Study on Nonlinear Vibration Analysis of Gear System with Random Parameters

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Abstract. In order to study the dynamic characteristics of gear nonlinear vibration system and the influence of random parameters, firstly, a nonlinear stochastic vibration analysis model of gear 3-DOF is established based on Newton's Law. And the random response of gear vibration is simulated by stepwise integration method. Secondly, the influence of stochastic parameters such as meshing damping, tooth side gap and excitation frequency on the dynamic response of gear nonlinear system is analyzed by using the stability analysis method such as bifurcation diagram and Lyapunov exponent method. The analysis shows that the stochastic process can not be neglected, which can cause the random bifurcation and chaos of the system response. This study will provide important reference value for vibration engineering designers.

1. Introduction

There are many nonlinear factors in the gear transmission system, such as the gear meshing stiffness, transmission error, bearing clearance, tooth side gap and so on. These coupling factors will cause the strong nonlinear vibration of the gear system and affect the vibration reliability of the gear system. Studies show [1-5] that the system will change from the periodical response to a chaotic vibration state with chaotic, disorder and aperiodic when the parameters of the gear system changed a little. Generally, the gear system response is not sensitive to the small changes of the initial conditions in the periodic response state, however, slight changes will make the system vibration response produce unpredictable results when the gear's system enters the chaotic state.

As we all known, for the gear system with nonlinear vibration, the change of gear's parameters will cause the system into a chaotic vibration state. Traditionally, chaotic vibration state is avoided by the conventional method (such as Lyapunov and bifurcation method), but its dynamic state still changes due to the randomness of gear's parameters. When the system is in chaotic or near-chaotic state, random bifurcation and random chaos [6] of the gear's system response, which affects the vibration and noise of the gear system and determines the vibration reliability of the gear system [7].

In this paper, the aim is to avoid random chaotic vibration of gear drive system, the random characteristics of gear's parameters must be considered, so as to better control or avoid such irregular chaotic vibration characteristics. Therefore, in this paper, the stability analysis of gear's nonlinear vibration with random parameters will be conducted, and it provides a reference and theoretical basis for the control and judgment of gear's nonlinear vibration with random parameters.

2. Numerical simulation of gear nonlinear vibration system with random parameters

2.1. Establishment of gear nonlinear vibration model with random parameters
To simulate the gear's nonlinear vibration with random parameters, the random parameter is expressed as the combination of the determined value and the disturbed value. For example, the excitation frequency is equivalent to $\omega_m + \omega_{m\Delta}$, where ω_m is the determined value of the excitation frequency, $\omega_{m\Delta}$, is the disturbed value of the excitation frequency. And all parameters are assumed as independent random variable in each time period, namely, the dynamic response of gear vibration is regarded as a Gaussian random process. The three-degree-of-freedom nonlinear torsional-torsional coupled dynamic model (as shown in Fig. 1, the specific derivation is shown in Ref. [8]) is taken as the study object. The static transmission error is obtained the first-order components, and the gear nonlinear vibration model can be expressed as a random parameter

$$
\begin{cases}
\dot{y}_1 = y_2 \\
\dot{y}_2 = -2\xi_p \dfrac{dy_p}{dt} - 2\xi_{mp} \dfrac{dy}{dt} - k_{11}f_p(y_p) - k_{13}f_m(y) \\
\dot{y}_3 = y_4 \\
\dot{y}_4 = -2\xi_g \dfrac{dy_g}{dt} + 2\xi_{mg} \dfrac{dy}{dt} - k_{22}f_g(y_g) + k_{23}f_m(y) \\
\dot{y}_5 = y_6 \\
\dot{y}_6 = F - (\omega_m + \omega_{m\Delta})^2 \cdot \dfrac{e_1}{b}\sin\big[(\omega_m + \omega_{m\Delta})t\big] \\
\quad + \dot{y}_2 - \dot{y}_4 - 2(\xi_m + \xi_{m\Delta})\dfrac{dy}{dt} - k_{33}f_m(y)
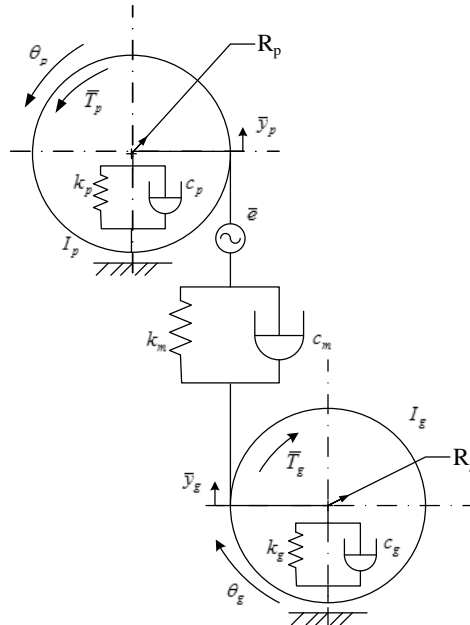\end{cases}
\tag{1}
$$

