

# Analysis of Fracture Characteristic of a Gas Main Pipe on the Basis of the Additive Mathematical Model of the Cyclic Random Process and Polynomial Function

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**Abstract.** The use of a mathematical model to describe the phenomenon of dynamic fracture of the gas main in a broad range of crack propagation rates is substantiated, taking into account the stochastic and cyclic nature of this process. Particular attention is paid to the reliability of the proposed approach, which is evaluated based on comparison with known natural experiments.

## 1. Introduction

Experimental studies of the dynamic fracture of oil and gas mains allow establishing the basic regularities in deformation processes in steels of different strength classes at high crack propagation rates [1]. However, at the moment, a single standard has not yet been developed, which would consider the main effects that are crucial for describing the elastic-plastic properties and the dynamic strength of materials under impact loads.

That is why a mathematical model to describe the dynamic deformation and fracture, evaluate the kinetics of crack propagation in gas mains is being actively developed, in order to create simple and effective models that would allow for a physically correct and simple description of these complex phenomena [2, 3]. Particular attention is paid to the choice or development of effective numerical methods and algorithms for their optimization in order to ensure the reliability of calculations.

However, as well as in case of other complex phenomena, the issue of identifying parameters and substantiating the reliability of the proposed models causes some difficulties in the mathematical description of these processes. This work is aimed at extending and supplementing the existing research [5, 6], in order to provide a more accurate description of the kinetics of crack propagation in case of dynamic fracture of the gas main based on a deeper analysis of the rate fluctuations caused by branching of the crack tip as it propagates along the body of the pipe.

The proposed approach allows establishing the physical relationship between the elastic-plastic fracture of a pipe and rate fluctuations at different distances from the place of origin of a crack of dynamic fracture. The development of such methods for modeling the kinetics of crack propagation in the pipe is the basis for the development of new technological approaches for the manufacture of high strength pipes with the preservation of high ductility of the material [6, 7].

The purpose of this work is to use a mathematical model of the cyclic random process to describe the dynamic fracture of a gas main pipe, taking into account the stochastic and cyclic nature of this phenomenon.



## 2. Description of the crack propagation kinetics by a polynomial

A peculiarity of the proposed approach is the interpretation and model description of the regularities in dynamic fracture of the pipe taking into account the relaxation mechanisms at the crack tip due to the nonlinear behavior of the main defect, as well as structural-mechanical relaxation, inelastic deformation, and plastic flow.

So, several stages of fracture of the gas main pipe are known:

- initiation of the primary crack and its start - a rapid propagation of the crack under the influence of internal pressure in the pipe;
- crack growth, at which its length increases and the propagation rate decreases, due to the change in the stress-strain state;
- “stabilization” of the crack propagation rate due to the “self-organization” of the fracture process of the pipe material.

- loss of stability and ultimate fracture of the pipe.

However, since the description of the dynamic fracture of the pipe must be physically grounded, on the one hand, and, on the other hand, provide an acceptable accuracy and correctness, it was assumed that all of these sections of the pipe fracture could be described by a single power dependence. This allows, in addition to the above conditions, to provide sufficient simplicity of computing and avoid determining a significant number of parameters. It should be emphasized that such simplification is a compulsory step, necessitated by the complexity of the macro- and micromechanisms of the dynamic fracture of the pipe.

Figure 1 shows the approximation of experimental data by a polynomial. The proposed approach consists in using an additive mathematical model in the following form:

$$V(\omega, l) = f(l) + \xi(\omega, l), \quad \omega \in \Omega, l \in \mathbf{R}, \quad (1)$$

where  $\xi(\omega, l)$  is the cyclic random process as a model of the cyclic component of the signal;  $f(l)$  is the deterministic function, which reflects the trend of the pipe fracture process,

$$f(l) = \sum_{n=0}^3 c_n \cdot l^n, \quad l \in \mathbf{R}, \quad (2)$$

where  $c_n$  is the coefficient of polynomial function (2).

Function (2) describes well the experimental data along the entire length of the analyzed signal.

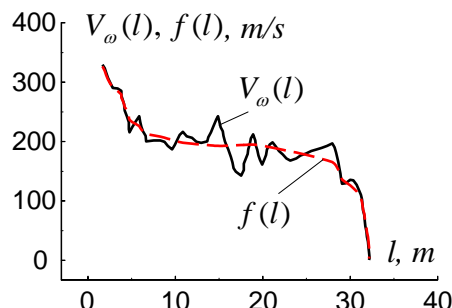
## 3. Approach to the analysis and modeling of the dynamic propagation of the crack

We considered the kinetics of the dynamic crack propagation in a gas main pipe in the form of an additive model - the sum of the polynomial and the cyclic random process. This allowed us to apply the methods of statistical analysis of the crack propagation kinetics in the gas main pipe for the purposes of its analysis, modeling and possible further forecasting.

