Copula Entropy coupled with Wavelet Neural Network Model for Hydrological Prediction

Yin Wang[1]**, JiGuang Yue**[1]**, ShuGuang Liu**[2] **and LiWang**[1]

[1]School of Electronics and Information Engineering, Tongji University, Shanghai 201804,

[2]School of Civil Engineering, Tongji University, Shanghai 201804,China,

Correspondence: tj_wangyin@163.com, Tel.: +86-131-6219-9852

Abstract. Artificial Neural network(ANN) has been widely used in hydrological forecasting. in this paper an attempt has been made to find an alternative method for hydrological prediction by combining Copula Entropy(CE) with Wavelet Neural Network(WNN), CE theory permits to calculate mutual information(MI) to select Input variables which avoids the limitations of the traditional linear correlation(LCC) analysis. Wavelet analysis can provide the exact locality of any changes in the dynamical patterns of the sequence Coupled with ANN Strong non-linear fitting ability. WNN model was able to provide a good fit with the hydrological data. finally, the hybrid model(CE+WNN) have been applied to daily water level of Taihu Lake Basin, and compared with CE ANN, LCC WNN and LCC ANN. Results showed that the hybrid model produced better results in estimating the hydrograph properties than the latter models.

1. Introduction

A hydrological forecast is the estimation of future states of hydrological phenomena. They are essential for the efficient operation of water infrastructure and the mitigation of natural disasters such as floods and droughts. Combined with the current researches in hydrological prediction at home and abroad, neural networks have been widely used in hydrological forecasting [1]. There are two key problems in using hydrological network for hydrological forecasting. (1) Predictors (choice of input variables), The most commonly used method at present is linear correlation coefficient method, but it must meet two conditions, one is the correlation between two variables Must be linear; Second, the variable must be subject to multivariate normal distribution, so there are some limitations. Another method is the mutual information method, which represents the amount of information about another random variable contained in a random variable. (2) The neural network has some shortcomings such as the local minima caused by non-uniqueness of network searching and the slow convergence rate of training times due to the randomness of given initial weights. Hydrological sequences, as the output of watershed system, exhibit highly complicated characteristics such as highly nonlinearity and multi-time scale features, making it very urgent to further improve traditional neural networks. Wavelet transform is a signal time-frequency analysis method with good function of time, frequency and multiresolution. Through the multi-scale analysis of time series, it can effectively identify the main frequency components and extract the local information. Therefore, it is a powerful tool to process hydrological time series [2-4].

In this paper, the wavelet neural network based on Copula entropy is proposed. The Copula entropy is used to calculate the mutual information to select the forecasting factor. The wavelet neural network is used as the forecasting model. Finally, the method is applied to the hydrological forecast in Taihu

Lake Basin, and the rationality of the method is verified through comparative experiments.

2. Copula Entropy theory

Copula entropy combines Copula function with entropy theory, measures the correlation between variables by the entropy of Copula function, and Copula function and entropy theory are two kinds of multivariable and nonlinear hydrological analysis tools that can comprehensively consider the correlation between variables [5]. At present, it has been widely used in hydrology. Copula entropy is constructed by a combination of Copula function and information entropy theory. It expands the theory of information entropy and replaces the unmanageable joint entropy and mutual information by the combination of one-dimensional entropy function and Copula function. Copula entropy combines Copula function And the advantages of information entropy theory.

2.1. Shannon entropy

In 1948 Shannon introduced the Boltzmann concept of entropy into information theory and used entropy as a measure of the uncertainty or amount of information of a random event. Therefore, the size of the information can be used to eliminate the uncertainty of how much, and the size of the random event uncertainty can be used to describe the probability distribution function [6].

Most hydrological variables are continuous, such as rainfall, water level and so on. Therefore, this paper focuses on the entropy of continuous random variables. For continuous variables, Shannon's information entropy can be expressed as:

$$H(x) = -\int_a^b f(x) \ln f(x) dx \tag{1}$$

2.2. Copula

One method of modelling dependencies which has become very popular recently is the copula. The word copula is a Latin noun which means 'a link, tie or bond', and was first employed in a mathematical or statistical sense by Sklar [7].

Sklar's theorem, which is the foundation theorem for copulas, states that for a given joint multivariate distribution function and the relevant marginal distributions, there exists a copula function that relates them. In a bi-variate setting: Mathematically, a copula is a function which allows us to combine univariate distributions to obtain a joint distribution with a particular dependence structure.

