

# Collaborative en-route and slot allocation algorithm based on fuzzy comprehensive evaluation

Shangwen Yang<sup>\*</sup>, Baohua Guo, Xuefei Xiao, Haichao Gao

State Key Laboratory of Air Traffic Management System and Technology, Nanjing  
Research Institute of Electronics Engineering, Nanjing, China

\*Corresponding author e-mail: swyang@aliyun.com

**Abstract.** To allocate the en-routes and slots to the flights with collaborative decision making, a collaborative en-route and slot allocation algorithm based on fuzzy comprehensive evaluation was proposed. Evaluation indexes include flight delay costs, delay time and the number of turning points. Analytic hierarchy process is applied to determining index weights. Remark set for current two flights not yet obtained the en-route and slot in flight schedule is established. Then, fuzzy comprehensive evaluation is performed, and the en-route and slot for the current two flights are determined. Continue selecting the flight not yet obtained an en-route and a slot in flight schedule. Perform fuzzy comprehensive evaluation until all flights have obtained the en-routes and slots. MatlabR2007b was applied to numerical test based on the simulated data of a civil en-route. Test results show that, compared with the traditional strategy of first come first service, the algorithm gains better effect. The effectiveness of the algorithm was verified.

## 1. Introduction

En-route and slot resource allocation which assigns available en-routes and time slots in coming time period to flights is one of the key technologies of collaborative en-route management. There have been some achievements, typical ones are formulated below. Goodhart studied the preferences of airlines during en-route resource allocation when the airspace was confined [1]. In 2005, AFP (Airspace Flow Program) was proposed by FAA (Federal Aviation Administration) [2] and was applied to alleviating air traffic pressure and reducing the influence of adverse weather. In AFP, flights are allowed to flexibly choose suitable en-route. According to real capacity of confined airspace, AFP allocates en-route and slot resource using traditional RBS (Ration-by-Schedule) algorithm in accordance with FCFS (first come first service). RBS well embodies the equity of resource allocation. Ball et al. proposed RBD (ration-by-distance) algorithm of which the efficiency was better than RBS and the equity was worse [3]. Hoffman et al. proposed an en-route resource allocation method considering airspace users' preferences and air traffic management decisions [4]. In the context of AFP, Pourtaklo et al. proposed an algorithm equitably allocating the en-routes and slots according to flight operators' preferences and their randomness [5]. Since 2006, AFP has been applied to reducing flight delay and the effect has been verified by practice [6]. However, CTOP (Collaborative Trajectory Options Program) optimally allocating en-route resource with CDM (collaborative decision making) was proposed by FAA and carried out in 2014 [7]. CTOP uses RBS algorithm and assigns flights holding



on the ground or rerouting before entering confined airspace [8]. Market mechanisms such as bid and auction were introduced in, and equitable resource allocation was performed according to the need of airlines [9-11]. Kim et al. proposed an evaluation model for en-route resource allocation effect [12]. In 2015, Kim et al. proposed an en-route sequential resource allocation model based on game theory [13].

Generally speaking, the aim of en-route resource allocation is minimizing various delay costs in existing achievements. Although the evaluation ways of delay costs are different, single objective is one-sided. Multi-objective model is complex and is difficult to be applied to system tools. In the paper, we introduced fuzzy comprehensive evaluation into en-route and slot resource allocation and proposed a collaborative en-route and slot allocation algorithm. We verified the effectiveness of the algorithm by performing computational test. The remainder is organized as follows. In Section 2, the algorithm is formulated, and necessary processes and assumptions are given. In Section 3, a simulation test based on the simulated data of a civil en-route is made to illustrate the efficiency of the algorithm, and necessary analyses are performed. Finally, we conclude in Section 4.

## 2. Algorithm Description

En-route and slot resource allocation should consider the needs of air traffic control department, airlines and passengers and gain satisfactory solutions. With collaborative decision making conception, there are various factors related to en-route and slot resource allocation. Each factor has different effect on en-route and slot resource allocation strategy. Fuzzy comprehensive evaluation could balance various factors and make decision based on comprehensive analyses. Fuzzy comprehensive evaluation has been widely applied to the fields such as traffic and transportation, economics and system engineering.

First, the set of evaluation indexes is established, and the weights are determined. Then, the remark set is established, and evaluation matrix is determined. The current two flights not yet obtained en-routes and slots in flight schedule would be selected, and fuzzy comprehensive evaluation would be performed. The en-route and slot for the current two flights are determined, and present allocation strategy is created. Continue selecting the flight not yet obtained an en-route and a slot in flight schedule. Perform fuzzy comprehensive evaluation until all flights have obtained en-routes and slots. The main process is shown in Fig. 1.

