

A Novel Methodology for Charging Station Deployment

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Abstract. Lack of charging stations has been a main obstacle to the promotion of electric vehicles. This paper studies deploying charging stations in traffic networks considering grid constraints to balance the charging demand and grid stability. First, we propose a statistical model for charging demand. Then we combine the charging demand model with power grid constraints and give the formulation of the charging station deployment problem. Finally, we propose a theoretical solution for the problem by transforming it to a Markov Decision Process.

1. Introduction

One critical issue holding back the widespread of electric vehicles (EVs) is the scarcity of charging facilities. For example, EVs in ShenZhen had increased by 6958 during 2009-2014, but only 3091 new chargers were constructed during this period. Even so, the utilization of chargers is less than 33%. On the other hand, numerous EVs charging synchronously will shake the power networks seriously. K. Clement-Nyns [1] shows that, when the EV penetration reaches 30%, the uncoordinated EV charging will cause 5% total power loss and 10% voltage deviation. Thus, when deploying charging stations, both the charging demand in traffic networks and the stability in power grid should be considered.

There are some papers focusing on the charging station deployment. L. Feng [2] models the charging demand by a weighted voronoi diagram to minimize the detours. But it only considers the size and location in traffic networks. C. Upchurch [3] and M. Kuby [4] model the charging demand as a flow refueling location model to maximize the captured flows. A. Lam [5] provides a structure for charging station placement including the formulation, complexity and solutions. In spite of the various models and solutions proposed, the impacts on power grid are untouched.

There are also some papers focusing on deploying charging stations in power networks. M. Aghaebrahimi [6], P. Sadeghi-Barzani [7], Z. Liu [8] locate charging stations in distribution systems to minimize costs. But mapping the power network constraints to costs is not proper, because violating the constraints may cause disasters. N. Ariyapim [9] proposes an ant colony optimization method to optimize the charging station locations to minimize the feeder loss in distribution systems. Although these papers study the deployment of charging stations in power networks, none of them consider the spatio-temporal features of charging demand.

2. The system model

The charging demand of EVs has remarkable spatio-temporal characteristics and varies randomly over time. Thus, a statistical model for charging demand is proposed in this section.



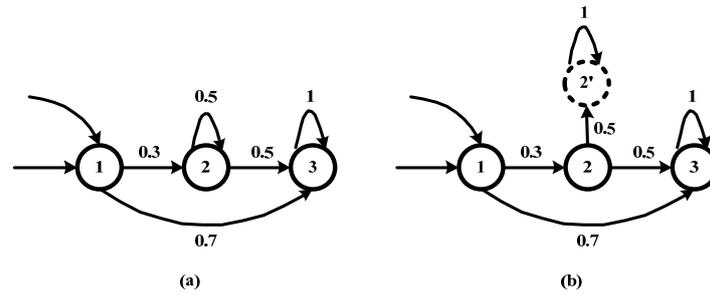


Figure 1. A illustration of graph and the conversion of hybrid node

2.1. The statistic model for charging demand

Let a directed graph $\mathcal{G} = \langle \mathcal{N}, \mathcal{A} \rangle$ denote the traffic network, where $\mathcal{N} = \{n_1, n_2, \dots, n_N\}$ is the set of nodes including the endpoints and crossing points of all roads. \mathcal{A} is the set of road segments connecting two nodes in \mathcal{N} . Typically, a node can play three roles, i.e. source node, transit node and destination node. A source node is the one where EVs will enter the map. A destination node is the one where EVs will stop the trips. A transit node is the one where EVs will certainly visit another node without stopping. A hybrid node is the one that plays more than one role.

For a road segment $(n_i, n_j) \in \mathcal{A}$, let t_{ij} denote the probability that EVs left n_i will visit n_j , here $t_{ij} = 0$ if $(n_i, n_j) \notin \mathcal{A}$. Then, for any n_i , we have $\sum_{j=1}^N t_{ij} = 1$. Let a_i denote the amount of EVs enter the map at n_i . In practice, we can get the knowledge about t_{ij} , a_i with historical traffic statistics. As shown in Fig.1 (a), the weight of an arc denotes the transition probability t_{ij} , node 1 is a transit node, node 2 is a hybrid node and node 3 is a destination node.

