

# Study on Fluid-solid Coupling Mathematical Models and Numerical Simulation of Coal Containing Gas

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**Abstract.** Based on coal seam gas migration theory under multi-physics field coupling effect, fluid-solid coupling model of coal seam gas was build using elastic mechanics, fluid mechanics in porous medium and effective stress principle. Gas seepage behavior under different original gas pressure was simulated. Results indicated that residual gas pressure, gas pressure gradient and gas low were bigger when original gas pressure was higher. Coal permeability distribution decreased exponentially when original gas pressure was lower than critical pressure. Coal permeability decreased rapidly first and then increased slowly when original pressure was higher than critical pressure.

## 1. General instructions

Coal seam gas seepage under multi-physics field coupling effect is one of the hot topics of gas migration researches. It has great significance to mine gas disaster prevention and improving gas extraction efficiency. At present, many researchers has studied fluid-solid coupling of coal seam gas seepage based on temperature and crustal stress<sup>[1-4]</sup>. But these researches didn't take enough influencing factors into consideration. Especially the influence of crustal stress, gas pressure and adsorption swelling stress on gas seepage needs to be studied further. So, based on influence of crustal stress, gas pressure and adsorption swelling stress on gas seepage, fluid-solid coupling model of coal seam gas was built and gas migration behavior under gas extraction was obtained using numerical simulation, in the hope of providing guidance to gas extraction and gas disaster prevention in deep coal seam.

In order to establish the model to make the following basic assumptions: (1) That gas containing coal is isotropic linear elastic body and elastic small deformation obeys the generalized Hooke's law. (2) Fluid in coal seam is single-phase fluid and saturated, the gas is ideal gas and gas adsorption obeys modified Langmuir adsorption equation. (3) Gas flow in coal seam obeys the Darcy law.

## 2. Dynamic permeability equation

### 2.1. Porosity equation of coal

According to definition of porosity  $\phi$ <sup>[5]</sup>, porosity equation expressed by bulk strain can be obtained.

$$\phi = 1 - \frac{1 - \phi_0}{1 + \varepsilon_V} \left( 1 + \frac{\Delta V_s}{V_{s0}} \right) \quad (1)$$



Where,  $\varphi$  is currently porosity,  $\varphi_0$  is original porosity,  $\Delta V_s$  is volume change quantity of coal skeleton,  $V_{s0}$  is original volume of coal skeleton,  $\varepsilon_v$  is volumetric strain (volume expansion is positive and volume compression is negative).

Change of matrix block volume strain  $\Delta V_s/V_{s0}$  is formed by 3 parts.

$$\frac{\Delta V_s}{V_{s0}} = -C_s \Delta p + \frac{\Delta \varepsilon_p}{1 - \varphi_0} \quad (2)$$

Where,  $C_s$  is compression coefficient of coal skeleton,  $\Delta p$  is gas pressure variation,  $\Delta p = p - p_0$ ,  $p_0$  is original gas pressure,  $p$  is current gas pressure,  $\Delta \varepsilon_p$  is adsorption expansion dependent variable of unit volume.

Expansion dependent variable caused by gas pressure can be expressed as follow<sup>[6]</sup>.

$$\Delta \varepsilon_p = \frac{2a\rho RT(1-2\nu)}{EV_m} \ln(1+bp) \quad (3)$$

Where,  $E$  is elastic modulus,  $\nu$  is poisson ratio,  $\rho$  is apparent density,  $V_m$  is molar volume,  $a$ ,  $b$  adsorption constant,  $R$  is gas content.

## 2.2. Effective stress equation

Relation between effective stress and volume strain of gas contained coal obeys the generalized Hooke's law. As coal body is always affected by stress and gas pressure, volume strain can be expressed when crustal stress and gas pressure changed.

$$\varepsilon_v = \frac{\sigma'_m}{K} = \frac{2a\rho RT(1-2\nu)^2}{EV_m} \ln(1+bp) - \frac{1-2\nu}{E} (\sigma - \sigma_0) + \frac{3(1-2\nu)\alpha_p}{E} \Delta p \quad (4)$$

Where,  $\sigma$  is current stress,  $\sigma_0$  is original stress,  $\alpha_p$  is pore pressure coefficient,  $\sigma'_m$  is average effective stress,  $K$  is bulk modulus,  $K = E / [3(1-2\nu)]$ .