Now, we elaborate the algorithm below.

Step 1: Establish the set of evaluation indexes for collaborative en-route and slot allocation.

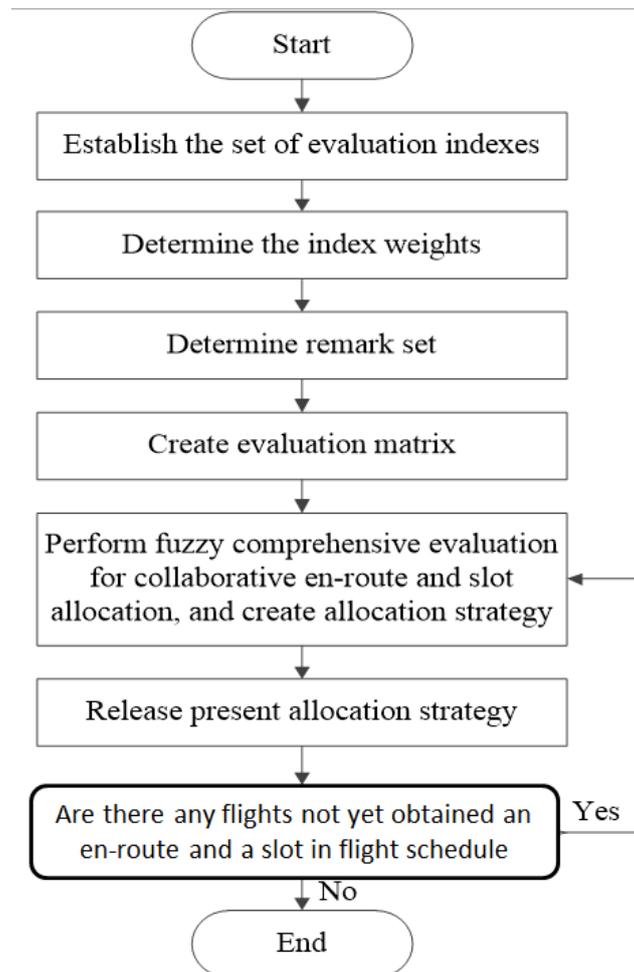
According to the efficiency, equity and effectiveness principles of collaborative en-route and slot allocation, delay costs, delay time and the number of turning points could represent the preferences of airlines, passengers and air traffic control department. Then, the evaluation set could be formulated as follows.

$$U = \{t, c, n\} \quad (1)$$

Where  $t$  represents the total delay time of the two flights intending to obtain current slot, and  $c$  and  $n$  are respectively the total delay costs and the number of turning points in selected en-route.

Step 2: Determine the index weights.

Generally speaking, there are two ways to determine the index weights. One is to consult relative experts and gain empirical value, the other is well-known analytic hierarchy process. In the paper, we use analytic hierarchy process, and the detail of analytic hierarchy process would not be formulated here due to the limitation of length.



**Figure 1.** Main process.

Step 3: Establish remark set as follows.

$$V = \{ \text{planned en-route for flight 1, temporary en-route 1 for flight 1, } \dots, \text{temporary en-route } z \text{ for flight 1, planned en-route for flight 2, temporary en-route 1 for flight 2, } \dots, \text{temporary en-route } z \text{ for flight 2} \} \quad (2)$$

Where  $z$  represents the number of temporary en-routes.

Step 4: Create evaluation matrix.

According to evaluation indexes and remark set, initial evaluation matrix could be formulated as  $R_p = \{r_{ij}\}$ , where  $r_{ij}$  represents the value of evaluation index  $i(1 \leq i \leq 3)$  with remark  $j(1 \leq j \leq 2z)$ . Then, each element of  $R_p$  would be divided by the sum of the row, and final evaluation matrix  $R$  could be created.

$$t = \sum_{k=1}^2 t_k, \text{ where } t_k \text{ represents the delay time of flight } k.$$

$$c = \sum_{k=1}^2 d_k t_k, \text{ where } d_k \text{ represents the delay costs of flight } k \text{ in corresponding en-route. The remark}$$

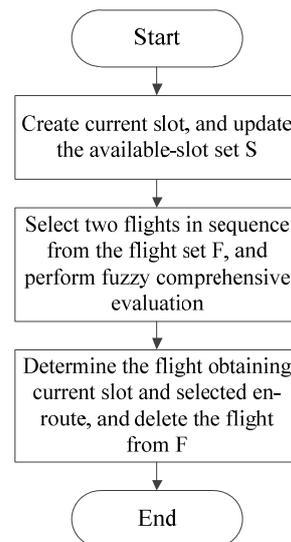
that is en-route  $x$  for flight  $k$  means that flight  $k$  obtains current slot and en-route  $x$ . Then, the other flight would be considered as being postponed until the following slot, and the delay costs are on the basis of the cost in planned en-route.

