

A Calculation Method of Electric Distance and Subarea Division Application Based on Transmission Impedance

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Abstract. The widely used method of calculating electric distances is sensitivity method. The sensitivity matrix is the result of linearization and based on the hypothesis that the active power and reactive power are decoupled, so it is inaccurate. In addition, it calculates the ratio of two partial derivatives as the relationship of two dependent variables, so there is no physical meaning. This paper presents a new method for calculating electrical distance, namely transmission impedance method. It forms power supply paths based on power flow tracing, then establishes generalized branches to calculate transmission impedances. In this paper, the target of power flow tracing is S instead of Q . Q itself has no direction and the grid delivers complex power so that S contains more electrical information than Q . By describing the power transmission relationship of the branch and drawing block diagrams in both forward and reverse directions, it can be found that the numerators of feedback parts of two block diagrams are all the transmission impedances. To ensure the distance is scalar, the absolute value of transmission impedance is defined as electrical distance. Dividing network according to the electric distances and comparing with the results of sensitivity method, it proves that the transmission impedance method can adapt to the dynamic change of system better and reach a reasonable subarea division scheme.

1. Introduction

In recent years, many domestic and foreign power systems have appeared voltage instability or voltage collapse causing the local power outages, so the voltage stability issues have attracted more and more attention [1-3]. Voltage classification control is an effective method of reactive power/voltage control. The premise of voltage classification control is subarea division, which is based on the electrical distance [4]. In 1989, P. Lagonotte presented the concept and formula of electric distance based on sensitivity and applied it to subarea division for the first time [5]. Now most of common methods are based on the idea of P. Lagonotte, but only improve in some ways for accuracy and rationality [6-8].

Sensitivity method can obtain the relation of the dependent variable to the independent variable by calculating derivatives [9]. The sensitivity matrix is the result of linearization and based on the hypothesis that the active power and reactive power are decoupled, so it is inaccurate. The ratio of two partial derivatives is not the relationship between two dependent variables, so there is no physical meaning. In addition, most papers do not form a unified standard for calculating sensitivity indexes and lack of mathematical proof.

Therefore, this paper presents a new method of calculating electric distances called transmission impedance method. It forms power supply paths based on power flow tracing, then establishes



generalized branches to calculate transmission impedances. The electrical distances are calculated by sensitivity method and transmission impedance method, and are applied to subarea division of IEEE three machines nine nodes system. Through the comparison and analysis of two results, the accuracy and feasibility of the new method can be seen.

2. Sensitivity method

The most widely used method of calculating electrical distance is the sensitivity method. The calculation process is as follows:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta U / U \end{bmatrix} \quad (1)$$

Deformed the above matrix equation as follows:

$$\begin{bmatrix} \Delta \theta \\ \Delta U / U \end{bmatrix} = \begin{bmatrix} A & B \\ C & S \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (2)$$

There into, S is called reactive power/voltage sensitivity matrix, and its solution is as follows:

$$S = \left[\frac{1}{U} \frac{\partial U}{\partial Q} \right] = [L - JH^{-1}N]^{-1} \quad (3)$$

According to the reactive/voltage sensitivity, the electrical distance is defined as follows:

$$\Delta U_i = \alpha_{ij} \cdot \Delta U_j \quad (4)$$

$$\alpha_{ij} = \left(\frac{\partial U_i}{\partial Q_j} \right) / \left(\frac{\partial U_j}{\partial Q_j} \right) \quad (5)$$

In order to ensure the positive definiteness and symmetry of the electric distance matrix, the following mathematical treatment is carried out:

$$D_{ij} = -\log(\alpha_{ij}) \quad (6)$$

$$D_{ij} = D_{ji} = -\log(\alpha_{ij} \cdot \alpha_{ji}) \quad (7)$$

From above we can see that there are several problems:

(1) In the process of linearization of the power flow equation, the accuracy is affected caused by omitting the higher order term.

(2) There is no physical meaning for the ratio of two partial derivatives. $\frac{\partial U_i}{\partial Q_j}$ represents that

changes of Q_j cause changes of U_i ; $\frac{\partial U_j}{\partial Q_j}$ represents that changes of Q_j cause changes of

U_j ; $\frac{\partial U_i}{\partial U_j}$ represents that changes of U_j cause changes of U_i . The sensitivity method considers

$\left(\frac{\partial U_i}{\partial Q_j} \right) / \left(\frac{\partial U_j}{\partial Q_j} \right) = \frac{\partial U_i}{\partial U_j}$ correct. However, in fact, because the independent variable is eliminated,

the calculation result is only a ratio, which cannot explain the causal relationship between U_i and U_j .

