Two tradeoffs between economy and reliability in loss of load probability constrained unit commitment

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Abstract. Spinning reserve (SR) should be scheduled considering the balance between economy and reliability. To address the computational intractability cursed by the computation of loss of load probability (LOLP), many probabilistic methods use simplified formulations of LOLP to improve the computational efficiency. Two tradeoffs embedded in the SR optimization model are not explicitly analyzed in these methods. In this paper, two tradeoffs including primary tradeoff and secondary tradeoff between economy and reliability in the maximum LOLP constrained unit commitment (UC) model are explored and analyzed in a small system and in IEEE-RTS System. The analysis on the two tradeoffs can help in establishing new efficient simplified LOLP formulations and new SR optimization models.

1. Introduction

Power systems need spinning reserve (SR) to avoid unforeseen events such as a sudden outage of generators/lines or a sudden increase of load. How to set optimal SR amount is an important issue and it has been analyzed extensively.

Traditionally, SR requirement is determined based on deterministic criteria. However, this deterministic method does not consider the stochastic nature of system behavior and component failures. The probabilistic methods have been proposed considering the stochastic nature of system behavior and component failures. Most of these methods are based on reliability metrics such as loss of load probability (LOLP), and expected energy not supplied (EENS), etc. With the help of the reliability metrics, it become possible to schedule suitable SR amount to make an elaborate balance between reliability and economy.

There are two ways to incorporate the reliability metrics into the UC model. In [1-2], LOLP or EENS which must be below a fixed threshold is used as a constraint in the UC model. In [3-4] the balance between reliability and economy is decided by minimizing the operating cost and the expected interruption cost (EIC). The EIC is equal to EENS multiplied by the average value of lost load (VOLL).

This paper optimizes SR based on LOLP since LOLP is more simple and easy to understand. However, LOLP is very hard to calculate when various contingency events are considered because it is highly nonlinear and combinatorial. The computer may run out of memory soon when high order outage events and many optimization periods are considered. References [5-6] express LOLP as a function of system spinning reserve (SSR) of that period using curve fitting. So that the LOLP formulation is simplified but the accuracy of the model cannot be guaranteed. In [7], the simulated annealing (SA) algorithm is used to solve LOLP constrained UC model. A heuristic algorithm for

LOLP constrained UC model is proposed and solved in [8]. SSR amount is gradually updated until the given LOLP threshold is satisfied.

From the methods mentioned above, they focus on how to make LOLP formulation easy to calculate. There is no comparison between the results obtained by the original model and the results obtained by simplified model. Actually, power and SR can be distributed among units in different ways as SSR increases, and different ways of distribution affect the tradeoff between economy and reliability. This paper focuses on two tradeoffs between economy and reliability in LOLP constrained UC model. The other sections are organized as follows: LOLP constrained UC model is expressed in section 2, the two tradeoffs are analyzed in Section 3. The case studies on the IEEE-RTS system are researched and results are reported in section 4, Section 5 gives the conclusion.

2. Unit commitment model with the maximum LOLP constraint

The objective function of SR optimization is to minimize the operating cost (including the fuel cost and the start-up cost) and reserve cost:

$$\min\left\{\sum_{t=1}^{N_T}\sum_{i=1}^{N_G}\left[C_{i,t}\left(P_{i,t},U_{i,t}\right)+SUC_iK_{i,t}\right]+\sum_{t=1}^{N_T}\sum_{i=1}^{N_G}q_{i,t}R_{i,t}\right\} \tag{1}$$

where

N_T: number of periods in the optimization horizon; NG: number of available generating units; $U_{i,t}$: status (0/1) of unit i during period t; $P_{i,t}$: power output of unit i during period t; $q_{i,t}$, $R_{i,t}$: reserve bid price and reserve bid amount of unit i during period t; $C_{i,t}(P_{i,t}, U_{i,t})$: running cost of unit i during period t, represented by a three-segment piecewise linear function; SUC_i: start-up cost of unit; $K_{i,t}$: a binary variable that satisfies

$$\begin{cases} K_{i,t}\geq 0 \\ K_{i,t}\geq U_{i,t}-U_{i,t-1} \end{cases} \tag{2}$$

A number of constrains must be obeyed during the optimization:

Power balance constraint

$$\sum_{i=1}^{N_G}P_{i,t}=P_t^D \tag{3}$$

where

P_t^D : load demand during period t.