(Sklar's theorem) Let F_{xy} be a joint distribution with margins F_x and F_y. Then there exists a function $C:[0,1]^2 \to [0,1]$, such that：

$$F(x_1, \cdots, x_n) = C(F(x_1), \cdots, F(x_2)) \tag{2}$$

If X and Y are continuous, then C is unique; otherwise, C is uniquely determined on the (range of X) x (range of Y). Conversely if C is a copula and F_y are distribution functions, then the function F_{xy} defined by 2 is a joint distribution with margins F_x and F_y.

2.3. Copula entropy

Let X_1, X_2 be random variables with marginal functions $F(x_1)$, $F(x_2)$, and $U_1 = F(x_1)$, $U_2 = F(x2)$. Then U_1 and U_2 are uniformly distributed random variables; and u_1 and u_2 will denote a specific value of U_1 and U_2, respectively. We define the entropy of copula function as CE in this study, which can be expressed as:

$$H_c(U_1, U_2, \cdots, U_d) = -\int_0^1 \cdots \int_0^1 c(u_1, u_2, \cdots, u_d) \log \int_0^1 c(u_1, u_2, \cdots, u_d) du_1 du_2 \cdots du_d \tag{3}$$

where $c(u_1, u_2, \cdots, u_d)$ is the probability density function of copulas and is expressed as

$\dfrac{\partial C(u_1, u_2, \cdots, u_d)}{\partial u_1 \partial u_2 \cdots \partial u_d}$ [8-9].
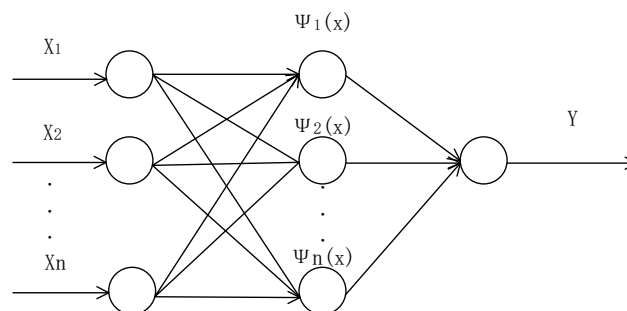
3. Wavelet neural network

Neural network is a very superior nonlinear function approximation method, which can describe any complicated nonlinear process and has been used successfully in many fields of nonlinear time series model [10]. However, there are some shortcomings of neural network, such as the local minimum caused by non-unique network optimization and the slow convergence rate of training times due to the randomness of given initial weights. Wavelet transform is a signal time-frequency analysis method with good function of time, frequency and multiresolution. Through the multi-scale analysis of time series, it can effectively identify the main frequency components and extract the local information [11]. Therefore, it is a method to process hydrological time series Signal powerful tool. Based on the good time-frequency localization of wavelet transform and the combination of some excellent features of neural network, the two neural networks are combined to form a wavelet neural network to inherit and amplify the superiority of the former two and make up for the other at the same time, Which makes the wavelet neural network has been widely developed and applied.

Wavelet neural network is based on BP neural network topology structure, using wavelet neurons instead of neural network neurons, namely regarding the wavelet basis function as the transfer function of hidden layer nodes [12].

Defining $x_1, x_2, \cdots, x_n$ as the input parameters of wavelet neural network, namely the input neurons; $y_1, y_2, \cdots, y_n$ respectively as the prediction output value of wavelet neural network; g(x) as the wavelet basis function; w_{ij} and w_{jk} as the wavelet neural network weights [13-14].

When the sequence of the input signal is $x_1 = (1, 2, \cdots, k)$, the output calculation formula of the hidden layer is as follows[15]:

$$h(j) = g\left[\left(\sum_{i-1}^{k} w_{ij} x_i - b_j\right) / a_j\right] \tag{4}$$
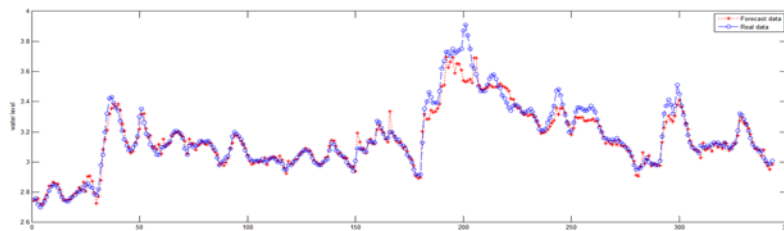