This paper aims to find a charging station deployment scheme to maximize the trips captured. A trip is captured iff it passes one or more charging stations. This objective elicits two issues: (1) how to count the trips captured at a hybrid node; (2) how to avoid repeat count, i.e. a trip passes multiple charging stations should be counted as captured only once.

For the first issue, we adopt the following transformation to eliminate the hybrid nodes. For any hybrid node $n_i \in \mathcal{N}$, by adding a redundant node $n_{i'}$ ($n_{i'} \notin \mathcal{N}$), and let $t_{i'i} = t_{ii}$, $t_{i'i'} = 1$ and $t_{ii} = 0$, we can eliminate a hybrid node from the map. With this step, n_i becomes a transit node and the new $n_{i'}$ is a destination node. By this transformation, we can obtain a *pure* map without hybrid node. Fig.1 (b) is the pure version of Fig.1 (a). The remaining parts of this paper will base on such a *pure* map. We use \mathcal{N}_p to denote the set of nodes of the pure map.

For the second issue, we adopt the following approach to avoid repeat count. If a node n_i is selected to deploy a charging station, then we set n_i as a “new” destination node by assigning $t_{ii} = 1$ and $t_{ij} = 0, \forall j \neq i$. It is obvious that, by doing so, any trip that passes n_i will not be captured by other node. Finally, the summation of the trips that ending at all charging stations nodes is the trips be captured by a deployment scheme. Basing on the above transformations, the quantized benefit of a charging station deployment scheme is modeled by using the standard Markov chain methodology [10] as follows.

A charging station deployment scheme is defined as a column vector $\mathbf{X} = (x_1, x_2, \dots, x_N)^T$, where $x_i = 1$ if a charging station is located at n_i , otherwise $x_i = 0$. Let \mathcal{D} denote the set of destination nodes on the pure map and \mathcal{D}_X denote the set of charging station nodes of \mathbf{X} . For a given deployment scheme \mathbf{X} , let $f_{ij}(\mathbf{X})$ denotes the probability that EVs entering the map at n_i , will leave the map at n_j . Note that an EV can only leave the map at a charging station node or a destination node. If an EV left the map at a destination node, the corresponding trip is not captured. If an EV left the map at a station node, the corresponding trip is captured. Then, for any $n_j \in \mathcal{D} \cup \mathcal{D}_X$ and any $n_i \in \mathcal{N}$ and $i \neq j$, we have

$$f_{ij}(\mathbf{X}) = \sum_{k \in \mathcal{N}_p} t_{ik'} f_{kj}(\mathbf{X}), \quad (1)$$

where $t_{ik'}$ is defined as

$$t_{ik'} = \begin{cases} 1, & i = k, x_i = 1 \\ 0, & i \neq k, x_i = 1 \\ t_{ik}, & x_i = 0 \end{cases} \quad (2)$$

Note that, in function (1), $f_{ij}(\mathbf{X}) = 1$ if $i = j$. Enumerating all the possible i and j , function (1) can compose a $N \times |\mathcal{D} \cup \mathcal{D}_x|$ linear equations. We can get the value of each $f_{ij}(\mathbf{X})$ easily by solving these equations. Then the amount of trips that is captured by \mathbf{X} is

$$F(\mathbf{X}) = \sum_{n_j \in \mathcal{D}_x} \sum_{n_i \in \mathcal{N}} a_i f_{ij}(\mathbf{X}). \quad (3)$$

2.2. The model of the power grid

For the charging station deployment problem, we should not only consider the locations on the map to meet the charging demand, but also consider their locations on the grid to meet the constraints of power grid. Thus, we give the model of power grid in this subsection.