$n$  is determined by the en-route  $x$  of the remark.

Step 5: Establish fuzzy comprehensive evaluation model for collaborative en-route and slot allocation.

According to general principles of air traffic control, there are three assumptions. First, real time of arrival (RTA) at airspace sector downstream would not be earlier than estimated time of arrival (ETA) at airspace sector downstream. Second, there are only an en-route and a slot for a flight. Third, the separation between leading flight and following flight in an en-route would not be less than miles-in-trail in the en-route.

The process of fuzzy comprehensive evaluation could be shown in Fig. 2.



**Figure 2.** The process of fuzzy comprehensive evaluation.

(a) Set the first slot in the available-slot set  $S$  as the current slot, and delete it from  $S$ .

(b) Select two flights in sequence from the flight set  $F$ , and perform fuzzy comprehensive evaluation. According to weight vector  $W$  and evaluation matrix  $R$ , comprehensive evaluation vector  $A$  could be formulated as  $A=W \cdot R$ . For the flight not meeting the first assumption, the elements in evaluation matrix resulted from the remark for the flight are set as a large number, which means the flight could not obtain an en-route and a slot at this time. Meanwhile, the corresponding values of the flight in other elements are zero, which means the flight could not affect the other flight. When the third assumption is not met, corresponding index values are set as a large number, which means the en-route is unavailable. Add a fictitious flight after the last flight. The fictitious flight is considered as the flight not meeting the first assumption, which ensures the integrity of the allocation.

(c) Determine the flight obtaining current slot and selected en-route according to minimum membership and then delete the flight from  $F$ . Allocation strategy would be created. When two or more memberships are equal for one of flights, the en-route with minimum number of turning points would be selected. When two or more memberships are equal for both two flights, the flight with larger delay time would be selected. If delay time is equal, the flight with larger delay costs would be selected.

Step 6: Continue performing fuzzy comprehensive evaluation model.

Release created allocation strategy. Determine whether the flight set  $F$  is null. If so, the algorithm terminates. Otherwise, continue performing fuzzy comprehensive evaluation model.

### 3. Numerical Tests

Set the simulated data of a civil en-route as an example. The capacity of the en-route decreased due to adverse weather, and flight delay would be resulted in. According to available airspace and air traffic control experiences, two temporary en-routes were set. The parameters of the three en-routes are shown in Table 1 where delay costs per hour are for light (L) flights in each en-route. Medium (M) and heavy (H) flights are respectively two times and four times the values of light flights according to experiences. The capacity of airspace sector downstream is ten flights per hour.

**Table 1.** The parameters of the en-routes.

No.	En-route	Miles-in-trail (minute)	Delay costs per hour (yuan/hour)	The number of turning points
1	Planned en-route	12	3000	1
2	Temporary en-route 1	5	3500	2
3	Temporary en-route 2	10	3200	3

Compare and score the evaluation indexes. Establish the judgment matrix  $\begin{pmatrix} 1 & 1/3 & 1/2 \\ 3 & 1 & 2 \\ 2 & 1/2 & 1 \end{pmatrix}$ . The

weight vector W for evaluation indexes could be calculated using analytic hierarchy process and formulated as W=(0.16, 0.54, 0.30).

According to the capacity of airspace sector downstream, the available-slot set S could be formulated as S={10:00, 10:06, 10:12, ...} where each element is the initial time of a slot and represents the slot. The length of a slot is six minutes. Flight schedule is listed in Table 2. As formulated in Section 1, FCFS is the basis of en-route and slot allocation such as CTOP [7] and market mechanisms [13]. The difference lies in how to process the need of flight operators on the en-route and slot. MatlabR2007b was applied to performing both the algorithm proposed in the paper and FCFS. Test results are shown in Table 2.