Experimental data on the gas main fracture (F. Oikonomidis et al.), published in [2, 3], were taken as input data at the first stage of processing the experimental dependence “the rate of dynamic propagation of the crack - the length of fracture”. Of these, a trend component  $\{f(l), l \in \mathbf{R}\}$  and a cyclic component  $\{\xi(\omega, l), \omega \in \Omega, l \in \mathbf{R}\}$  were identified. The procedure for extracting the trend consisted of two steps: describing the data by a polynomial and finding the corresponding coefficients.

The cyclic component was determined by subtracting the trend function  $\{f(l), l \in \mathbf{R}\}$  from the signal of the input process  $V(\omega, l)$ , in particular:

$$\xi(\omega, l) = V(\omega, l) - f(l), \quad \omega \in \Omega, l \in \mathbf{R}, \quad (3)$$



**Figure 1.** Experimental dependence of a decrease in the growth rate of the crack along the trajectory of its growth [2, 3] and its approximation by a polynomial.

#### 4. Some results of statistical processing of the data on the gas main fracture.

$V_\omega(l)$  is the input realization of the investigated process of the crack growth rate.  $\xi_\omega(l)$  is the realization of the cyclic component of the investigated process of the crack growth rate.

To analyze this component, it is necessary to use the methods of segmentation [8], define the discrete function of the rhythm, interpolate it [8, 9], and use the obtained results when applying the methods of statistical evaluation [10].

Methods of statistical analysis of cyclic random processes are adapted to the problems of analyzing the fracture rate of the gas pipeline to evaluate such probabilistic characteristics as the initial moment function of the first order (mathematical expectation), and the central moment function of the second order (dispersion).

The realization of the estimation of mathematical expectation is given in the form:

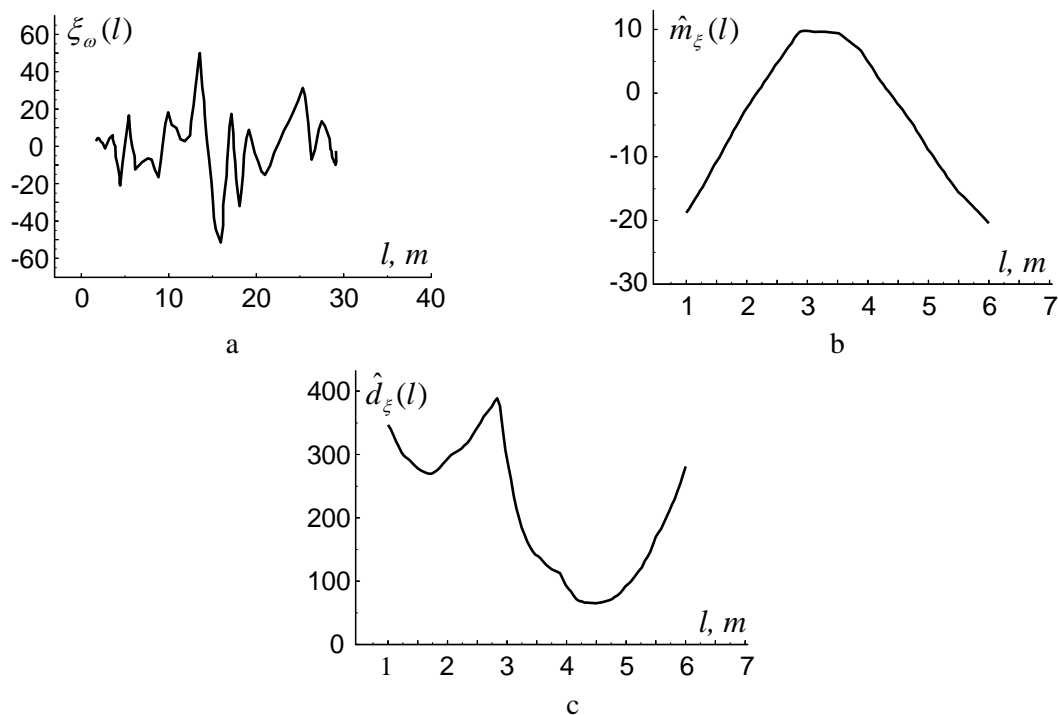
$$\hat{m}_\xi(l) = \frac{1}{M} \sum_{n=1}^M \xi_\omega(l + L(l, n)), \quad l \in \mathbf{W}_1 = [L_1, L_2], \quad (4)$$

where  $L_1 \neq 0$  - in the general case,  $M$  is the number of cycles in the cyclic process (cyclic component),  $\xi_\omega(l)$  is the realization of the cyclic component of the crack propagation rate (the realization of a cyclic random process),  $L(l, n)$  is the function of the rhythm of a cyclic random process,  $\mathbf{W}_1$  is the definition area of the first cycle of the process.