SR constraint is

$$\begin{cases} R_{i,t}\leq P_{i,}^{\max}U_{i,t}-P_{i,t} \\ R_{i,t}\leq U_{i,t}\left(\tau R_i^{up}\right) \end{cases} \tag{4}$$

where

$P_i^{\max}$: maximum power output of unit i; $R_{i,t}$: ramp up rate of unit i; τ : amount of time available for the units to ramp up their output for delivery of reserve capacity . In this paper, τ is assumed to be 0.5 h.

Unit operating constraints

$$\left(P_{i,t},U_{i,t}\right)\in\psi,\forall i,\forall t \tag{5}$$

Unit operating constraints include upper and lower generation limits, minimum up-time and ramp-up and ramp-down rate constraints, initial conditions.

LOLP constraint

$$LOLP_t\leq \mathrm{LOLP}^{\max} \tag{6}$$

In the formulation of LOLP, only random outages of units are considered. Load forecast errors and the effect of transmission and distribution networks are not taken into account. First order outage means outage of one unit; second order means simultaneous outage of two units. LOLP can be explicitly formulated as below where superscripts'1', '2' represent first and second outage events, respectively. The explicit LOLP can be expressed below. For simplicity, LOLP caused by higher-order outage events are not shown here.

$$LOLP_t \approx \sum_{i=1}^{N_G} p_{i,t}^1 b_{i,t}^1 + \sum_{i=1}^{N_G} \sum_{j>i}^{N_G} p_{i,j,t}^2 b_{i,j,t}^2 \tag{7}$$

where

$p_{i,t}^1$: probability when unit *i* occurs outage during period *t*; $p_{i,j,t}^2$: probability when simultaneous outage of units *i* and *j* occurs during period *t*;

Binary variables $b_{i,t}^1, b_{i,j,t}^2$, satisfy

$$b_{i,t}^1 = \begin{cases} 1, & if \quad P_{i,t} + R_{i,t} - SSR_t > 0 \\ 0, & otherwise \end{cases} \tag{8}$$

$$b_{i,j,t}^2 = \begin{cases} 1, & if \quad P_{i,t} + R_{i,t} + P_{j,t} + R_{j,t} - SSR_t > 0 \\ 0, & otherwise \end{cases} \tag{9}$$

SSR_t: SSR during period *t*, that is,

$$SSR_t = \sum_{i=1}^{N_G} R_{i,t} \tag{10}$$

Equation (8)-(9) can be linearized as [2]. For example, (8) can be linearized to (10):

$$\frac{P_{i,t} + R_{i,t} - SSR_t}{\sum_{i=1}^{N_G} P_{i,}^{\max}} \le b_{i,t}^1 \le 1 + \frac{P_{i,t} + R_{i,t} - SSR_t}{\sum_{i=1}^{N_G} P_{i,}^{\max}} \tag{11}$$

The outage probabilities $p_{i,t}^1, p_{i,j,t}^2$ can be formulated as [2]:

$$p_{i,t}^1 = u_i U_{i,t} \prod_{j=1, j\ne i}^{N_G} \left(1 - u_j U_{j,t}\right) \tag{12}$$

$$p_{i,j,t}^2 = u_i u_j U_{i,t} U_{j,t} \prod_{k=1, k\ne i,j}^{N_G} \left(1 - u_k U_{k,t}\right) \tag{13}$$

Where u_i is the outage replacement rate. During ΔT, it is equal to $\gamma_i \cdot \Delta T$. Here γ_i is the failure rate of unit *i*.