Fig.1 Dynamic model of a spur gear pair

in which the dimensionless nonlinear function of gap is expressed as

$$
f_m(y) = \begin{cases}
y - (b_m + b_{m\Delta})/b, & y > (b_m + b_{m\Delta})/b \\
0, & -(b_m + b_{m\Delta})/b \le y \le (b_m + b_{m\Delta})/b \\
y + (b_m + b_{m\Delta})/b, & y < -(b_m + b_{m\Delta})/b
\end{cases}
\tag{2}
$$

where ξ_m and $\xi_{m\Delta}$ are the determined value and the disturbed value of meshing damping ratio, respectively；ω_m and $\omega_{m\Delta}$ are the determined value and the disturbed value of excitation frequency, respectively；b_m and $b_{m\Delta}$ are the determined value and the disturbed value of the tooth backlash, respectively；$\xi_{m\Delta}$、$\omega_{m\Delta}$、$b_{m\Delta}$ are similar to Gauss white noise with zero mean.

2.2. Nonlinear vibration numerical solution of random parameters gear

For nonlinear random vibration analysis, the most effective method is numerical integration method [6,7]. The numerical simulation of nonlinear random vibration is based on numerical integration. The step-by-step integration method is always used to solve the system dynamics equation, so that the solution of the system in the time domain is obtained. There are many kinds of step-by-step integration methods. At present, linear acceleration method, Runge-Kutta method, Newmark- β method and Wilson- θ method are widely used. In this paper, the Runge-Kutta method is used to solve the dynamic differential equations of the system. The basic steps are:

(1)Determination of the basic random variables and the distribution functions；

(2)Let t=0, and give the initial value $x(0), \dot{x}(0)$;

(3)Sampling the basic parameters

(4)Establishing dynamic equations of deterministic gear system from sampling results;

(5)Solving the deterministic dynamics equation (4) in the [t, $\Delta t + t$] moment vibration displacement and velocity by Runge-Kutta method.

3. Analysis of nonlinear vibration with random parameters

3.1. Parameters of gear pair

In order to study the effect of random parameters on the response of the gear's nonlinear system, the bifurcation diagram and Lyapunov exponents method [9] is used to analyze gear's nonlinear vibration systems with random parameters. And it provides the reference for the desiger. In this paper, an external meshing spur gear is taken as the study object, the gear parameters and working conditions are shown in table 1.

Table.1 Parameters of gear pair

Parameter's name	pinion	wheel
Modulus m (mm)	4	4
Tooth number z	20	30
Addendum coefficient h_a^*	1	1
Tip clearance coefficient c^*	0.25	0.25
Pressure angle α_0 (°)	20	20
tooth width B(mm)	16	16
Young modulus E(MPa)	2.07×10^5	2.07×10^5
Poisson's ratio ν	0.259	0.259
torque T(N·m)	500	750
rotate speed (r/min)	3000	2000