The realization of the dispersion was defined as:

$$\hat{d}_\xi(l) = \frac{1}{M-1} \cdot \sum_{n=1}^{M-1} [\xi_\omega(l + L(l, n)) - \hat{m}_\xi(l + L(l, n))]^2, \quad l \in \mathbf{W}_1 = [L_1, L_2]. \quad (5)$$

Thus, according to the simulation results, a description of the fracture process of the gas main pipe is proposed, which is based on in-depth study of the results of measuring the kinetics of the main crack propagation. For this, continuum-kinetic assumptions were used, in which fracture was considered using certain averaged parameters, such as the continuous process of accumulation and coalescence of defects that cause crack propagation. At the same time, a decrease in the intensity of deformation processes was noted in the local sections of the material in the vicinity of the crack tip, which propagates in conditions of dynamic fracture of the gas pipeline. This reduces fluctuations of the crack propagation rate, causing changes in the parameters presented in Fig. 2a.



**Figure 2.** Cyclic component of the crack growth (a), statistical estimates of its mathematical expectation (b) and dispersion (c) in the analysis of the crack propagation rate in the main pipe on the basis of the proposed model

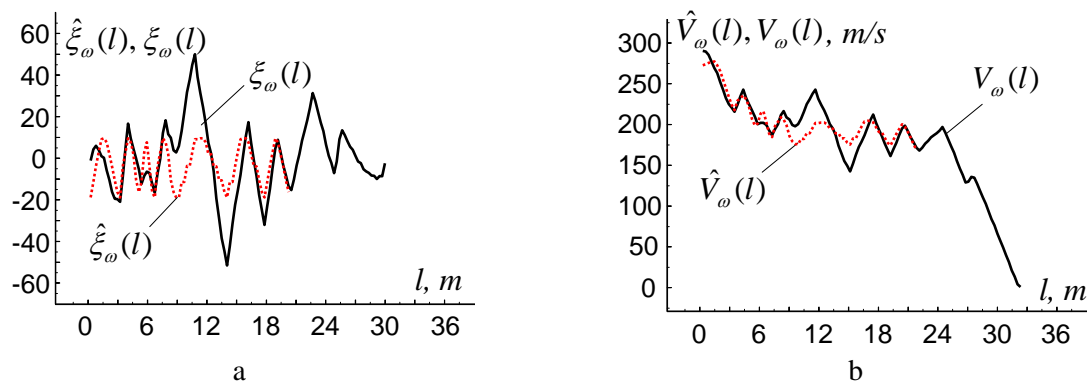
It should be emphasized that the obtained estimates of the mathematical expectation and dispersion, Fig. 2b,c are sensitive to fluctuations in the fracture rate of the main gas pipeline, which indicates the possibility of using these numerical features in automated systems for technical diagnostics.

In order to approbate the mathematical model and the method of analysis used in this work, a series of simulation experiments was carried out to compare actual and simulated realizations of fracture rate processes that occur in the main gas pipeline, taking into account the obtained statistical estimates and the trend component.

### 5. Imitation modeling of the dynamic propagation of the crack

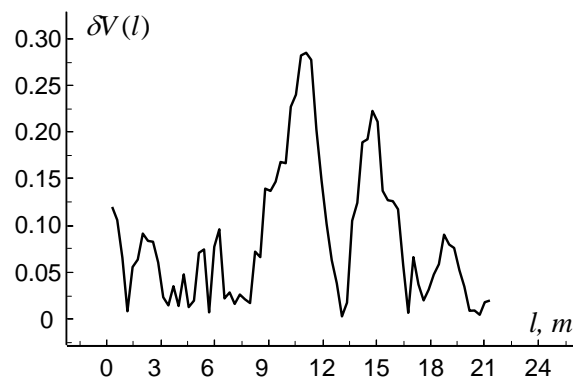
In order to predict fracture of the gas pipeline and develop the methods to arrest the dynamic propagation of the crack, it is necessary to perform an unambiguous mathematical description of the results obtained on the natural structures. The results of estimating changes in the fracture rate of the pipe allowed to quantitatively analyze the fracture process and indirectly estimate the coordinates of defects along the trajectory of the crack propagation, which is important for the analysis of macro- and micromechanisms of fracture. The data of computer simulation of the fracture rate of the main gas pipeline by the proposed method were compared with the experimental ones. As is seen from Fig. 3a, b, the proposed methods of statistical processing of the fracture rate of the gas pipeline describe very well the nature of growth of the main crack [2, 3].