3. Two tradeoffs between economy and reliability

Considering two ways to reach the goal of making LOLP level below the given maximum LOLP. One way is to simply increase SSR without LOLP constraint, till the computed LOLP satisfies the requirement. In this way, power and SR are distributed among units according to the marginal cost of units. Power and SR are firstly distributed to the units with low marginal cost. For this method, economy is more preferred, the decease of LOLP is only the by-product of SSR increase. With the increase of SSR, when the corresponding LOLP is below the given $LOLP^{max}$, economy and reliability reach a tradeoff, which can be called the primary tradeoff.

The other way is considering LOLP constraint in the problem of LOLP constrained UC, as the model proposed in this paper. This way can get the least SSR and total cost in the condition of satisfying LOLP constraint after optimizing. The manner of power and SR distribution here is different from that in the first way, considering that the power and SSR should satisfy LOLP constraint and minimize the operation cost at the same time.

For a fixed SSR, to achieve a more reliable system, power and SR trend to be distributed dispersely among units. This is because SR provided by a unit can only be used for others events, it cannot be used for the unit itself. As a result, the economy of the system is not the best. On the contrary, for a fixed SSR, to achieve a better economy level, power and SR trend to be distributed to large units since large units used to be more economical due to their economy of scale [9]. There is a tradeoff between economy and reliability when SSR is fixed, which can be called the secondary tradeoff.

The two tradeoffs can be specifically shown through the following test. Considering a three units system, the maximum output, marginal cost, SR cost and outage probability of units are shown in table 1. The load is 200 MW. For simplicity, other constraints are not considered.

Table 1. Parameters of three units system.

Unit	$P_i^{\max}$ (MW)	$P_i^{\min}$ (MW)	Marginal cost （$/MW）	SR cost （$/MW）	Outage probability
U_1	100	10	30	3	0.001041
U_2	200	10	20	2	0.001041
U_3	400	20	10	1	0.001041

There are 5 models are considered. They are shown below.

$$\text{Model 1}\begin{cases} objective \quad (1) \\ constraint: \\ \qquad (3) \\ SSR = 100MW \end{cases} \quad \text{Model 2}\begin{cases} objective \quad \min \ LOLP \\ constraint: \\ \qquad (3)\, and\, (7) \sim (13) \\ SSR = 100MW \end{cases} \quad \text{Model 3}\begin{cases} objective \quad (1) \\ constraint: \\ \qquad (3)\, and\, (6) \sim (13) \\ SSR = 100MW \end{cases}$$

Model 1 is a SR constrained UC model. In this model, the reliability is implicitly respected by SR constraint. LOLP level is not explicitly considered. When distributing the fixed SSR and load, only economy aspect is considered by objective (1). Results of output power and SR of units are shown in figure 1.

Model 2 tries to get the best reliability level when a fixed SSR is considered. Different from model 1, the objective function is to minimize LOLP level. LOLP is expressed in (7). When distributing the fixed SSR and load, only reliability aspect is considered. The results of output power and SR are shown in figure 2. After optimization, the computed LOLP is 0.0000033.

Model 3 is a LOLP constrained UC with fixed SSR. In (6) of model 3, $LOLP^{max}$ is set as 0.0000033, which is the same as the computed LOLP in model 2. Model 3 yield a balance between model 1 and model 2. The results are shown in figure 3.

$$\text{Model 4}\begin{cases} objective \quad (1) \\ constraint: \\ \qquad (3)\, and\, (6) \sim (13) \end{cases} \quad \text{Model 5}\begin{cases} objective \quad (1) \\ constraint: \\ \qquad (3) \\ SSR \le SSR^{\max},\ increase\ SSR^{\max}\ by\ 1MW \end{cases}$$

Model 4 is a LOLP constrained UC. It is the model proposed in section II. In model 4, $LOLP^{max}$ is set as 0.0000033. After optimization, the computed SR is 200MW. The results are shown in figure 4.