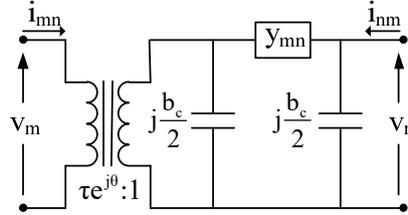


Figure 2. The model of a branch that connects the buses m and n [11].

A power network contains a set \mathcal{B} of nodes and a set \mathcal{L} of lines connecting these nodes. A node, also referred to as a bus in the power engineering nomenclature, can represent a generator or a load substation. A line, also known as a branch, can stand for a transmission or a transformer.

Consider first a power system module of two nodes, m and n , connected through a line. Two-node connections can be represented by the equivalent π model, with the line series impedance $z_{mn} = 1/y_{mn}$ and total charging susceptance $b_{c,mn}$, in series with an ideal phase shifting transformer whose tap ratio has magnitude τ and phase shift angle θ . The π model is shown in Fig. 2.

Denote the complex voltage at m by v_m and the complex current flowing from m to n by i_{mn} . Invoking Ohm's and Kirchoff's laws on the circuit of Fig. 2 yields

$$\begin{bmatrix} i_{mn} \\ i_{nm} \end{bmatrix} = \begin{bmatrix} y_{mn} + j \frac{b_{c,mn}}{2} & -y_{mn} \\ y_{mn} & y_{mn} + j \frac{b_{c,mn}}{2} \end{bmatrix} \begin{bmatrix} v_m \\ v_n \end{bmatrix}. \quad (4)$$

A small shunt susceptance $b_{s,mm}$ is typically assumed between the node and the ground, yielding the current $i_{mm} = j b_{s,mm} v_m$. In this paper, we denote the complex current at m by i_m . Then according to KCL, we have

$$i_m = Y_{mm} v_m + \sum_{n \in \mathcal{B}_m} Y_{mn} v_n, \quad (5)$$

where \mathcal{B}_m is the set of buses directly connected to m ; $Y_{mm} = j(b_{s,mm} + \sum_{n \in \mathcal{B}_m} b_{c,mn}/2) + \sum_{n \in \mathcal{B}_m} y_{mn}$; $Y_{mn} = -y_{mn}$ if $n \in \mathcal{B}_m$, and zero otherwise.

If a transformer of property $\tau_{mn} = |\tau_{mn}| e^{j\phi_{mn}}$ exists from m to n , where $|\tau_{mn}|$ is the tap ratio and ϕ_{mn} is the phase shift, then the Y_{mm} , Y_{nn} and Y_{mn} in (5) should be replaced by Y'_{mm} , Y'_{nn} and Y'_{mn} respectively, where $Y'_{mm} = Y_{mm} - Y_{mn} - y_{c,mn}/2 + (Y_{mn} + y_{c,mn}/2)/|\tau_{mn}|^2$, $Y'_{nn} = Y_{nn}/\tau_{mn}^*$, $Y'_{mn} = Y_{mn}/\tau_{mn}$.

Denote the complex power injected at m by $s_m = p_m + jq_m$ and the conjugation by $*$, it holds that $s_m = v_m i_m^*$. By writing the complex bus admittance as $Y_{mn} = G_{mn} + jB_{mn}$ and the complex bus voltage as $v_m = |v_m| e^{j\theta_m}$, then for any $m \in \mathcal{B}$, we have

$$p_m = \sum_{n \in \mathcal{B}_n} |v_m| |v_n| (G_{mn} \cos \theta_{mn} + B_{mn} \sin \theta_{mn}), \quad (6)$$

$$q_m = \sum_{n \in \mathcal{B}_n} |v_m| |v_n| (G_{mn} \sin \theta_{mn} - B_{mn} \cos \theta_{mn}), \quad (7)$$

where $\theta_{mn} = \theta_m - \theta_n$. The $2|\mathcal{B}|$ equations in (6) and (7) involve $4|\mathcal{B}|$ variables $\{p_m, q_m, |v_m|, \theta_m\}_{m \in \mathcal{B}}$. Among the variables, 1) the reference bus has fixed $(|v|, \theta)$; 2) $(p, |v|)$ are controlled at generator buses; 3) (p, q) are predicted at load buses. Fixing the $2|\mathcal{B}|$ variables, the remaining ones can be obtained by solving (6) and (7).