It is established, Fig. 3, that the relative mean square error of simulation does not exceed (in experiments) 15%, which is sufficient for engineering evaluation. It should be noted that the comparative analysis of the accuracy of reproducing the fracture rate of the pipe is individual for each realization of the dynamic crack propagation process. Figure 3 shows the results of realization of the crack propagation rate in the main pipe: the experimental and simulated data.



**Figure 3.** Comparison of simulation results (a, b) of the crack propagation rate in the main pipe with experimental data:  $\hat{\xi}_\omega(l)$  is the simulated cyclic component, which takes into account statistical estimates;  $\xi_\omega(l)$  is the cyclic component defined from the input signal;  $\hat{V}_\omega(l)$  is the simulated realization of the crack propagation process in the main pipe, taking into account statistical estimates of the cyclic component and the trend component;  $V_\omega(l)$  is the input realization of the crack growth rate process.

The results of simulation of the dynamic fracture of the main gas pipeline are satisfactorily consistent with the experimental data. They allow to numerically describe the rate variations along the trajectory of the crack propagation, which result from fracture localization in the conditions of dynamic accumulation of defects.



**Figure 4.** Relative mean square error of the simulation results of the crack propagation rate in the main pipe compared to the experimental data.

The results obtained in the work are the basis for developing the method of computer simulation of the gas main fracture. They are also used to develop the criteria that prevent the dynamic fracture. These criteria are then used in specific techniques of fracture prediction based on the results of the preliminary evaluation of the crack growth kinetics in natural structures [8-10]. The fact of repeatability (approximate to cyclic repeatability) of statistical estimates obtained for probability characteristics of processes and the variability of their rhythm functions is confirmed experimentally. This indicates the adequacy of the developed mathematical model and the method, as well as the appropriateness of their use in the problems of modeling, analysis and forecasting of processes of dynamic fracture of the gas main pipe.

## 6. Conclusions

A mathematical model is proposed, which allows taking into account the structural and mechanical features of fracture of gas mains. This model reflects the relationship between the nonlinearity of the crack growth rate in a pipe metal and relaxation mechanisms, and stages of its dynamic fracture. The fracture model of the main gas pipeline is considered as the sum of the deterministic polynomial function and the cyclic random process. Computer software is created based on the proposed mathematical model for analyzing the two-dimensional crack growth structure along its trajectory.

The proposed methods and approaches are substantiated on the basis of comparison with literary experimental data. The parameters are identified and the model is verified based on comparison with known natural experiments. Relative mean modeling error does not exceed 15 %.

In further research, it is planned to study the application of the exponential function as a model of the trend component, and compare the simulation results of the crack propagation rate in the main pipe on the basis of two approaches using the polynomial and exponential trend components.

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## References

- [1] Nordhagen H O, Kragset S, Berstad T, Morin A, Dorum C, Munkejord S T *Computers and Structures*, 2012, **94-95** 13-21.
- [2] Oikonomidis F., Shterenlikht A., Truman C. E. *Int. J. of Pressure Vessels and Piping* 2013, **105-106** 60-68.
- [3] Oikonomidis F., Shterenlikht A., Truman C. E. *Int. J. of Pressure Vessels and Piping* 2014 **122** 15-21.
- [4] Demofonti G., Mannucci G., Spinelli C.M., Barsanti L., Hillenbrand H.G. Large-diameter X 100 gas line pipes: Fracture propagation evaluation by full-scale burst test, Pipeline Technology, Vol. 1, R. Denys (Editor), Elsevier Science, 2000, 509-520.
- [5] Zhuang Zhuo, O'Donoghue P E *Int. J. of Fracture* 2000 **101** 269-290.
- [6] Ren Z J, Ru C Q *Engineering Fracture Mechanics* 2013 **99** 214-222.
- [7] Mirzaei M, Najafi M, Niasari H *Int. J. of Impact Engineering* 2015 **85** 27-36.
- [8] Lytvynenko I, Maruschak P, Lupenko S, Panin S *Applied Mechanics and Materials* 2015, **770** 288-293.
- [9] Lytvynenko I V *J. of the Ternopil National Technical University* 2016 **84** 131-138.
- [10] Lytvynenko I V, Maruschak P O, Lupenko S A, Popovych P V 2016 *Mat. Sci.* **52** 113-122.