Model 5 belongs to a SR constrained UC model too. But SSR^{max} is gradually increased until the computed LOLP value is less than 0.0000033. The results are shown in figure 5.

The Total cost and computed LOLP of five models are listed in table 2.

From figure 1 and table 2, it can be found that SR and power are totally distributed to one unit. Although the total cost is the lowest among all the models and SR requirement is satisfied, the scheduled SR is invalid if this unit breaks down.

From figure 2 and table 2, it is found that SR and power are distributed dispersedly to gain a better reliability level when SSR is fixed, but the total cost is highest.

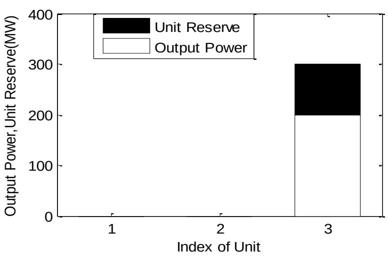

Figure 1. Results of model 1.

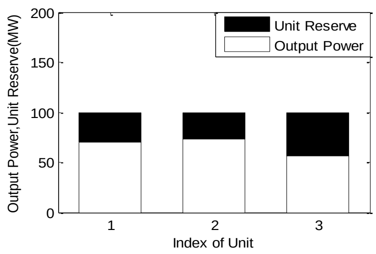

Figure 2. Results of model 2.

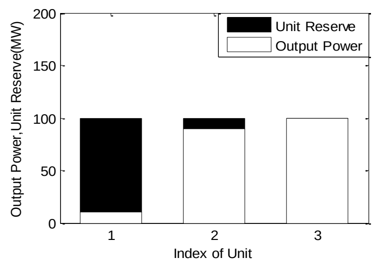

Figure 3. Results of model 3.

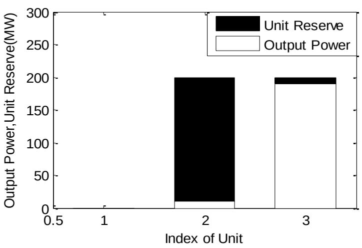

Figure 4. Results of model 4.

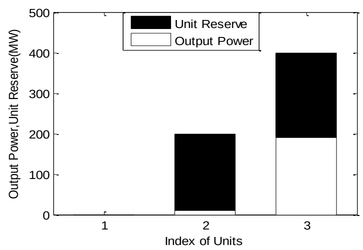

Figure 5. Results of model 5.

Table 2. Total cost and LOLP of five models.

	Total cost ($)	SSR (MW)	Computed LOLP
Model 1	2100.00	100	0.00104
Model 2	4320.00	100	0.0000033
Model 3	3390.00	100	0.0000033
Model 4	2490.00	200	0.0000011
Model 5	2690.00	400	0.0000011

Model 3 justifies the existence of secondary tradeoff. Through model 3, the system can achieve the same reliability level as that in model 2, and the total cost is $3390, which is between the total cost of model 1 and the total cost of model 2. The redistribution of SR and power just reflects the effect of the secondary tradeoff.

Model 4 is a LOLP constrained UC model. Compared model 4 with model 3, from table 2 it can be found that relaxing the reserve constraint yields a better tradeoff between reliability and economy. Since all the tradeoffs are considered in model 4, although a larger SSR is scheduled the total cost of model 4 is lower than that of model 3, and a lower LOLP level is achieved in model 4 at the same time.

If the primary tradeoff is only considered, the model can be changed to be model 5. When SSR amount increases to 399MW, the LOLP is 0.00104. When SSR amount increases to 400MW, LOLP is changed to 0.0000011 which starts to smaller than 0.0000033. The drastic decrease of LOLP is caused by that SR on unit two just can remedy the outage capacity of unit three. Compared figure 5 with figure 4, it can be seen that much SR is distributed on unit three which is the largest unit. That's why it's no effect on system reliability though the SSR increases to 399MW. From table II, the total cost in model 5 is also more than that in model 4 because SR is distributed ineffectively without considering the secondary tradeoff.