## 2.3. Dynamic permeability equation

According to relation between coal permeability and porosity in Kozeny-Carman equation<sup>[7]</sup>, coal permeability equation under multi-physics field coupling effect can be obtained using equation (1), (2), (3) and (4),

$$k = k_0 \left( \frac{1}{\varphi_0} \right)^3 \left[ 1 - \frac{(1-\varphi_0)(1-C_s \Delta p) + \Delta \varepsilon_p}{1 + \varepsilon_v} \right]^3 \quad (5)$$

Where,  $k_0$  is original permeability,  $k$  is permeability after change.

## 3. Fluid-solid coupling mathematical model of gas contained coal

### 3.1. Deformation field equation of coal skeleton

**3.1.1. Equilibrium equation** According to effective stress theory and ignoring free gas seepage, inertia force of coal deformation and body force of gas, differential equation of mechanical equilibrium of gas contained gas can be expressed as follow,

$$\sigma'_{ij,j} + (\alpha p \delta_{ij})_{,j} + F_i = 0 \quad (6)$$

**3.1.2. Geometric equation** Coal deformation obeys the generalized Hooke's law and the follow equation can be obtained,

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (i, j = 1, 2, 3) \quad (7)$$

**3.1.3. Constitutive equation** Total strain of gas contained coal is composed by strain caused by crustal stress, strain caused by gas pressing coal body and strain caused by coal adsorption expansion, that is,

$$\varepsilon = \frac{1}{2G} \left( \sigma' - \frac{\nu}{1+\nu} \Theta' \right) + \frac{C_s}{3} p - \frac{2a\rho RT(1-2\nu)}{3EV_m} \ln(1+bp) \quad (8)$$

Using Lamé constant in equation (8), the equation is reorganized and expressed by tensor,

$$\sigma_{ij} = \lambda \varepsilon_{ij} + 2G \varepsilon_{ij} + \alpha_{PY} p \delta_{ij} + \alpha_{PX} aT \ln(1+bp) \delta_{ij} \quad (9)$$

Where,  $\lambda$ ,  $G$  is Lamé constant,  $\alpha_{PY}$  and  $\alpha_{PX}$  is stress coefficient caused by gas pressure and gas adsorbing,  $\lambda = \frac{2G\nu}{(1-2\nu)}$ ,  $\alpha_{PY} = 1 - \frac{(3\lambda + 2G)C_s}{3}$ ,  $\alpha_{PX} = \frac{2\rho R(1-2\nu)(3\lambda + 2G)}{3EV_m}$ ,  $G = \frac{E}{2(1+\nu)}$ . Equation (9) is constitutive equation of gas contained coal expressed by strain.

**3.1.4. Stress field equation** Putting constitutive equation (9) expressed by strain into equilibrium equation (6) and equation (6) can be reorganized and expressed by tensor,

$$Gu_{i,jj} + \frac{G}{1-2\nu} u_{j,ji} + \alpha_{PY} p_{,i} + \alpha_{PX} [aT \ln(1+bp)]_{,i} + F_i = 0 \quad (10)$$

Equation (10) is fluid-solid coupling stress field equation of gas contained coal.

### 3.2. Seepage field equation

Gas flow obeys the Darcy law and coupling seepage field equation considering terms of source and sink can be obtained by ignoring body force of gas,

$$\nabla \cdot (\rho_g v_g) + \frac{\partial X}{\partial t} = q \quad (11)$$

Where,  $\rho_g$  is gas density,  $v_g$  is absolute speed of gas flow,  $X$  is gas content in coal seam.

Gas content in coal can be calculated by following equation,

$$X = \left( \frac{abp}{1+bp} e^{n(t_0-t)} B + \varphi \frac{p}{p_n} \right) \cdot \rho_n \quad (12)$$

Where,  $e$  is base of natural logarithm,  $e=2.718$ ,  $T_0$  is temperature of adsorption constant testing lab,  $T$  is coal temperature,  $n$  is coefficient which can be calculated by  $n = 0.02/(0.993+0.07p)$ ,  $B$  is coefficient,  $B = \frac{\gamma_m}{1+0.31W} \cdot \frac{100-A-W}{100}$ ,  $A$ ,  $W$  is ash and water content, %,  $\gamma_m$  is coal bulk density,  $p_n$  is atmospheric pressure,  $\rho_n$  is density under atmospheric.

Putting equation (12) into equation (11) and the equation (11) can be shorten as follow,

$$\left[ 2\varphi + \frac{2abBp_n}{(1+bp)^2} \right] \frac{\partial p}{\partial t} + 2p \frac{\partial \varphi}{\partial t} - \nabla \cdot \left( \frac{k}{\mu} \nabla p^2 \right) = q \quad (13)$$

Porosity change  $\frac{\partial \varphi}{\partial t}$  in gas fluid-solid coupling problem can be expressed by following equation<sup>[8]</sup>,

$$\frac{\partial \varphi}{\partial t} = \left( 1 - \frac{C_s}{C_f} \right) \frac{\partial \varepsilon_v}{\partial t} + C_s (1 - \varphi) \frac{\partial p}{\partial t} \quad (14)$$

Where, is whole compression coefficient of coal body and  $C_f = \frac{3(1-2\nu)}{E}$ .