3.2. Effect of meshing damping

To study the influence of random process on the motion state of the system of the meshing damping value ξ_m, the largest Lyapunov exponent and bifurcation diagram were drawn in the case of the stochastic process and non-stochastic process, respectively, which is shown in Figure 2 and 3. From Figure 2 (a) and figure 3 (a) shows that in ξ_m=0.03, the largest Lyapunov exponent is greater than zero, and the bifurcation diagram is a number of discrete points, so this phenomenon indicates the system into chaos; in ξ_m =[0.03, the range of 0.037], the largest Lyapunov exponent is less than zero, and the

system response begins bifurcate, the system response in this time can be judged as period doubling motion; chaotic motion appeared after a certain interval system; ξ_m increased gradually, and the system from chaotic motion to quasi periodic motion when the maximum Lyapunov system refers to 0. it becomes periodic motion. Finally, it becomes periodic motion, at this moment, the bifurcation diagram shows a curve with parameters, and the maximum Lyapunov exponent is less than 0. The change of the whole process is slow and asymptotic. The conclusion drawn from the largest Lyapunov exponent map is consistent with the bifurcation diagram's. The state of motion is chaotic-doubling chaotic-quasi periodic-periodic motion.

It can be seen from Fig. 2 (b) and Fig. 3 (b) that due to the influence of ξ_m random process, it is different in $\xi_m = [0.03, 0.037]$ and is no longer periodic but chaotic.
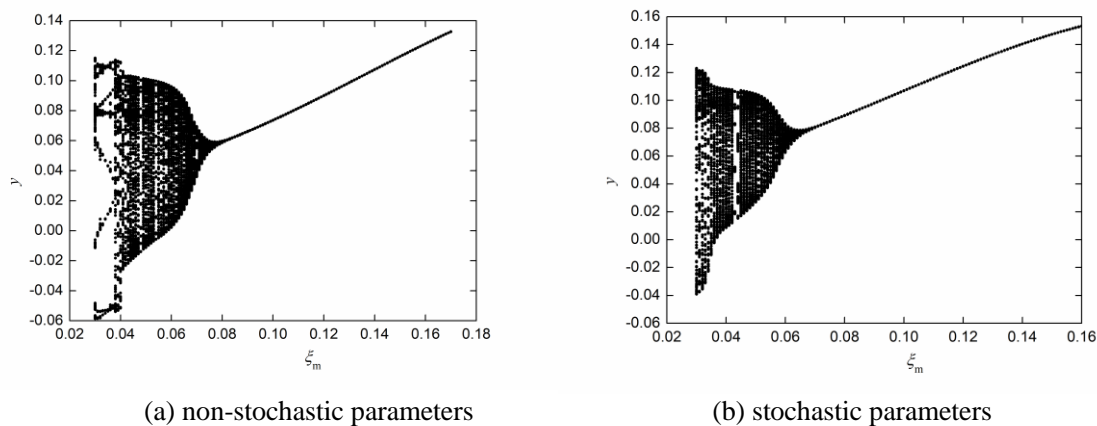

(a) non-stochastic parameters   (b) stochastic parameters

Fig.2 Bifurcation diagram

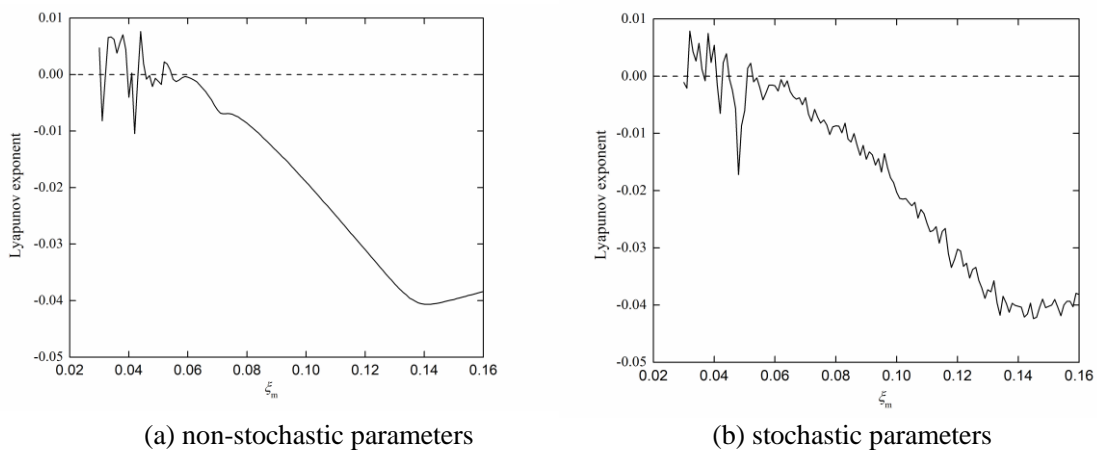(a) non-stochastic parameters   (b) stochastic parameters