(3) For the same transmission process, the electric distance of forward direction should be equal to the one of inverse direction. The sensitivity method lacks proof only by mathematical processing to ensure symmetry.

3. Transmission impedance method

3.1. Generalized branch and transmission impedance

The generalized branch is relative to the narrow branch, of which two nodes are not necessarily directly connected [10]. As we all know, for a branch, the injected current is equal to the output current. So, if two nodes are not directly connected but meet the condition, we may can describe the electrical relationship between the two nodes from a generalized perspective. As shown in Figure 1:

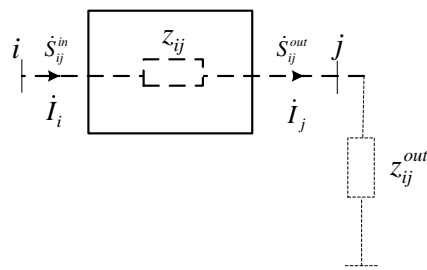


Figure 1. Generalized branch

If $\dot{I}_i = \dot{I}_j$, then there is a virtual branch between node i and node j , which is called generalized branch. The impedance of the generalized branch can characterize the voltage loss caused by the current flowing from node i to node j , which is called transmission impedance.

3.2. Power flow tracing

Power flow tracing is an effective method to establish generalized branches and obtain transmission impedances. Based on the power flow results, firstly form multiple single-power supply networks by downstream tracking. Then, for each single-power supply network, do upstream tracking to find the power supply paths from each power source to each load. Combining with voltages, we can calculate the transmission impedances between every two nodes on a power supply path [11-13]. The transmission impedance between two nodes in original network is equal to the parallel result of transmission impedances between the two nodes of all power supply paths. Obviously, the transmission impedance is a finite value only when there is power transmission between nodes. If there is no transmission, the impedance is infinite.

In this paper, the target of power flow tracing is \dot{S} instead of Q , partly because Q itself has no direction. In addition, the grid delivers complex power so that \dot{S} contains more electrical information than Q .

3.3. The bidirectional description of power transmission and the definition of electrical distance

3.3.1. Forward power transmission

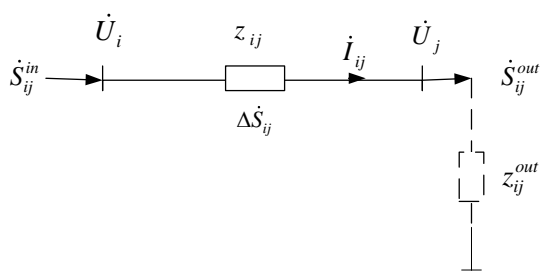


Figure 2. Forward power transmission

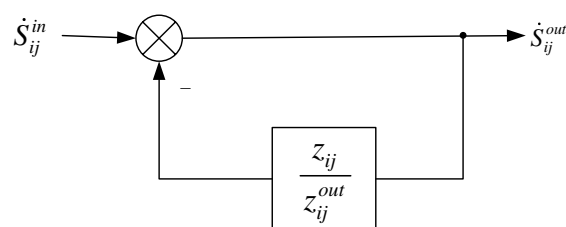


Figure 3. Block diagram of forward transmission

According to Figure 2, there are

$$\dot{S}_{ij}^{in} = \Delta \dot{S}_{ij} + \dot{S}_{ij}^{out} \quad (8)$$

$$\dot{S}_{ij}^{out} = \dot{U}_j \cdot (\dot{I}_{ij})^* = z_{ij}^{out} \cdot \dot{I}_{ij} \cdot (\dot{I}_{ij})^* \quad (9)$$

$$\Delta \dot{S}_{ij} = (\dot{U}_i - \dot{U}_j) \cdot (\dot{I}_{ij})^* = z_{ij} \cdot \dot{I}_{ij} \cdot (\dot{I}_{ij})^* \quad (10)$$

According to formulas(9)(10), there is

$$\Delta \dot{S}_{ij} = \frac{z_{ij}}{z_{ij}^{out}} \cdot \dot{S}_{ij}^{out} \quad (11)$$

According to formulas(8)(11), there is

$$\dot{S}_{ij}^{in} = \left(1 + \frac{z_{ij}}{z_{ij}^{out}} \right) \cdot \dot{S}_{ij}^{out} \quad (12)$$

Deform the formula(12), there is

$$\frac{\dot{S}_{ij}^{out}}{\dot{S}_{ij}^{in}} = \frac{1}{1 + \frac{z_{ij}}{z_{ij}^{out}}} \quad (13)$$

It can be seen that the formula(13) describes the relationship between the input power and the output power of branch $i - j$. Draw the block diagram as Figure 3. We can see that the block diagram contains a negative feedback part that represents the ratio of the loss power to the output power. The numerator of feedback part is the transmission impedance of branch $i - j$.

