, However, based on fatigue cumulative damage principle and rain-flow method, the fatigue life of cable is short and fatigue damage is easy to be found.

Acknowledgments

References

[1]   Qingkuan Liu,Yunfei zheng,Xiaobing Liu,Wenyong Ma.*Wind Load ,Wind Induced Vibrations and Control of Cables of Cable-stayed Bridges*[J].Engineering Mechanics,2015,32(09):1-8.(In Chinese)

[2]   Tingting Liu. *Study on Wind-induced Response and Fatigue of Flexible Structure* [D].Dalian University of Technology, 2012. (In Chinese)

[3]   Zdravkovich M. *Different Modes of Vortex Shedding: An Overview* [J]. Journal of Fluids & Structures, 1996, 10(5):427-437.

[4]   Ji Liang. *The Wind-excited Vibration of The Cable-stayed Spatial Latticed Structure* [D].Zhejiang University, 2005. (In Chinese)

[5]   Zifan Fang, Tao Qin, Chen Zhu, Zhihui Chen, Yiliang Huai. *Transmitting Antenna Structure's Dynamic Time History Response Analysis Under Fluctuating Wind Load Environment.* [J/OL].Journal of China Three Gorges University (Natural Sciences), 2015, (01):80-84. (In Chinese)