

# Nonlinear Talbot Effect and Its Applications

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**Abstract:** Talbot effect, a lensless self-imaging phenomenon, was first discovered in 1836 by H.F. Talbot. The conventional Talbot effect has been studied for over a hundred years. Recently, the rapid development of optical superlattices has brought a great breakthrough in Talbot effect research. A nonlinear self-imaging phenomenon was found in the periodically poled LiTaO<sub>3</sub> (PPLT) crystals. [1][2][3] This nonlinear Talbot effect has applications not only in optics but also in many other fields. For example, the phenomenon is realized by frequency-doubled beams, which offers people a new way to enhance the spatial resolution of the self-images of periodic objects. And by observing the self-image of the second harmonic (SH) field on the sample surface, people can detect the domain structure in the crystal without damaging the sample. Throughout this review paper, an overview of nonlinear Talbot effect and two applications of this phenomenon is presented. Breakthroughs like achieving a super-focused spot and realizing an acousto-optic tunable SH Talbot illuminator will be introduced as well.

## 1. Introduction

Talbot effect has been discovered for over a hundred years. It has important applications in imaging processing and synthesis, optical metrology, optical analysis, as well as in electronic optics and microscopy. However, the current researches on Talbot effect mostly focus on studying the properties of input beams. Furthermore, in these researches, real gratings are required during the imaging process. In a very recent report, self-imaging caused by the SH field generated in PPLT crystals has been discovered for the first time. The nonlinear Talbot effect has extended the scopes of the conventional research.

In 1836, H.F. Talbot [4] found the self-images of periodic objects without using any optical lens. In 1881, Lord Rayleigh [5] first explained Talbot effect analytically, classifying it as a phenomenon based on Fresnel diffraction and the interference of diffracted beams. He also gave the explicit expression of the Talbot length for plane-wave input beams:  $z = \frac{2d^2}{\lambda}$ , where  $z$  is the Talbot length, at multiples of which self-images occur,  $d$  is the period of the grating and  $\lambda$  is the wavelength of the incident light. In other words, the process of Talbot effect can be divided into two parts: First, the incident light passes through the periodic structure and produces diffracted beams. Second, these diffracted beams interfere in the space behind. However, in the self-imaging phenomenon observed in PPLT crystals, the diffracted beams are replaced by the periodic SH field generated in PPLT crystals. Since lattice distortion will appear during the poling process, domain walls in the crystal become nonideal, resulting in the changes of nonlinear coefficients near them. Then the SH intensity generated from the domain walls and that inside domains will be different, thus forming the periodic SH field on the output surface. These frequency-doubled beams with different intensity interfere with each other in the



space behind, and ultimately produce the self-imaging phenomenon.

Unlike the conventional Talbot effect, the self-imaging phenomenon in PPLT crystals has the following characteristics:

1. It is not directly produced by the incident light, but by the SH field on the output surface of PPLT crystals.

2. In the self-imaging phenomenon, there is no real object involved in self-imaging, the self-images are copies of the SH field on the output surface of PPLT crystals, the pattern represents the distribution of SH coefficient of the sample surface.

3. This phenomenon is realized by frequency-doubled beams, whose wavelength is half of the input light, so that the spatial resolution in imaging is enhanced by a factor of 2.

Since the nonlinear Talbot effect can reflect the SH coefficient distribution of the optical superlattice surface, it is possible to investigate the domain structure in the optical superlattice with this method. Comparing with other nonlinear instruments, nonlinear Talbot effect is a lensless self-imaging effect, and it does not require a reference SH wave. Comparing with the chemical etch method, it works without damaging the sample surface. Observations in this way will be much easier. On the other hand, nonlinear Talbot effect contributes to the enhancement of the spatial resolution, so it also plays an important role in lithography field. In this review, the application of nonlinear Talbot effect in these areas will not be discussed. In the following, a method of using nonlinear Talbot effect to break the diffraction limit and an optimization of Talbot illuminator will be introduced.

## 2. Superfocusing with Nonlinear Talbot Effect

### 2.1 Diffraction limit

The concept of diffraction limit was first proposed by Abbe, Rayleigh et al.[6] at the end of the 19th century. For an ideal point, illuminated through the optical system, there will be a Fraunhofer diffraction image whose diameter is determined by the wavelength of the incident light and the numerical aperture (NA) of the optical system,  $D \propto \frac{\lambda}{NA}$ . Since the Airy disks of each point are difficult to distinguish when they are close, the resolution of the optical system is limited. This limitation of resolution is essentially caused by the uncertainty of quantum mechanics. For a certain frequency of photons, its resolution is fixed when it is given an exact range of momentum in one direction. In the case of Fourier analysis, the information limited by the diffraction limit is carried by the high-frequency waves emitted by the object, and these subwavelength high-frequency waves are exponentially decayed evanescent-waves. They will decay in a very short distance, so we cannot get the details they carried after the distance of several wavelengths, which makes the imaging imperfect.

### 2.2 Super-oscillation and Super-gain antennas

For the realization of super-resolution, an important technology utilizes the evanescent-wave to go beyond the diffraction limit. As mentioned above, these subwavelength high-frequency waves carry information limited by the diffraction limit, so it is feasible to realize super-resolution in near field with evanescent waves. However, since the decay speed of these evanescent waves is extremely rapid, the working distances of such optical means are limited to within a wavelength.

At present, by combining the nonlinear Talbot effect and the Super-oscillation effect as well as super-gain antennas theory, there is a new way to form a sub-wavelength focused spot at a distance of several hundred SH wavelengths from the surface of the sample.