**Table 2.** Flight schedule and test results.

No.	Aircraft type	ETA	The algorithm proposed				FCFS			
			RTA	Delay time (min)	Delay costs (yuan)	Selected en-route	RTA	Delay time (min)	Delay costs (yuan)	Selected en-route
1	H	10:00	10:00	0	0	1	10:00	0	0	1
2	H	10:04	10:06	2	466.67	2	10:06	2	466.67	2
3	M	10:08	10:24	16	1600	1	10:12	4	426.67	3
4	H	10:12	10:12	0	0	1	10:18	6	1200	1
5	H	10:16	10:18	2	466.67	2	10:24	8	1866.67	2
6	M	10:20	10:42	22	2346.67	3	10:30	10	1066.67	3
7	H	10:24	10:30	6	1280	3	10:36	12	2400	1
8	H	10:28	10:36	8	1600	1	10:42	14	3266.67	2
9	L	10:32	10:54	22	1173.33	3	10:48	16	853.33	3
10	M	10:36	10:48	12	1200	1	10:54	18	1800	1
	Total			90	10133.34	18		90	13346.68	19

In Table 2, equal total delay time illustrates that the capacity of airspace sector downstream is the bottleneck for en-route operation. The total delay time maintains unchanged no matter which

allocation strategy is adopted. It can be seen from Table 2 that, the algorithm is better than FCFS. It distributes delay time for each flight on the basis of decision preferences and reduces total delay costs and the number of turning points.

The algorithm establishes evaluation indexes on the basis of the efficiency and equity of en-route operation and allocates en-route and slot resource effectively. Decision preferences are reflected in index weights. The algorithm overcomes the limitation of single objective and properly reduces the problem size. It is fit for real time operation, and index weights could be adjusted to decision core. The algorithm is practical and the operation is good. It would be well applied to system tools.

#### 4. Conclusion

Introducing fuzzy comprehensive evaluation into the en-route and slot allocation problem, we proposed a collaborative en-route and slot allocation algorithm based on fuzzy comprehensive evaluation. Mathematical software was applied to performing the algorithm with the simulated data of a civil en-route as an example. Numerical test verified the performance of the algorithm as well as the comparison with FCFS. Collaborative en-route management refers to various stakeholders and complex airspace. Then, we will consider more decision objectives and propose mathematical models or algorithms on flexible allocation of en-route resource.

#### References

- [1] J. Goodhart, *Increasing Airline Operational Control in a Constrained Air Traffic System*, University of California, Berkeley, 2000.
- [2] M. Libby, J. Buckner, M. Brennan, *Operational concept for airspace flow programs (AFP), Version 1*, Washington, DC: FAA Air Traffic Organization, 2005.
- [3] M. O. Ball, R. Hoffman, A. Mukherjee, *Ground delay program planning under uncertainty based on the ration-by-distance principle*, *Transportation Science*. 44 (2010) 1-14.
- [4] R. Hoffman, J. Burke, T. Lewis, A. Futer, M. Ball, *Resource allocation principles for airspace flow control*, AIAA Guidance, Navigation and Control Conference and Exhibit, San Francisco, 2005.
- [5] N. V. Pourtaklo, M. Ball, *Equitable allocation of enroute airspace resources*, Proc. Eighth USA/Europe Air Traffic Management Research and Development Seminar, Napa, California, 2009, pp.1-8.
- [6] Information on FAA Greatly Expands Air Traffic Program to Minimize Summer Delays, <[http://www.faa.gov/news/press\\_releases/news\\_story.cfm?newsID=8889](http://www.faa.gov/news/press_releases/news_story.cfm?newsID=8889)>(retrieved 08.10), 2007.
- [7] Information on AC 90-115 – Collaborative Trajectory Options Program (CTOP): Document Information, Federal Aviation Administration. June 24, 2014. <[http://www.faa.gov/documentLibrary/media/Advisory\\_Circular/AC-90-115.pdf](http://www.faa.gov/documentLibrary/media/Advisory_Circular/AC-90-115.pdf)> (accessed 24.11.14).
- [8] Information on FAA Industry Forum 2012 Post Event Website. FAA – CTOP Presentation for Aviation Forum 2012 – Final.pdf, <<http://faaindustryforum.org/4>> (accessed 01.09.13).
- [9] S. L. Waslander, R. L. Raffard, C. J. Tomlin, *Market-based air traffic flow control with competing airlines*, *Journal of Guidance, Control and Dynamics*. 31 (2008) 148-161.
- [10] L. Castelli, R. Pesenti, A. Ranieri, *The design of a market mechanism to allocate air traffic flow management slots*, *Transportation Research Part C*. 19 (2011) 931-943.
- [11] A. Kim, *Collaborative Resource Allocation Strategies for Air Traffic Flow Management*, University of California, Berkeley, 2011.
- [12] A. Kim, M. Hansen, *A framework for the assessment of collaborative en route resource allocation strategies*, *Transportation Research Part C*. 33 (2013) 324-339.
- [13] A. Kim, M. Hansen, *Some insights into a sequential resource allocation mechanism for en route air traffic management*, *Transportation Research Part B*. 79 (2015) 1-15.