

A Direction of Arrival Estimation Algorithm Based on Orthogonal Matching Pursuit

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Abstract: The results show that the modified DSM is able to predict local buckling capacity of hot-rolled RHS and SHS accurately. In order to solve the problem of the weak ability of anti-radiation missile against active decoy in modern electronic warfare, a direction of arrival estimation algorithm based on orthogonal matching pursuit is proposed in this paper. The algorithm adopts the compression sensing technology. This paper uses array antennas to receive signals, gets the sparse representation of signals, and then designs the corresponding perception matrix. The signal is reconstructed by orthogonal matching pursuit algorithm to estimate the optimal solution. At the same time, the error of the whole measurement system is analyzed and simulated, and the validity of this algorithm is verified. The algorithm greatly reduces the measurement time, the quantity of equipment and the total amount of the calculation, and accurately estimates the angle and strength of the incoming signal. This technology can effectively improve the angle resolution of the missile, which is of reference significance to the research of anti-active decoy.

1. Introduction

With the rapid development of array signal processing technology^[1], the research of high precision direction finding technology such as zero formation, beamforming and spatial spectrum estimation is also developing. And the estimation of spatial spectrum can effectively solve the estimation problem of the direction of Arrive (DOA), which gradually becomes the hotspot of people's research. This paper presents a DOA estimation algorithm based on orthogonal matching pursuit. Through the compression sensing (CS) technology, the algorithm significantly reduce the amount of measurement data, effectively reduce the signal processing time, achieve high-resolution signal acquisition purposes, and effectively suppress the noise interference. And the power spectrum of the signal is estimated to obtain accurate signal azimuth and intensity information.

2. Algorithm Design

2.1 A model of the array antenna receiving signal

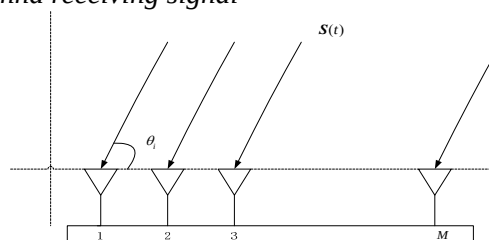


Figure 1 DOA of Array Signal

In this paper, the antenna array^[4,12] is composed of M antennas with the same direction function, and each antenna is arranged in line equidistant, and the distance between two adjacent elements is d . The

signals propagating in the airspace are represented as $N \times 1$ dimensional vectors, that is

$$\mathbf{S}(t) = [s_1(t) \ s_2(t) \ \cdots \ s_N(t)] \quad (1)$$

In the ideal case, the array element of the same direction function does not take into account the influence of the channel gain, and the observation vector of $M \times 1$ dimension is $\mathbf{X}(t)$, that is

$$\mathbf{X}(t) = [x_1(t) \ x_2(t) \ \cdots \ x_M(t)]^T \quad (2)$$

2.2 A Fast Direction Finding Algorithm For Signal Processing

The compression perceptual technique is the sparse representation of the signal through the orthogonal transform base Ψ ^[3]. The algorithm design of a measurement matrix Φ satisfies a non-correlation with Ψ and is sampled at a sampling frequency well below Nyquist to obtain a data measurement containing the main information. Finally, the optimal signal is solved by designing the corresponding signal reconstruction algorithm, which reconstructs the original signal with high probability, extracts the main information contained in the signal, reduces the measurement time, reduces the number of equipment, reduces the total amount of calculation.

When measuring a limited number of radiation sources on a given area Q , the signal received by the seeker satisfies sparsity because the target is located only a few of them at a limited angle. Therefore, according to the need of the seeker resolution^[5], the area is divided into P partition, then $Q \in [\theta_1, \theta_2, \dots, \theta_P]^T$, can be constructed to represent the signal attributes of the vector:

$$\mathbf{Y} = [y_1, y_2, \dots, y_P]^T \quad (3)$$

In Eq.3, an element represents a radiation source, the numerical size represents the energy intensity of the signal, and the ordinal number indicates the partition where the radiation source is located, that is, the angle at which the signal arrives. Because there are only a limited number of targets in the region, that is, there are only L elements with strong energy in $\mathbf{Y}$. Other $P-L$ elements that the region of the noise signal elements, the value is smaller or close to zero, then the signal vector $\mathbf{Y}$ to meet the sparseness.