In 1943, Schelkunoff proposed a super-gain antennas design. In 1952, G. Toraldo Di Francia [7] applied the antenna theory to the optical, he believes that the classical limit is just a practical limit rather than a theoretical limit. By constructing a series of concentric rings, controlling the amplitude and phase of the light passing through each ring, we can get as small Airy disks as desired, thus breaking the diffraction limit. Since the Fraunhofer diffraction image of a very narrow ring of any diameter can be written as a 0-order Bessel function, by making the circle with certain diameters pervious to light, and the other regions not transparent, we can form a spot much smaller than the

Abbe diffraction limit in the far field.

On the other hand, it is traditionally assumed that the oscillation speed of the band-limited function is impossible to exceed its highest Fourier component. In 1985, however, Bucklew and Saleh constructed an ideal band-limited function that oscillates faster in local-region than its highest Fourier component. In 2006, Berry et al. explicitly proposed the concept of super-oscillation and applied it to optical focusing. As the super-oscillation can be seen as a wave function superimposed by several different wave vectors, it happens to reflect some sub-wavelength characteristics, and its ability to carry sub-wavelength details is better than evanescent-wave's, so it is feasible to use the super-oscillation effect to reflect subwavelength details [8].

In the nonlinear Talbot effect, the phases between SH generated by the negative domain and by the positive domain in the PPLT crystal have a difference of  $\pi$ , so that the modulation of the optical phase on the plane can be achieved in the second harmonic generation. So, we can observe the sub-wavelength focusing phenomenon in a way similar to GTD Francia's theory. The phase-matching condition of the second harmonic generation in PPLT crystals limits nonlinear Talbot effect to a small bandwidth. The generated SH has a limited bandwidth of the spatial frequency and therefore satisfies the realization condition of super-oscillation effect. By now, relevant experiments have been successful in achieving focusing spots with a diameter within one-eighth of the size of the incident light hundreds of wavelengths away from then sample surface.

The use of non-linear Talbot effect to realize super-focusing can produce Airy disks of any desired size. The technology does not rely on the evanescent-wave, so the working distance can be much longer than the near-field super-resolution technique. And it does not need structure smaller than the wavelength of the incident light, so the process is also simpler. It is believed that this technology will have important application in lithography, medical imaging, biological imaging and cell imaging etc.

### 3. Second-Harmonic Talbot Illuminator

The optical element, which divides the incident beam into an array of micro-beams, is called an array generator, and the Talbot illuminator is capable of converting uniform plane waves into periodic intensity patterns. However, conventional Talbot illuminators are fabricated by spatially selective etching or deposition, and illuminators made in this way are usually only fixed for a particular wave band. Although spatial light modulators based on liquid crystal can be used to achieve a tunable array, the commercial spatial light modulator typically has a big pixel, thus limiting its application.

Since the periodically polarized  $\text{LiNbO}_3$  (PPLN) crystals have excellent nonlinear optical properties, as well as large electro-optic, acousto-optic and piezoelectric coefficients, there have been many studies on using PPLN crystals to achieve tunable arrays in recent years. In 2006, M. Paturzo et al. [9] proposed an electro-optic tunable phase array in a PPLN crystal. However, the study of M. Paturzo did not involve the nonlinear Talbot effect. In 2012, an acousto-optic tunable SH Talbot illuminator based on PPLN crystals was presented [10]. Thanks to the maturation of the polarization technology, the period of PPLN crystals can be reduced down to submicron, coupled with the increase in spatial resolution caused by frequency doubling, this theory can be applied to high-density illumination array.

In the SH Talbot illuminator, the acoustic waves can be generated either by the PPLN crystal itself or induced by an external transducer. When the radio frequency signal is used to excite the acoustic field in the PPLN crystal, the period is the same as the period of the SH field. Compare the SH field and the acoustic field in the PPLN crystal as two mutually parallel gratings  $G_1$ ,  $G_2$  and calculate the subsequent light field distribution, it is found that a series of bright fine lines can be recognized at the first Talbot distance when the sound intensity is large. It has been successful to use the acoustic field in the PPLN crystal to modulate the phase of the light passing through. On the other hand, if the transducer is used to stimulate the acoustic field, we can get a variety of periodic sound field. It has been confirmed in the simulation that the period of the phase grating  $G_2$  has an effect on the thin line pattern at the first Talbot distance. Therefore, a more convenient method of modulating the phase of the second harmonic array can be obtained. And when the direction of the grating  $G_1$  is along x-axis

and the direction of the phase grating  $G_2$  is along z-axis, we can even observe a two-dimensional diffraction pattern, which makes the SH Talbot illuminator capable for more applications.

#### 4. Conclusion

Nonlinear Talbot effect is an extension of the conventional Talbot effect. It has many interesting properties and many potential applications in characterization, lithography, optical communication and many other nonlinear processes. For example, the imaging resolution can be doubled because the phenomenon is realized by frequency-doubled beams. Also, the self-imaging of the SH field makes it a useful way to test domain structures in samples without damaging them. There is no doubt that the discovery of nonlinear Talbot effect has greatly improved and promoted the conventional theory. This new effect makes many test processes more convenient and simplifies many optical instruments. However, the current research on the nonlinear Talbot effect is still in the initial stage, further researches on problems like achieving the n-order nonlinear Talbot effect and reducing the size of the super-focused spot will be carried out in the future. When more disciplines are involved, nonlinear Talbot effect will certainly play an important role in many fields.

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