Fig.3 The largest Lyapunov exponents

3.3. Effect of Tooth gap

In order to study the influence of random process on system motion state when each value of the tooth-side clearance b_m is taken, the bifurcation diagram and maximum Lyapunov exponent graph without bm stochastic process and bm stochastic process are plotted respectively, as shown in Figs. 4 and 5 .
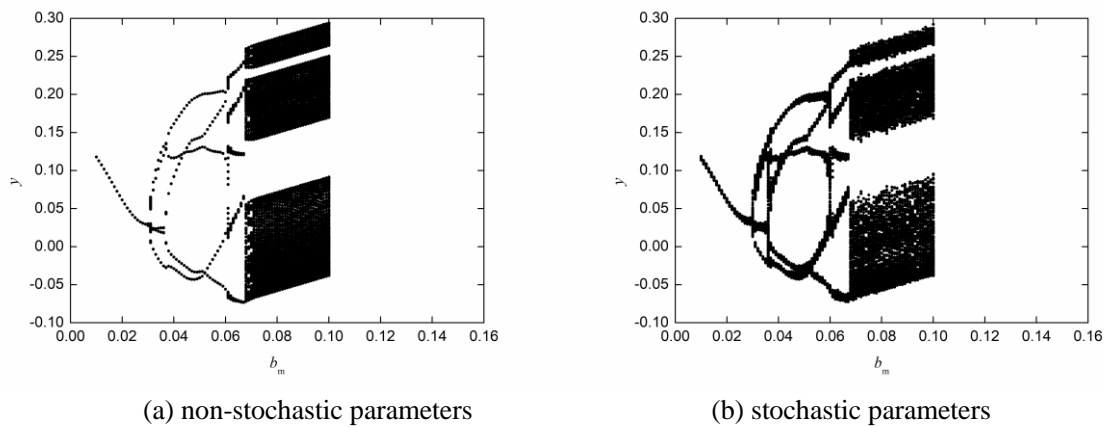
(a) non-stochastic parameters   (b) stochastic parameters

Fig.4 Bifurcation diagram

It can be seen from Fig. 4 (a) and 5 (a) that when the tooth gap b_m is in the interval of [0.01,0.32], the system is a single period of motion. After b_m = 0.32, the system begins to fork. After b_m=[0.32, 0.61], the system is double periodic; b_m is in the range of [0.61, 0.68], and the system is quasi-periodic. After that, the system enters quasi-periodic motion and chaos state. With the increase of the tooth gap, the system is bifurcated into a quasi-periodic state by a single cycle and a double cycle, and finally enters the chaotic state.