Putting equation (14) into equation (13) and fluid-solid coupling field equation of gas contained coal can be expressed by

$$\left[ 2\varphi + \frac{2abBp_n}{(1+bp)^2} \right] \frac{\partial p}{\partial t} + 2p \left( 1 - \frac{C_s}{C_f} \right) \frac{\partial \varepsilon_v}{\partial t} + 2p C_s (1 - \varphi) \frac{\partial p}{\partial t} - \nabla \cdot \left( \frac{k}{\mu} \nabla p^2 \right) = q \quad (15)$$

### 3.3. Definite condition

**3.3.1. Definite condition of coal skeleton deformation field** Boundary conditions, surface force is known and  $\sigma = F_i$ .  $F_i$  is distribution function of coal surface force. Internal boundary namely borehole wall is free boundary.

$$\text{Initial condition is } u|_{t=0} = \bar{u} \text{ or } \left. \frac{\partial u}{\partial x_i} \right|_{t=0} = F_i.$$

**3.3.2. Definite condition of seepage field** Boundary condition: external boundary is  $p=p_0$ , internal boundary is  $p=P$ ,  $P$  is extraction pressure in borehole. Original condition is  $p=p_0$  when  $t=0$ .

## 4. Numerical simulation of coal gas seepage under extraction

### 4.1. Numerical simulation model

Software in this simulation is automatic generated system (FEPG) by finite element program. Based on gas borehole extraction characteristics, numerical simulation model is build, shown in Figure 1. Radius of borehole is 0.1m, gas seepage area radius is 3m. Unite type in computational domain is quadrangle and unit around borehole is concentrated. There are 1578 units in this model (shown in Figure 1). Calculation time is 10 days that is  $t=864000s$  and time step is  $\Delta t=8640s$ .

Basic parameters in this simulation are from 3# coal seam in Wuyang Coal mine Lu'an Group. These parameters are shown in Table 1.

**Table 1.** Coal mechanical parameters and gas seepage parameters of 3# coal seam in Wuyang Coal mine Lu'an Group

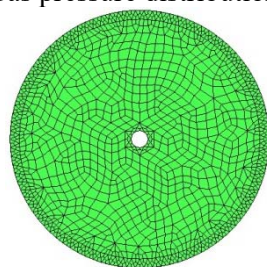
parameters	$\rho/t \cdot m^{-3}$	$E/MPa$	$\sigma_0/MPa$	$a/m^3 \cdot t^{-1}$	$b/MPa^{-1}$	$\varphi_0$	$k_0/10^{-3} \mu m^2$	$p_0/MPa$
value	1.24	150	10	20	1.08	0.06	4.8	0.5
parameters	$\nu$	$W/\%$	$A/\%$	$p_n/MPa$	$R/J \cdot mol^{-1} \cdot K^{-1}$	$\rho_n/kg \cdot m^{-3}$	$T_0/^\circ C$	$C_s/MPa^{-1}$
value	0.3	1	10	0.1	8.314	0.059	30	0.001

In this simulation, external boundary stress in coal deformation field is  $\sigma_0 = 10MPa$ . Internal boundary is free boundary. Original displacement is  $u|_{t=0} = 0$ . External boundary of gas seepage field defined as gas pressure boundary which is the same with original gas pressure. In internal boundary, gas pressure in borehole is  $P=0.1MPa$ . In order to have gas seepage behavior under different gas pressure, original gas pressure of gas seepage field is set as 0.5MPa, 1MPa, 1.5MPa, 2MPa and 2.5MPa.

