

# A Decomposition Method for Security Constrained Economic Dispatch of a Three-Layer Power System

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**Abstract.** This paper proposes a new decomposition method for the security-constrained economic dispatch in a three-layer large-scale power system. The decomposition is realized using two main techniques. The first is to use Ward equivalencing-based network reduction to reduce the number of variables and constraints in the high-layer model without sacrificing accuracy. The second is to develop a price response function to exchange signal information between neighboring layers, which significantly improves the information exchange efficiency of each iteration and results in less iterations and less computational time. The case studies based on the duplicated RTS-79 system demonstrate the effectiveness and robustness of the proposed method.

## 1. Introduction

Optimizing power system operation under multi-regional scale is increasingly important because it benefits the system in multiple ways, such as achieving less operation cost, accommodating more renewable energy and requiring less operating reserve [1]. For example in the State Grid area of China, there are three layers of dispatching centers which are national layer, regional layer and provincial layer from the highest layer to the lowest. If the operational decisions, such as security constrained economic dispatch (SCED), can be made with all the details in the national layer, it could result in hundreds of millions of operational cost reduction every year [2].

However, because of the ultra-large size of power interconnections in the highest layer, it is very challenging to develop methods or algorithms in a centralized manner due to the following reasons [3]. First, the computational difficulty will increase significantly because of the increased dimensionality. Second, the high communication requirement between the layers is hard to meet. Third, each area in the lowest layer has its own features which may need operation mode customized to some extent. A centralized optimization will not allow that.

Extensive literature can be found on decomposing multiple-layer SCED problems to reduce computational difficulty, using methods such as topology configuration [4, 5], Dantzig-Wolfe decomposition (DWD) [6-8], Lagrange Relaxation (LR) [9, 10] and some other decentralized optimization algorithms. DW decomposition and LR methods are the most popular ones. However, DWD has difficulty in tackling many coupled constraints and generator's second-order cost function. LR method updates the Lagrange multiplier at each iteration to guide the optimization process. But the update strategy is hard to choose to avoid oscillations.



This paper is motivated by the challenges described above. We aim at designing a practical SCED method in ultra-large, multiple-layer systems so that the benefits of optimizing in larger area is obtained while the computation and communication burden is acceptable. To specify the problem, we target a multi-regional system like the State Grid system in China, which has three-layers. And we call them national, regional and provincial layer respectively in this paper. The highlights of the proposed method are:

1) The method is based on a three-layer architecture. Between neighboring layers, closed loop is designed to achieve the decomposition with acceptable accuracy.

2) The information exchange between neighboring layers is achieved using a new concept proposed in this paper, termed as price response function (PRF). PRF contains the information of the relationship between the tie-line power and LMP at tie-line bus in the respectively lower layer. Using PRF, the optimization in the respectively upper layer will be more precise. More importantly, PRF is calculated dynamically throughout the whole iterative process.

3) When optimizing the SCED in the respectively upper layer, Ward equivalencing-based network reduction is used to compress the network information in the respectively lower layer. This significantly reduce the computational complexity.

The remainder of this paper is organized as follows. Section 2 summarizes the procedure of the proposed method. Section 3 introduces Ward equivalencing-based network reduction and PRF training. Section 4 presents the SCED models used in each layer. Section 5 carries out case studies and Section 6 concludes the paper.

## 2. Procedures of the Proposed Method

The procedure of the proposed method is shown in Fig.1. The inner loop is for coordination between the regional layer and provincial layer, while the outer loop is for coordination between the national layer and the regional layer.

Here we summarize the procedures of the inner loop.

Step 0) Perform Ward equivalencing-based network reduction in each province and each region, then the equivalent networks for each region and national layer are obtained. Initialize province-to-province (P2P) and region-to-region (R2R) tie-line power flow to be zero. The network reduction approach is presented in Section 3.1.