According to the relationship between the direction of arrival θ_i and the phase difference ϕ_i , the signal in the designated region Q is divided into P partitions by the phase difference, and the signal phase difference corresponding to each partition is

$$\phi_i = \phi_1 + \frac{1}{P}(i-1), \quad i = 1, 2, \dots, P \quad (4)$$

In Eq.(4), $\phi_1 = 2\pi d \sin \theta_1 / \lambda$ is the phase difference of the signal in the first partition, so the signal steering vector for each partition can be expressed as $\mathbf{a}_i(\theta_i) = [1, e^{-2\pi j \frac{d \sin \theta_i}{\lambda}}, \dots, e^{-2\pi j \frac{(P-1)d \sin \theta_i}{\lambda}}]^T$. It can be seen that the steering vectors in the designated area Q satisfy the conditions of orthogonal to each other, that is

$$\mathbf{a}^H(\theta_i) \mathbf{a}(\theta_j) = \begin{cases} 0, & i \neq j \\ P, & i = j \end{cases} \quad (5)$$

At this time there are orthogonal matrix

$$\Psi = 1/\sqrt{P} [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_P)] \quad (6)$$

And it can satisfy $\Psi^H \Psi = \mathbf{I}_{P \times P}$. By means of the orthogonal basis matrix, $\mathbf{Y}$ is sampled much lower than the Nyquist frequency, then the incoming wave signal can be sparsely represented by the signal vector $\mathbf{Y}$, which is equivalent to sampling the P elements of the antenna, that is

$$\mathbf{S} = \Psi \mathbf{Y} \quad (7)$$

Then Ψ is the orthogonal transformation of the signal.

The compression sensing technology^[6] is a way of collecting the main information from the less data by linear measurement, which effectively avoids the cumbersome process of sampling the high sampling rate and greatly shortens the signal processing time. For a homogeneous array antenna, the

advantage of a direction of arrival estimation algorithm based on orthogonal matching pursuit is to measuring the target region with less number of elements M as much as possible.

In general, the signal is linearly measured with a $M \times P$ dimensions of the measurement matrix Φ so that its dimension is reduced. To make the incoming wave signal S from the M measurements in the signal reconstruction, get the main data transmission, but also need to meet the conditions, that is, the measurement matrix Φ and orthogonal transform base Ψ is not related. And because Ψ is known, the measurement matrix Φ can satisfy the condition that the orthogonal transformation base Ψ is not similar. Therefore, the reception matrix A with the M elements is a row full rank matrix and $A \in \mathbf{D}^{M \times P}$, thus, A and Ψ are still relevant. At this time, using the two matrices $B \in \mathbf{D}^{M \times M}$ and $C \in \mathbf{D}^{P \times P}$ satisfying the Gaussian white noise distribution and performing random perturbations on the receiving matrix, the measurement matrix Φ can be expressed as

$$\Phi = BAC \quad (8)$$

The measurement matrix plays an important role in compression perception, and the correct selection of the appropriate measurement matrix contributes to the accurate reconstruction of the signal. The more commonly used measurement matrix is the Gaussian stochastic measurement matrix. In this kind of measurement matrix, all nonzero elements are independent and identically distributed random variables, most of which are not related to sparse signals, and the number of measurements required for reconstruction is relatively small, which is easier to meet the requirements of compression perception.

From Eq.(8) we can see that the measurement matrix and the orthogonal transform base satisfy the condition of dissimilarity, so

$$X = \Phi S = \Phi \Psi Y = \Theta Y \quad (9)$$

In Eq.(9), $\Theta = \Phi \Psi$ is the $M \times P$ dimensions perceptual matrix. Signal vector Y through the orthogonal transformation base, linear measurement projection, you can express the output observation vector X . The signal reconstruction algorithm is used to optimize the solution, and the partition of the L target radiation sources in Y is obtained, and the direction of the wave signal is estimated by the sequence number of the partition.