The charging power of EVs mainly comes from power networks. Thus we must make sure that the power network constraints are not violated when deploying charging stations. The main impacts on power networks are the voltage deviation and power limitation. Given a deployment scheme \mathbf{X} , the charging power needed at each candidate location is $s_c \mathbf{X}$ where s_c is the charging power of a charging station. It's reasonable to assume that a charging station will draw power from its local bus. So, we use a $|\mathcal{B}| \times |\mathcal{N}|$ matrix \mathbf{W} to denote the connections between candidate locations and buses, with $w_{ij} = 1$ if n_j draws power from the bus i , otherwise zero. Denote the charging power by a $|\mathcal{B}| \times 1$ vector s^B , then we have

$$s^B = s_c \mathbf{W} \mathbf{X}. \quad (8)$$

Connecting the charging power to the buses should not exceed the power capacity, i.e.

$$s + s^B \leq \bar{s}, \quad (9)$$

where \bar{s} is the maximum power capacity, s is the base load.

Gather the charging powers and based loads, the new bus voltages can be obtained by solving (6) and (7). The new bus voltages should meet the voltage constraint:

$$|v| \geq \underline{v}, \quad (10)$$

where \underline{v} is a $|\mathcal{B}| \times 1$ vector that denotes the lower bound of bus voltages.

2.3. The problem formulation

$$\max F(\mathbf{X}) \quad (11)$$

$$s.t. \quad \sum_{i=1}^N x_i \leq budget \quad (12)$$

$$|v| \geq \underline{v}, \quad (13)$$

$$s + s^B \leq \bar{s}, \quad (14)$$

$$x_i \in \{0, 1\}, 1 \leq i \leq N \quad (15)$$

$$w_{ij} \in \{0, 1\}, \forall n_i \in \mathcal{N}, \forall j \in \mathcal{B}. \quad (16)$$

The objective function (11) aims to find the optimal charging station deployment scheme \mathbf{X} to maximize the trips captured. The constraints (12) indicates the upper bound of budget. The constraints (13), (14) are grid constraints.

3. Solution

Since the computation of $F(\mathbf{X})$ involves matrix inversion, the problem formulated in section 2.3 is not easily amenable to the standard integer programming techniques. Thus, we transform the problem to a constrained average-reward Markov Decision Process (MDP).

3.1. Introduction of Markov Decision Process

A Markov process is observed at discrete time points t_1, t_2, t_3, \dots . At any time, the process is corresponded to a state. The *state-space* can be denoted by a finite set $\mathcal{E} = \{e_1, e_2, \dots, e_E\}$. At any time, an *action* should be chosen to switch to the next time and state. Let $A(e_i)$ denote the finite set of all possible *actions* in state e_i . If the system is in state e_i and action $a \in A(e_i)$ is chosen, then a reward r_{ia} is earned immediately. Let p_{iaj} denote the probability the system will switch to e_j next. Assume that $\{e_t | t=1, 2, \dots\}$ and $\{a_t | t=1, 2, \dots\}$ are the sequences of observed states and corresponding actions of a process.

A decision rule π_t is a function which assigns a probability to the event that action a is taken at time t . For a Markov policy $R = (\pi_1, \pi_2, \dots)$. For any policy R and initial state e_i , we denote the average expected reward by $\phi(R)$, i.e.

$$\phi(R) = \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^E \sum_{a \in A(j)} P_R(e_{t'} = e_j, a_{t'} = a | e_{t'} = e_i) r_{ja}, \quad (17)$$

where $P_R(e_{t'} = e_j, a_{t'} = a | e_{t'} = e_i)$ is the induced conditional probability, under policy R , that at time t the system is in state e_j and action a is taken, given that the system is in state e_i at the time t_1 . The policy R^* is said to be average optimal iff $\phi(R^*) = \sup_R \phi(R)$, $e_i \in \mathcal{E}$.