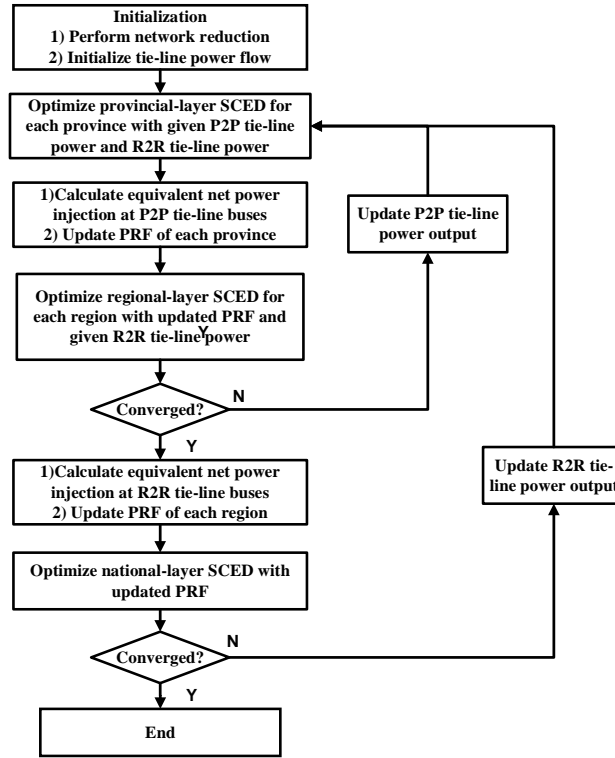
Step 1) Each province optimizes its SCED with given P2P and R2R tie-line power flow.

Step 2) Using provincial SCED solutions, each province calculate equivalent net power injection at P2P tie-line buses and update its PRF. The PRF is defined in Section 3.2.

Step 3) Using a SCED-based model, each region optimizes its incremental power injection at P2P tie-line buses over the equivalent regional network with updated PRF and given R2R tie-line power.

Step 4) If all incremental power injection at P2P tie-line buses are within a small tolerance, it is considered convergence; otherwise, return to step 1.

The outer loop shares the same idea, thus it is not elaborated here.



**Figure 1.** The procedures of the proposed method.

### 3. Network Reduction and PRF Training

#### 3.1. Network Reduction

Ward equivalencing-based network reduction is used to obtain a network model with only tie-line buses. The network equation of an area (either a province or a region) can be represented as follow.

$$\begin{bmatrix} \mathbf{B}_{TT} & \mathbf{B}_{TI} \\ \mathbf{B}_{IT} & \mathbf{B}_{II} \end{bmatrix} \begin{bmatrix} \theta_T \\ \theta_I \end{bmatrix} = \begin{bmatrix} \mathbf{P}_T \\ \mathbf{P}_I \end{bmatrix} \quad (1)$$

By eliminating  $\theta_I$ , we have

$$\tilde{\mathbf{B}}_{TT} \theta_T = \tilde{\mathbf{P}}_T \quad (2)$$

Where

$$\tilde{\mathbf{B}}_{TT} = \mathbf{B}_{TT} - \mathbf{B}_{TI} \mathbf{B}_{II}^{-1} \mathbf{B}_{IT} \quad (3)$$

$$\tilde{\mathbf{P}}_T = \mathbf{P}_T - \mathbf{B}_{TI} \mathbf{B}_{II}^{-1} \mathbf{P}_I \quad (4)$$

Combining the  $\tilde{\mathbf{B}}_{TT}$  of each area in a unified order of tie-line buses can generate global equivalent admittance matrix  $\tilde{\mathbf{B}}_G$ . Due to the space limitation, the formation of  $\tilde{\mathbf{B}}_G$ , which can be found in [11], is not explicitly shown in this paper. The dimension of  $\tilde{\mathbf{B}}_G$  is much smaller than the original global

admittance matrix. With accuracy maintained, the equivalent network could save a lot of computational effort.

Moreover, equation (4) shows how to calculate the equivalent net power injection at tie-line buses.

### 3.2. Price Response Function (PRF) Training

In the proposed method, the PRF plays a role of exchanging information between neighboring layers. The respectively upper layer model uses PRF from the lower layer as key pricing signals to achieve optimality.

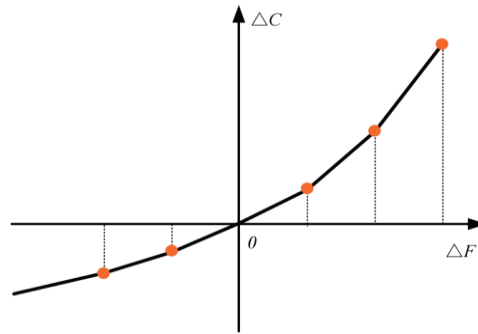
Take tie-line bus b in area A for example. The training process of its PRF is as follows.

Step 1) After solving the SCED associated with area A at the kth iteration, LMP at bus b, denoted by  $\lambda_b^k$ , can be obtained.

Step 2) Calculate the total generation of area A, denoted by  $P_A^k$ . Current operating point of bus b can be represented by point  $(P_A^k, \lambda_b^k)$ .