It can be seen from Fig. 4 (b) and 5 (b) that due to the influence of the parameter random process, the b_m ten-periodic motion begins to blur in the [0.32,0.61] interval and the maximum Lyapunov exponent close to 0. This phenomenon indicates that gear motion tend to enter the quasi-periodic or even chaotic movement trend.
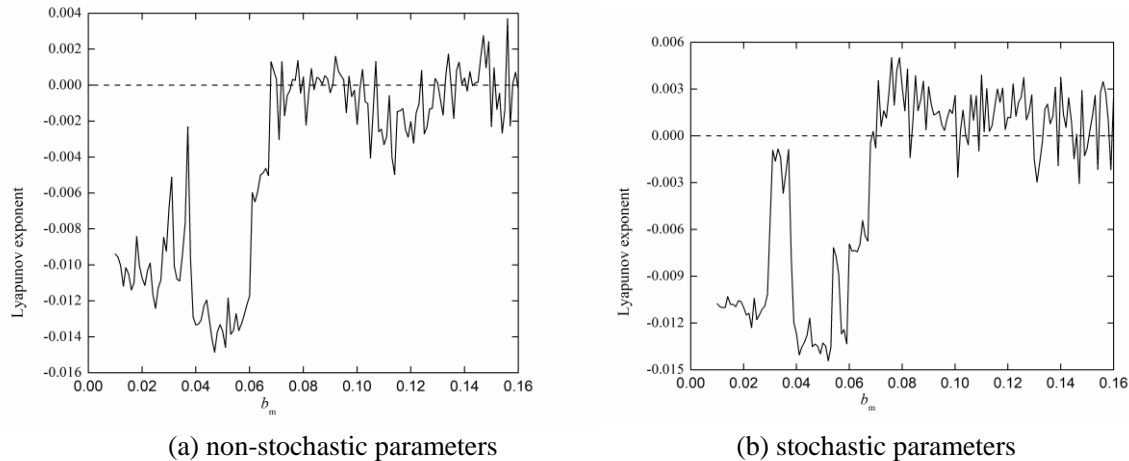

(a) non-stochastic parameters   (b) stochastic parameters

Fig.5 The largest Lyapunov exponents

3.4. Exciting frequency

To dynamically describe the variation of the vibration characteristics of gear system with the non-dimensional excitation frequency ratio ω_m, when ω_m is changed in the interval [0.5, 2], the bifurcation without ω_m stochastic process and ω_m stochastic process graphs and largest Lyapunov exponent graphs, as shown in Fig. 6 and Fig. 7.

As can be seen from Fig. 6 (a) and Fig. 7 (a), when ω_m is within [1.52, 1.59], the bifurcation diagram of the system shows that each parameter value corresponds to many points, and the corresponding maximum Lyapunov exponent is larger than 0. So the system of gear is in chaotic motion when ω_m is in this interval; when ω_m is in other intervals, the bifurcation graph shows as a curve changing with parameters or a parameter corresponds to multiple points, the corresponding maximum Lyapunov exponents are all less than 0, so it indicates that the system is periodic movement.
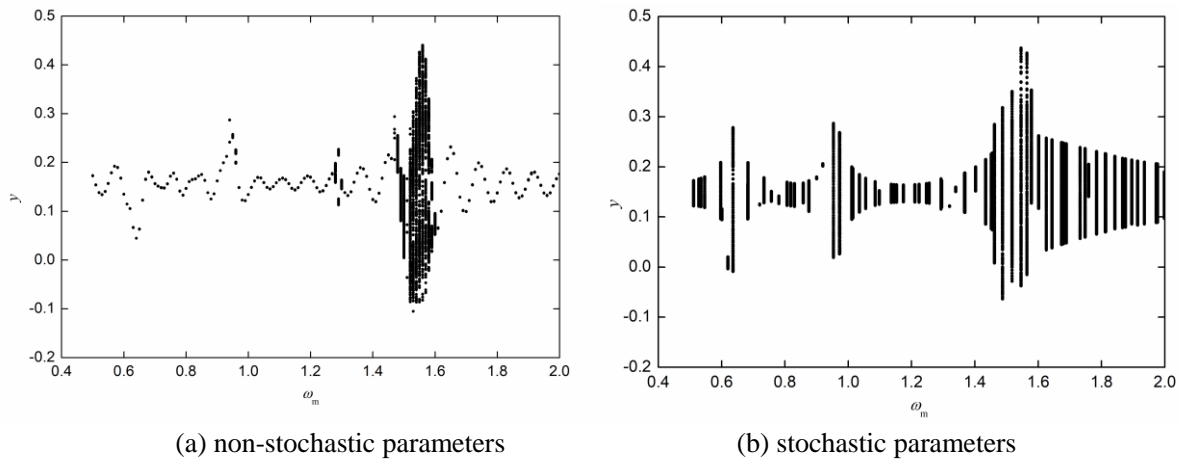
(a) non-stochastic parameters   (b) stochastic parameters