Figure 1. Wavelet neural network structure

## 4. Application

4.1. Study area and data

The data of daily Rainfall and water level in Fenhuang Station, Lintou Station, Xingqiao Station, Jingdong Station, Hehua Station and flood season from 2005 to 2013 in Taihu Lake Basin were used to calculate. The measured daily Rainfall and water level of Fenhuang Station, LinTou Station, XingQiao Station, Jingdong Station and Hehua Station for the first ten days are taken as possible input variables, and the water level of Fenhuang Station at time t is taken as the output variable.

*4.2. Comparison of predicted results with different input sets*

To forecast the water level at Fenhuang gauging station of Taihu River, the daily Rainfall and water level data of 10 years were used. The first Nine years (2005-2013) data were used for calibration of the model, and the remaining One years (2013) data were used for validation. Two input sets were used for establishing the hydrological model, one of which was obtained by the LCC method and the other by the CE method. ANN and WNN were used for forecasting water level. The forecasting performances are shown as follows. It can be seen that low RMSE values for WNN+CE models when compared to other models.
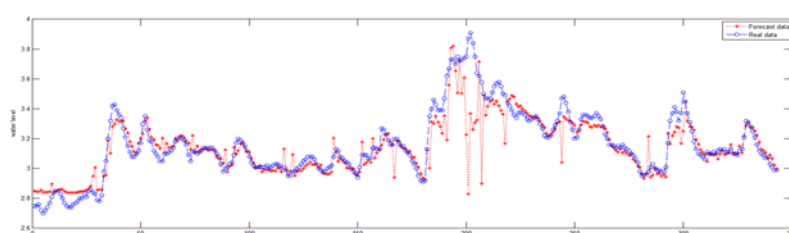

Figure 2. CE WNN model
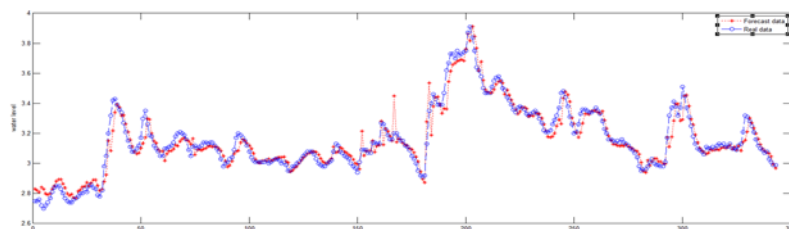
Figure 3. LCC WNN model
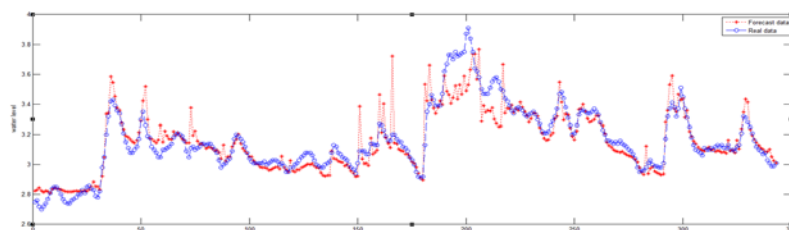
Figure 4. CE BP model

Figure 5. LCC BP model

5. Conclusions

Based on Copula entropy and wavelet neural network, this paper establishes a hydrological prediction model to adapt to multi-time series data in hydrology field. Compared with the traditional correlation coefficient method, the Copula entropy theory is introduced into the method of hydrological forecasting factor selection, which has the advantages of fast calculation, good scalability and application value. The wavelet neural network not only inherits the good time-frequency localization characteristics of wavelet transform, but also fully analysis hydrological variation rules and obtains the general trend and detail information in the process of variation, and possesses the self-learning function of neural network

and extremely strong nonlinearity Fitting ability and other advantages [16-17]. Based on the Copula entropy and the traditional linear correlation method as the predictor selection method, the BP neural network and the wavelet neural network are used as the forecasting models. The hydrological forecast is carried out for the Phoenix station of Shengze town in the Taihu Lake basin. The prediction results show that the wavelet neural network based on the Copula entropy Model forecasting accuracy is the highest, which can provide effective information for proper flood control decision-making [18-20].

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