Step 3) For each tie-line bus, a series of points  $(P_A^k, \lambda_b^k)$  can be obtained through iterations. They are the key points where the  $\lambda_b^k$  would change with respect to  $P_A^k$ .

Step 4) Shift the y-axis to a given total generation  $P_A^{sp}$ , integrate the curve and make the resulting curve pass through the point (0,0). Then, an incremental curve can be drawn with respect to incremental power output at bus b, as shown in Fig.2.



**Figure 2.** The incremental system cost curve with respect to incremental power output at tie-line bus b.

The curve shown in Fig.2 is defined as the PRF of this tie-line bus b, which quantifies the incremental generation cost of area A at a certain generation layer, with respect to power increment at bus b.

## 4. SCED Models in each layer

Based on the procedures presented in Section 2 and the decomposition techniques introduced in Section 3, this section formulates the mathematical SCED model used at each layer.

### 4.1. SCED model at the provincial layer

The SCED model at the provincial layer is to optimize inner-province generation outputs for minimum operating cost, with P2P tie-line power given. The constraints include the power balance constraint (6), transmission line flow constraints (7)-(8) and generator output limit constraints (9).

$$\min_{P_i} \sum_{i \in U_A} f_i(P_i) \quad (5)$$

s.t.

$$\sum_{b \in N_A} D_b = \sum_{i \in U_A} P_i + \sum_{t \in T_A^{\text{in}}} \bar{F}_t - \sum_{t \in T_A^{\text{out}}} \bar{F}_t \quad (6)$$

$$F_l = \sum_{i \in U_A} S_{l,i}^A P_i - \sum_{b \in N_A} S_{l,b}^A D_b + \sum_{t \in T_A^{\text{in}}} S_{l,t}^A \bar{F}_t - \sum_{t \in T_A^{\text{out}}} S_{l,t}^A \bar{F}_t, \quad \forall l \in L_A \quad (7)$$

$$|F_l| \leq F_l^{\max}, \quad \forall l \in L_A \quad (8)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad \forall i \in U_A \quad (9)$$

where,  $\bar{F}_t$  is the tie-line flow from the solution of the regional problem;  $S_{l,i}^A, S_{l,b}^A, S_{l,t}^A$  are provincial generation shift factor of tie-line  $t$  with respect to unit  $i$ , bus  $b$  and intra-province bus of tie-line  $t$ , respectively.

#### 4.2. SCED model at the regional layer

The SCED model at the regional layer is to optimize the equivalent power injection at P2P tie-line buses so as to achieve scheduled inter-province exchange in an optimal manner. The model is based on the equivalent regional network. The constraints include the power balance constraint (11), scheduled inter-province power exchange constraints (12)-(15) and incremental power injection limit constraints (16).

$$\min_{\Delta P_b} \sum_{b \in N_T} C_b \left( \sum_{t \in T_b^{\text{in}}} \Delta F_t - \sum_{t \in T_b^{\text{out}}} \Delta F_t \right) \quad (10)$$

s.t.

$$\sum_{b \in N_T} \Delta P_b = 0 \quad (11)$$

$$\Delta F_t = \sum_{b \in N_T} S_{t,b}^v \Delta P_b, \quad \forall t \in T \quad (12)$$

$$F_t = \tilde{F}_t + \Delta F_t, \quad \forall t \in T \quad (13)$$

$$E_j^{\min} \leq \sum_{t \in T_j^+} F_t - \sum_{t \in T_j^-} F_t \leq E_j^{\max}, \quad \forall j \in \Omega \quad (14)$$

$$|F_t| \leq F_t^{\max}, \quad \forall t \in T \quad (15)$$

$$\Delta P_b^{\min} \leq \Delta P_b \leq \Delta P_b^{\max}, \quad \forall b \in N_T \quad (16)$$

where,  $N_T$  is the set of P2P tie-line buses;  $S_{t,b}^v$  are the generation shifting factors computed over regional equivalent network formed using the method introduced in Section III.A, which has only P2P tie-line buses. In this model,  $\tilde{F}_t$  is from the SCED solution at the provincial layer, which is calculated based on the equivalent net injection at P2P tie-line buses submitted by each province.

#### 4.3. SCED model at the national layer

The SCED model at the national layer is similar to that at the regional layer. It is to optimize the equivalent power injection at R2R tie-line buses so as to achieve scheduled inter-region exchange in an optimal manner. The model is based on the equivalent national network. The formulation is similar to that of the regional layer therefore it is not elaborated here due to space limitation.