Fig.6 Bifurcation diagram

From Fig.6 (b) and Fig.7 (b), we can see that due to the influence of ω_m stochastic process, the bifurcation diagram of the system shows that each parameter value corresponds to a large number of points [0.61 0.63] and [0.93 0.96]. And the corresponding maximum Lyapunov exponent is equal to 0, and the system is quasi-periodic in these two intervals. It can also be seen that the bifurcation diagram of the system shows many points in [1.52 2], and the maximum Lyapunov exponent is larger than 0. So in this interval to determine the system for chaotic movement.
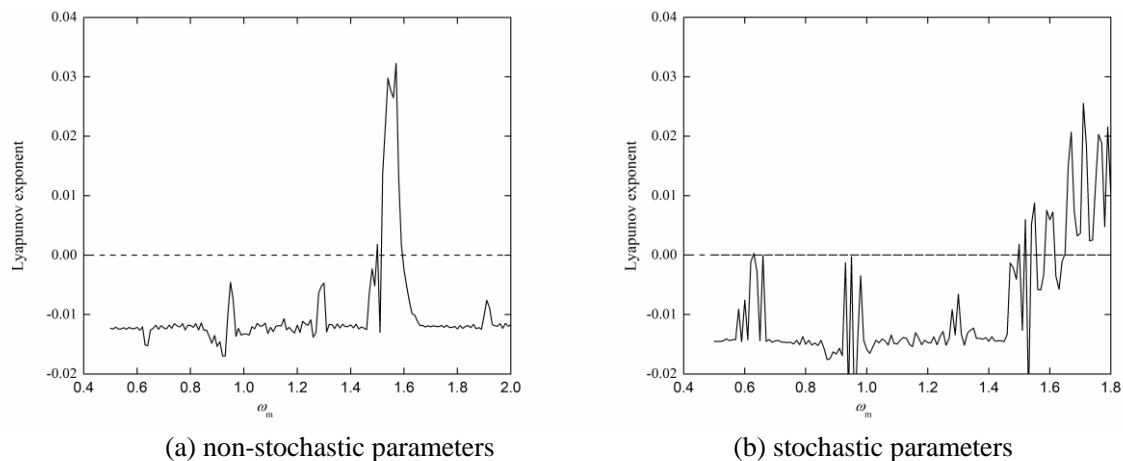

(a) non-stochastic parameters   (b) stochastic parameters

Fig. 7 The largest Lyapunov exponents

4. Conclusion

In this paper, the modeling and simulation of gear nonlinear vibration system with random parameters are carried out. The influence of the random parameters on the dynamic response of the nonlinear vibration system is analyzed by bifurcation diagram and Lyapunov exponent method. Through the analysis and comparison of the influence of the parameters (meshing damping ξ_m, the tooth-side clearance b_m and excitation frequency ω_m) with non-stochastic process and stochastic parameters, some conclusion is obtained as follow:

(1) For meshing damping ξ_m, due to the random process, when ξ_m is in the range of [0.03,0.037], it will cause the system to change from the period of double periodicity to chaos.

(2) For the tooth-side clearance b_m, due to the random process, when b_m is in the range of [0.32,0.38], the system changes from five times of periodic motion to fuzzy, and the motion of gear tends to be quasi-periodic and even chaos movement.

(3) For the excitation frequency, the stochastic process has the most severe impact on the system response. When ω_m is in the [1.59,2] interval, the system changes from single-period to chaos motion;

ω_m is between [0.61,0.63] and [0.93,0.96], the system changes from single-period to quasi-period and the periodic motion in other periods becomes fuzzy and no longer stable.

In conclusion, the stochastic process of the gear parameters makes the system from the periodic motion into quasi-periodic and even the chaotic motion. It can be shown that the value of random process parameters can not be ignored, the traditional deterministic model can no longer meet the engineering requirements. This study can provide the theoretical basis for the control and judgment of gear's nonlinear vibration with random parameters.

Acknowledgement

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