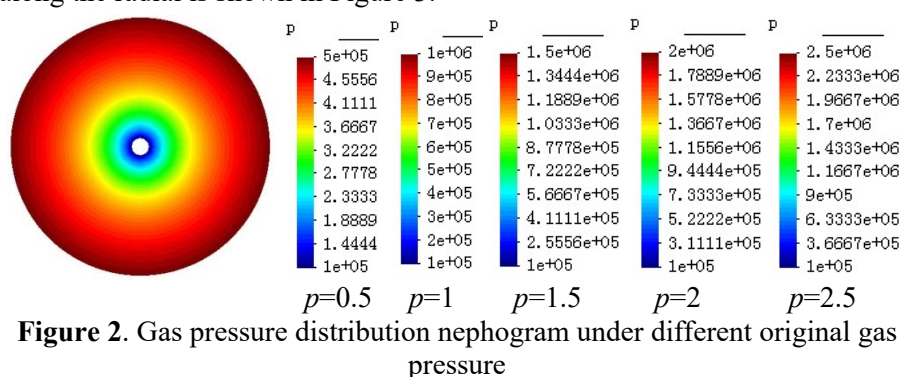
### 4.2. Numerical simulation results analysis

#### (1) Gas pressure distribution

Gas pressure distribution nephogram under different original gas pressure is shown in Figure 2. Gas pressure distribution along the radial is shown in Figure 3.



**Figure 1.** Numerical simulation model and meshing



**Figure 2.** Gas pressure distribution nephogram under different original gas pressure

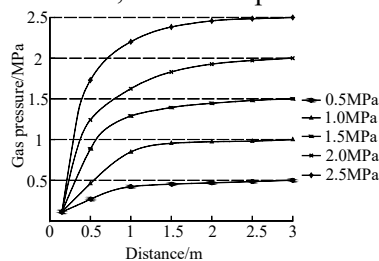
These figures showed that residual gas pressure is bigger with bigger original gas pressure when distance from extraction borehole is the same. Gas pressure gradient is bigger with longer distance. The bigger original gas pressure is, the easier the gas pressure gradient is going to forming. The gas pressure gradient is bigger with nearer distance from exposed coal wall at face. These explain coal and rock outburst tendency around face and provide a direction for outburst prevention. That is stress relief slot, deep hole standing shot and gas extraction through borehole. These measures can decrease gas pressure gradient and increase width of pressure relief zone, so then outburst can be prevented.

### (2) Gas flow distribution

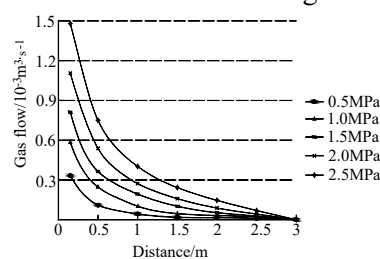
Gas flow distribution along the radial is shown in Figure 4. This figure showed that gas flow is bigger with higher original gas pressure. When gas flow is bigger, gas is more easily to gather as exposed coal wall, higher gas concentration is more easily to appear and gas concentration overrun accidents are easily to happen.

### (3) Permeability distribution

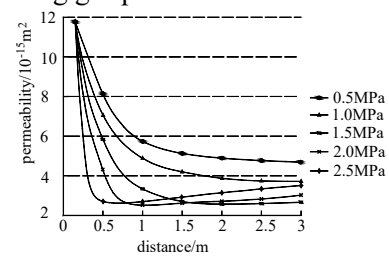
Permeability distribution along the radial is shown in Figure 5. This figure showed that permeability decreases exponentially with the distance from borehole increasing when original gas pressure is 0.5MPa, 1MPa and 1.5MPa. Permeability decrease rapidly first and then increase slowly and finally remain stable with distance from borehole increasing when original gas pressure is 2MPa and 2.5MPa. The author considers that there is a critical gas pressure and shrinkage of coal matrix is main effect on permeability when gas pressure is less than critical gas pressure. Pore gas pressure is main effect on permeability when gas pressure is bigger than critical gas pressure. According to calculation, critical gas pressure in this paper is 1.8MPa. In order to get better extraction effect, during extraction, coal seam permeability should be increased through decreasing gas pressure.



**Figure 3.** Gas pressure distribution along the radial



**Figure 4.** Gas flow distribution along the radial



**Figure 5.** Permeability distribution along the radial

## 5. Conclusions

(1) Based on porosity definition, dynamic seepage equation concluding multi influence factors. Using elasticity and fluid mechanics in porous medium, fluid-solid coupling mathematical model of gas contained coal is build.

(2) Residual gas pressure, pressure gradient and gas low get bigger and coal and gas outburst hazard get greater when gas pressure is higher.

(3) There is a critical gas pressure. Permeability decreases exponentially when gas pressure is less than critical gas pressure. Permeability decrease rapidly first and then increase slowly when gas pressure is bigger than critical gas pressure. Decreasing gas pressure is one of the ways to improve extraction efficiency and prevent coal and gas outburst effectively.

## Acknowledgements

This work was financially supported by National Natural Science Foundation of China (Program No. 51404189 and 51404190) and Natural Science Basic Research Plan in Shaanxi Province of China (Program No. 2015JQ5191)

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