Signal reconstruction is the process of reconstructing the P dimensions signal vector Y from the observed data vector X of the M dimensions, which is also the most essential part of the compression-sensing technique. When the noise is not present, the sparse vector can be reconstructed by solving the optimization problem, that is

$$\min \|y\|_0 \text{ s.t. } X = \Theta Y \quad (10)$$

In Eq.(10), $\|y\|_0$ is the zero norm of y , indicating the number of nonzero elements in y . However, in the modern electronic battlefield, the signal environment is complex, noise interference is inevitable, if the noise variance is ε , then the optimization problem can be expressed as

$$\min \|y\|_0 \text{ s.t. } \|X - \Theta Y\|_2^2 \leq \varepsilon \quad (11)$$

The process of solving the optimization problem is the process of the algorithm. In this paper, Orthogonal Matching Pursuit (OMP) algorithm^[8] is used to reconstruct the signal. The algorithm chooses a column to deal with the principle of Gram-Schmidt, which ensures the optimality of the iteration. Then the signal in the composition of these orthogonal basis of the space projection, obtained signal components and residuals.

The algorithm steps are as follows:

Enter: $M \times P$ dimensions perception matrix Θ , $M \times 1$ dimensions observation vector X , sparse degree L ;

(1) Initialization: residual $r_0 = S$, $\hat{X}_0 = 0$, index set $A_0 = \emptyset$, number of iterations $i = 1$;

(2) Take the product of the residual r and the perceptive matrix ϕ_j , and obtain the corresponding

index λ of the elements with the largest absolute value: $\lambda_i = \arg \max_{j=1,2,\dots,P} |\langle r_{i-1}, \phi_j \rangle|$;

(3) Update index set $\mathcal{A}_i = \mathcal{A}_{i-1} \cup \{\lambda_i\}$, update the set of atoms in the perception matrix $\boldsymbol{\Theta}_i = [\boldsymbol{\Theta}_{i-1}, \boldsymbol{\varphi}_i]$;

(4) The approximate solution is estimated by LS $\hat{\mathbf{X}}_i = \arg \min_i \|\mathbf{S} - \boldsymbol{\Theta}_i \hat{\mathbf{X}}_i\|_2$, update the residual $\mathbf{r}_i = \mathbf{S} - \boldsymbol{\Theta}_i \hat{\mathbf{X}}_i$, $i = i + 1$;

(5) If $i > L$, the iteration stop, else implementation step (2).

The algorithm is actually to calculate the signal vector $\mathbf{Y}$ in the L strong energy components and the corresponding serial number position, and the remaining $P - L$ signal components as zero ignored, the most critical is that the algorithm to ensure that each time The iteration is optimal, reducing the number of iterations required for convergence. The most critical is that the algorithm ensures that each iteration is optimal, reducing the number of iterations required for convergence. At the same time, the OMP algorithm can calculate the power of L strong energy in the signal vector to complete the estimation of the power spectrum of the radiation source signal. In summary, the OMP algorithm has certain advantages in computational complexity and accuracy, and can reconstruct the original signal with high probability, and the algorithm is much faster than the norm. In this paper, the simulation experiments are carried out using OMP algorithm for signal reconstruction.

3. Simulation verification

According to the spatial spectrum estimation scheme described above, the simulation is carried out to verify its feasibility. So that the length of the signal is 256, the size of the measured number M is 64. The designated area with the observation range $[0^\circ, 60^\circ]$ is divided into 100 divisions, and 10 signals are randomly distributed in the region, that is, the coefficient vector $\mathbf{Y}$ satisfies the sparseness of $L = 10$. The following is a single signal reconstruction as shown below.