### 5. Case Study

The IEEE Reliability Test System (RTS) 79 is used to test the performance of the proposed method. A computer with Intel Core i7 2.40 GHz is employed to perform all computations using Gurobi 5.6.2[12].

The RTS-79 system has 24 buses and 33 generators. Data of the RTS-79 can be found in [13]. Based on 4 duplicate RTS-79 systems, we create a three-layer national power system with two regions and two provinces in each region. Each province is formed based on RTS-79, but the generator cost function is modified to generate difference. Each pair of the two provinces in a region has one or two P2P tie-lines and two R2R tie-lines are set up between the two regions.

In order to test the effectiveness of the proposed method. Three scenarios are considered.

S1: The original data, which has no congestion.

S2: Change the capacity of inner-province line 116-117 from 500 MW to 300 MW.

S3: Change the capacity of inner-province line 116-117 from 500 MW to 300 MW. Change the capacity of P2P tie-line 113-215 from 500 MW to 400 MW.

The test results are shown in Table 1. In order to test the accuracy of the proposed method, the national-layer fully-centralized SCED model is also solved to provide the global optimal solution. The accuracy shown in Table 1 indicates the relative error of solution provided by the proposed method with respect to the global optimal solution.

**Table 1.** The Test Results of the Duplicated RTS-79 System

Case	Iterations	Computation time (s)	Total Cost (\$)	Accuracy (%)	Line 116-117 Flow (MW)	Tieline 113-215 Flow (MW)
S1	27	0.16	31219	99.96	-322.14	446.64
S2	31	0.21	31298	99.91	-300.00	445.13
S3	30	0.19	31336	99.93	-300.00	400.00

As shown in Table 1, the proposed method uses acceptable computational time and iterations to obtain high-accuracy solutions, which indicates that the proposed method is effective to solve SCED problems of large-scale three-layer power systems. The power flows of inner-province line 116-117 and P2P tie-line 113-215 in all cases are also shown in the table. Clearly those lines are congested in S2 and S3, which indicates that the proposed method fully utilizes the tie-line capacity to minimize the national-wise operating cost.

### 6. Conclusion

To tackle the computational challenge of solving SCED for large-scale multiple-layer power systems, this paper proposes a new decomposition method. The proposed method decomposes the national-layer fully-centralized SCED model into three layers. Using the Ward equivalencing-based network reduction and the proposed price response function (PRF), the price information is exchanged effectively between neighboring layers. The three-layer power system case studies created based on the RTS-79 system show that the proposed method is effective and robust.

The main contribution of the proposed method is to provide the concept of PRF, which digs deeper into the solution of the lower-layer model to provide a price curve instead of only one price as the Lagrange multiplier does. Future work will concentrate on further improving the practicality of the model. Heuristics for further reducing the computational time will also be investigated.

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**Nomenclature**

$A/R$	Province/region index
$M_A / M_R$	Number of province/region
$i$	Unit index
$U / U_A$	Set of units / Set of units in province $A$
$P_i$	Active power output of unit $i$
$P_i^{\max} / P_i^{\min}$	Upper/lower limit of active power output of unit $i$
$f_i(\cdot)$	Cost function of unit $i$
$b$	Bus index
$N / N_A$	Set of buses / Set of buses in province $A$
$D_b$	Active power demand of bus $b$
$l$	Intra-province transmission line index
$L$	Set of intra-province transmission lines
$L_A$	Set of transmission lines within province $A$
$F_l$	Active power flow of transmission line $l$
$F_l^{\max}$	Active power capacity of transmission line $l$
$j$	Inter-province power exchange schedule index
$\Omega$	Set of inter-province power exchange schedules
$E_j^{\max} / E_j^{\min}$	Maximum/minimum power exchange of schedule $j$
$t$	Tie-line index
$T$	Set of P2P tie-lines
$T_A^{\text{in}}$	Set of P2P tie-lines importing into province $A$
$T_A^{\text{out}}$	Set of P2P tie-lines exporting from province $A$
$T_b^{\text{fr}}$	Set of P2P tie-lines from bus $b$
$T_b^{\text{to}}$	Set of P2P tie-lines to bus $b$
$T_j^+$	Set of P2P tie-lines with consistent direction as schedule $j$
$T_j^-$	Set of P2P tie-lines with opposite direction as schedule $j$
$\Delta P_b^{\max} / \Delta P_b^{\min}$	Maximum/minimum incremental power injection at bus $b$
$C_b(\cdot)$	Price response function for tie-line bus $b$