3.2. The problem reformulation

Given a location problem with the node set \mathcal{N} and the initial customer distribution \mathbf{b} , we define the following MDP: the state space is set to $\mathcal{E} = \mathcal{N}$; the action is set to $A(e_i) = \{0, 1\}$ for any $e_i \in \mathcal{E}$; the transition probability p_{iaj} is set to $p_{iaj} = t'_{ij}$, where t'_{ij} is defined in function (2); the reward r_{ia} is set to $r_{ia} = a$. Specifically, taking action 1 at n_i means treating this node as a charging station node, and vice versa. Then we have the following result:

Theorem 1 Let vector \mathbf{u} be a deployment policy. Define a function h_u by $h_u(i) = u_i$. Then, according to function (3), we have

$$F(\mathbf{u}) = \sum_{i=1}^N b_i \phi(h_u) = \sum_{j=1}^N u_j \sum_{i=1}^N b_i f_{ij}(\mathbf{u}). \quad (18)$$

Furthermore, according to function (17), we have

$$\begin{aligned} \sum_{i=1}^N b_i \phi(h_u) &= \sum_{i=1}^N b_i \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^N \sum_{a \in A(j)} P_{h_u}(e_{t'} = e_j, a_{t'} = a | e_{t'} = e_i) r_{ja} \\ &= \sum_{i=1}^N b_i \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^N P_{h_u}(e_{t'} = e_j, a_{t'} = a | e_{t'} = e_i) \end{aligned} \quad (19)$$

The definitions of p_{iaj} , r_{ia} , and h_u imply that the Markov chain induced by policy h_u has a transition matrix whose elements are defined by equation (2). Thus,

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T P_{h_u}(e_{t'} = e_j, a_{t'} = a | e_{t'} = e_i) = \begin{cases} f_{ij}(\mathbf{u}), & \text{if } u_j = 1 \\ 0 & \text{if } u_j = 0 \end{cases} \quad (20)$$

According to the above theoretical derivation, we can get the following theorem 2, which is an immediate consequence of BERMAN [12].

Theorem 2 *Given the following mixed integer program (MIP),*

$$\begin{aligned} & \max \sum_{i=1}^N x_{i1} \\ \text{s.t.} \quad & \sum_{a \in A(e_i)} x_{ia} - \sum_{j=1}^N \sum_{a \in A(e_j)} p_{jai} x_{ja} = 0, \forall 1 \leq i \leq N \\ & \sum_{a \in A(e_i)} x_{ia} + \sum_{a \in A(e_i)} y_{ia} - \sum_{j \in N} \sum_{a \in A(e_j)} p_{jai} y_{ja} = b_i, \forall 1 \leq i \leq N \\ & x_{i1} \leq u_i, \forall 1 \leq i \leq N \\ & \sum_{i=1}^N u_i = m \\ & x_{ia}, y_{ia} \geq 0, u_i = \{0, 1\}, \forall 1 \leq i \leq N. \end{aligned}$$

We have the following results: (1) the MIP is solvable if and only if MDP is solvable; (2) The optimal value of MIP is equal to the optimal value of MDP; (3) if $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{u}^*)$ is the optimal solution of MIP, then \mathbf{u}^* is the optimal station deployment scheme.

The problem described in theorem 2 is a mixed integer program, which can be solved by existing solvers such as CVX, CPLEX etc. So, due to page limit, we do not show the detailed computing and evaluation in this paper.

4. Conclusion

This paper studies the deployment of charging stations in traffic networks considering grid constraints to balance the charging demand and grid stability. The main contribution of this paper include two-folds: (1) we propose a statistical model for charging demand, (2) we combine the charging demand model with power grid constraints and give the formulation of the charging station deployment problem, (3) we propose a theoretical solution for the problem by transforming it to a Markov Decision Process.

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