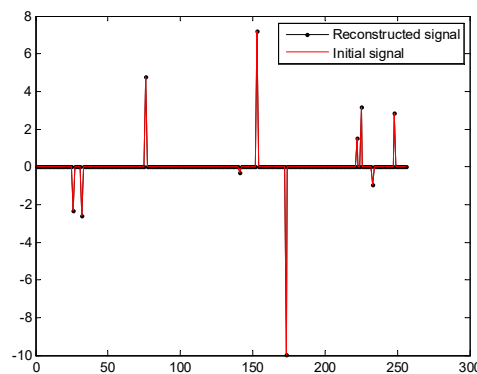


Figure 2 Single signal reconstruction

It can be seen from Figure 2, using the algorithm for signal reconstruction, with a lower sampling rate can be high probability to restore the main information signal. At the same time, the algorithm only collects L strong energy elements, and the other $P - L$ noise signal elements in this region are neglected due to the smaller ones, thus effectively avoiding the noise effect in the space and have some noise suppression function^[6,7]. It can be seen that the technology is feasible.

From the previous analysis of the algorithm, we can see that the number of antenna elements must meet $M \geq L \log_2(P/L)$, to ensure the optimality of multiple iterations, and thus to complete the signal reconstruction of great probability. It can be seen that the setting of the measurement number is quite critical to the whole algorithm, which directly affects the error of the spatial spectrum estimation.

The standard deviation of the angle estimation error in the simulation experiment is:

$$\mu = \sqrt{\frac{\sum_{i=1}^L \left(\frac{\hat{\theta}_i - \theta_i}{\theta_i} \right)^2}{L}} \quad (12)$$

Let the length S of signal N be 256 and the signal to noise ratio is $\mu = 20\text{dB}$. The designated area of the observation range $[0^\circ, 60^\circ]$ is divided into 40 divisions, and five signals are randomly distributed in the region, that is, the vector Y satisfies the sparseness of $L = 5$. The error of ESPRIT algorithm, MUSIC algorithm and CS algorithm^[9,10] is compared to analyze the influence of array element M on algorithm.

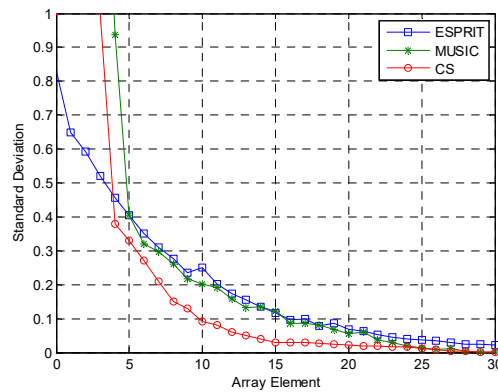


Figure 3 Estimation of the relationship between standard deviation and array element

As can be seen from Fig. 3, with the increase of the number of elements, the estimated standard deviation of the three algorithms is gradually reduced and the estimation accuracy is improved. By the above analysis of the algorithm, we can see that the number of elements must meet the conditions of $M \geq 15$ to ensure the optimal estimate of the above figure to verify this conclusion. Under the same conditions, the algorithm proposed in this paper has higher accuracy than ESPRIT algorithm and MUSIC algorithm.

When the number of elements is 25, the error of ESPRIT algorithm, MUSIC algorithm and CS algorithm is compared to analyze the influence of signal to noise ratio on the algorithm.

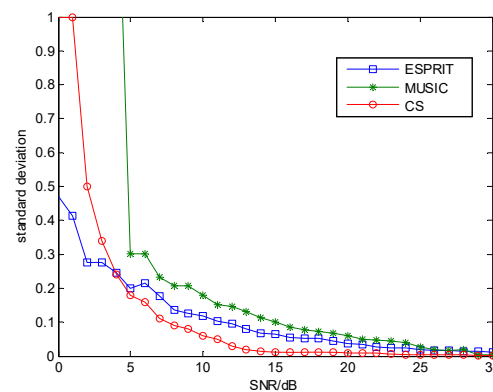


Figure 4 Estimation of the relationship between standard deviation and SNR

As shown in Fig. 4, the estimated standard deviation of ESPRIT algorithm, MUSIC algorithm and CS algorithm decreases with the increase of signal-to-noise ratio. When the SNR is greater than 14dB, the standard deviation of the CS algorithm is less than 0.012 and is decreasing, and the measured angle is quite correct. When the SNR is less than 7dB, the standard deviation of the CS algorithm is larger than 0.11 and increases. When the SNR is equal to 3, the estimated standard deviation is 0.34 and the