From the five models above, two tradeoffs in the SR optimization models can be observed clearly. The tradeoff between security and economy which is on the SSR amount has the largest effect, and it is called the primary tradeoff. The tradeoff between security and economy which focuses on the power and SR distribution under fixed SSR has relatively small effect, and that is why it is called secondary tradeoff. Under many occasions, the effect of secondary tradeoff is small, but not negligible. This conclusion can be used in LOLP simplification model.

4. Case studies on the IEEE-RTS system

Take the IEEE-RTS system as the example [10-11]. There are 26 units in this system. The UC date and ramp rate limits are from [10], and the start-up costs and reliability date are from [11]. The output of units at t=0 is given by the economic dispatch of the committed units for the first hour at a load level of 1700MW. The system lead time is 1 h. For simplicity, the prices of SR equal to 10% of the highest incremental cost of energy production. For simplicity, only the first hour is considered. The model is coded on GAMS and solved by a large-scale MILP solver CPLEX 11.2.

When $LOLP^{max}$ is 0.001, optimize (1) with LOLP constraint (6), which is the model proposed in section 2. Power and SR distribution are shown in figure 6. The results are shown in table 3 and the SSR, SR cost, total cost and $LOLP^{after}$ ($LOLP^{after}$ is calculated based on the computed unit schedule) are included. From table 3, the SSR is 333.50MW.

Set SSR as 333.5MW and run the basic UC model without (6). That is to say, run a reserve constrained UC. The power and SR distribution are shown in Fig.7. The results are shown in table 3.

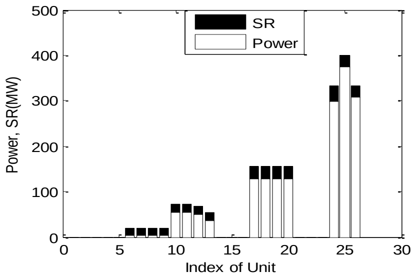

Figure 6. Output power, reserve after optimizing (1)with (6).

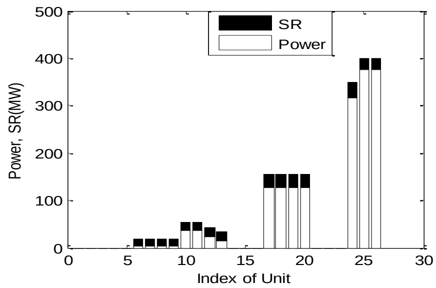

Figure 7. Output power, reserve after reserve constrained UC.

Table 3. Results comparison of optimizing (1) with (6) and a reserve constrained UC.

	SSR(MW)	SR cost($)	total cost($)	$LOLP^{after}$
Optimize (1)with (6)	333.5	563.55	20803.63	0.00094
Run basic UC with fixed SSR	333.5	563.55	20422.46	0.00264

Compared figure 6 with figure 7, It can be found that although SSR is the same, the power distribution among units are quite different. For example, the power of unit 10~13 in figure 6 is more than that in figure 7 and the power of unit 24, 26 in figure 6 is less than that in figure 7. It can be described that some power is transferred from unit 24, 26 to unit 10~13. The power distribution becomes more disperse in figure 6. As the result, $LOLP^{after}$ is decreased from 0.00264 to 0.00094 at the cost of $381.17. It can be found that SR distribution on the units in figure 6 is the same to that in figure 7, so that SR cost is equal in table 3. Actually, SR is also distributed dispersely along with the power distribution under many occasions to achieve a better reliability level.

It can be appreciated that satisfying LOLP constraint by running a SR constrained UC with gradually increased SR will cause a higher total cost. For example, in the case study, gradually increase SSR from 333.5MW and run reserve constrained UC until the given LOLP threshold 0.001 is satisfied. It will found that when SSR increases to 400MW, $LOLP^{after}$ starts to below 0.001 and the total cost is $23869.56 which increases 14.7% compared with $20803.63.

Two tradeoffs are realized by optimizing (1) with LOLP constraint (6), but the secondary tradeoff is not considered as SSR increases in SR constrained UC. The comparison of the total cost illustrates that the secondary tradeoff has an important effect on the distribution of power and SR, and finally affects the tradeoff between the system reliability and economy.

As we all know, two tradeoffs are considered in the model proposed in section 2. Many methods ignore the secondary tradeoff in order to calculate the model efficiently. References [5-6] express LOLP as a function of SSR using curve fitting. But LOLP(SSR) curve is difficult to be fitted precisely, because there are many parameters to be calculated and parameters are problem specific. Besides, power and SR can't be distributed to units as the optimal way even though the simplified LOLP(SSR) can be fitted precisely, because the secondary tradeoff is not embodied in this simplified formulation. Reference [8] updates SSR to participate in the next iteration, then $LOLP^{after}$ is calculated based on the solution of this iteration. The method also ignores the secondary tradeoff considering the way of SSR increase. A new method needs to be developed to compute LOLP efficiently and well respect the secondary tradeoff at the same time.

5. Conclusions

In this paper, two tradeoff between economy and reliability in LOLP constrained UC are proposed and analyzed. As SSR increases, LOLP varies differently when considering two tradeoffs and only considering the primary one. Besides, from the economy point of view, the total cost is compared between the conditions considering and not considering the secondary tradeoff. This paper points out that the second tradeoff have non-negligible effect and it cannot be ignored under many occasions.

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References

[1] Gooi H B, Mendes D P, Bell K R W and Kirschen D S 1999 Optimal scheduling of spinning reserve *IEEE Trans Power Syst* **14** 1485-90
[2] Farrokh A, Mahmud F F, and Mohammad S Unit commitment with probabilistic spinning reserve and interruptible load considerations *IEEE TransPower Syst* **24** 388-397
[3] Ortega-Vazquez M A and Kirschen D S 2007 Optimizing the spinning reserve requirements using a cost/benefit analysis *IEEE Trans Power Syst* **22** 24-33
[4] Wang M Q, Gooi H B and Chen S X 2011 Optimising probabilistic spinning reserve using an analytical expected-energy-not-supplied formulation *IET Generation, Transmission & Distribution* **5(7)** 772-780
[5] Chattopadhyay D and Baldick R Unit commitment with probabilistic reserve 2002 *IEEE Power Engineering Society Winter Meeting* **1** 280-285
[6] Li Z and Han X S 2009 Risk Constrained Unit Commitment Considering Uncertainty of Wind Power and Load *Proc. 2009 Sustainable Power Generation and Supply Conf* 1-5
[7] Simopoulos D N, Kavatza S D and Vournas C D 2006 Reliability constrained unit commitment using simulated annealing *IEEE Trans.Power Syst* **21(4)** 1699-1706
[8] Yang M, Cheng F L and Han X S 2012 A Fast Heuristic Algorithm for Maximum LOLP Constrained Unit Commitment *IEEEPES Innovative Smart Grid Technologies Conf* Tianjin, China
[9] Wang M Q, Gooi H B and Han X S 2015 A tradeoff between unit capacity and average production cost in spinning reserve optimization *International Journal of Electrical Power & Energy Systems* **71** 215-221
[10] Wang C and Shahidehpour S M 1993 Effects of ramp-rate limits on unitcommitment and economic dispatch *IEEE Trans. Power Syst* **8(3)** 1341-50
[11] Grigg C, Wong P and Albrecht P, et al 1999 The IEEE reliability test system-1996 *IEEE Trans. Power Syst* **14(3)** 1010-18