error is large. So the algorithm can not be used to estimate accurately. This is due to the signal power and noise power gradually approaching the size of the use of OMP has been unable to distinguish between signal and noise. It can be seen from the figure that under the same conditions, the standard deviation of the CS algorithm proposed in this paper is much lower than that of the other two algorithms and has higher accuracy. It can be seen that the CS algorithm has a certain noise suppression ability, can greatly improve the accuracy of parameter estimation.

In order to verify the effect of sparseness and measurement on the success rate, the length of the incoming wave signal is 256, so that the number of antenna elements is [50,99,152,204,250], and the number of elements is tested every 5 pairs of signal sparsity. As shown in Fig. 7. And the signal sparseness is [4,12,20,28,36], the signal sparsity is tested once every 5 pairs of elements, without all traversing, as shown in Fig. 8. Repeat iteration for 1000 times for each set of signal lengths, array elements, and sparseness^[11,15].

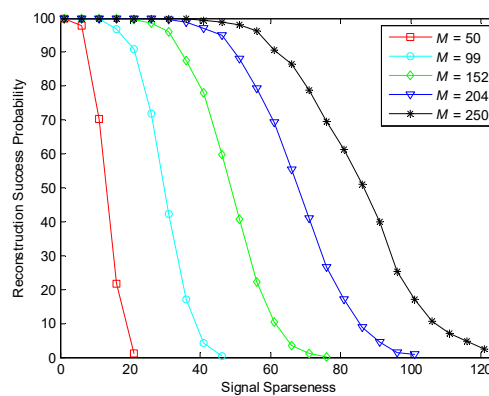


Figure 5 Signal Sparseness and Reconstruction Success Probability

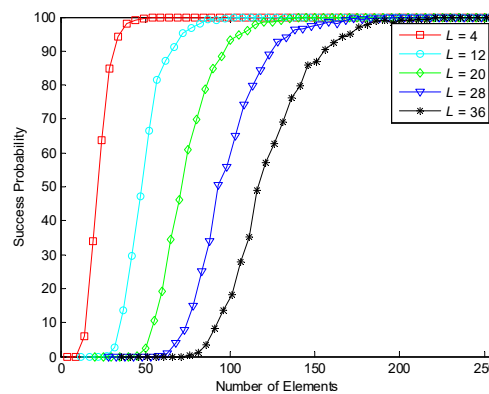


Figure 6 Number of Elements and Reconstruction Success Probability

It can be seen from Fig. 5 that when the number of elements M is constant, the signal reconstruction success rate decreases with the increase of signal sparseness L .

It can be seen from Fig. 6 that when the signal sparseness L is constant, the probability of signal reconstruction success increases with the increase of the number of elements M .

In summary, the signal sparseness and the number of elements have a direct impact on the success rate of signal reconstruction. After several experiments, it is proved that the probability of reconstruction is ideal when the signal reconstruction is carried out by using OMP. When the number of elements, signal sparseness and the total number of partitions satisfy $M \geq L \log_2(P/L)$.

4. Conclusion

In this paper, the compressed sensing technology is used to establish the spatial spectrum estimation model, and the orthogonal matching pursuit algorithm is used to reconstruct the signal accurately, and the optimal estimation value is obtained. The algorithm has a good noise suppression function, can greatly reduce the amount of required equipment, calculation of total savings, to a certain extent, the signal processing time, high speed missile rapid processing of signal. Simulation analysis shows that the estimation error and reconstruction success rate of the algorithm are greatly affected by the number of elements, and the number of array elements can effectively improve the performance of the algorithm. The application of compressed sensing in signal processing is the focus of current research. The speed of signal processing in this paper still needs to be strengthened, and further research is needed.

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