3.3.2. Reverse power transmission

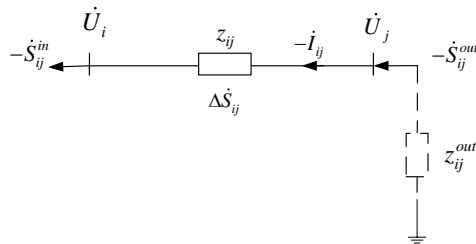


Figure 4. Reverse power transmission

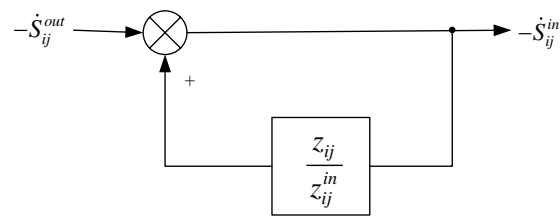


Figure 5. Block diagram of reverse transmission

According to Figure 4, imitate the derivation process of formulas(8)~(13) and we can get

$$\frac{-\dot{S}_{ij}^{in}}{-\dot{S}_{ij}^{out}} = \frac{1}{1 - \frac{z_{ij}}{z_{ij}^{in}}} \quad (14)$$

Similarly, it can be seen that the formula(14) describes the relationship between the input power and the output power of branch $j - i$. Draw the block diagram as Figure 5. We can see that the block diagram contains a positive feedback part that represents the ratio of the loss power to the input power. The numerator of feedback part is the transmission impedance of branch $j - i$.

It can be found that the numerators of feedback parts of two block diagrams are all the transmission impedances. Whereas distances are generally defined as scalars for comparison and applied to partitions, we take the absolute value of transmission impedance as electric distance.

4. Case study

In order to verify the feasibility of the proposed method, the IEEE three machines nine nodes system is taken as an example to apply this method and achieve reactive power partitioning. The shortest distance clustering method is adopted for the partition method [14].

4.1. Calculation results and subarea application

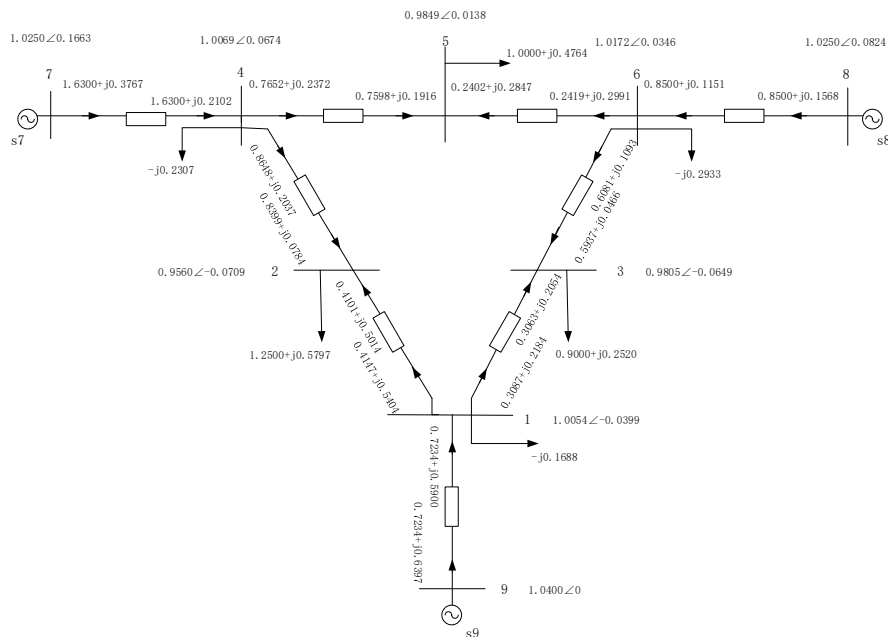


Figure 6. IEEE three machines nine nodes system

By downstream tracking and upstream tracking, we can find power transmission paths from each power source to each load and calculate the transmission impedances. Shown as follows:

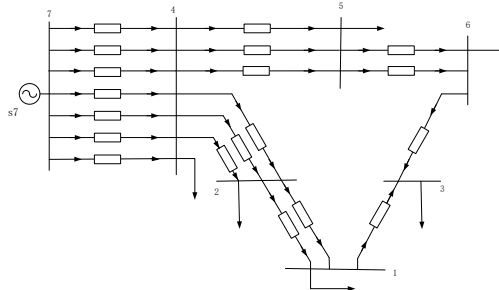


Figure 7. Transmission impedances by s 7

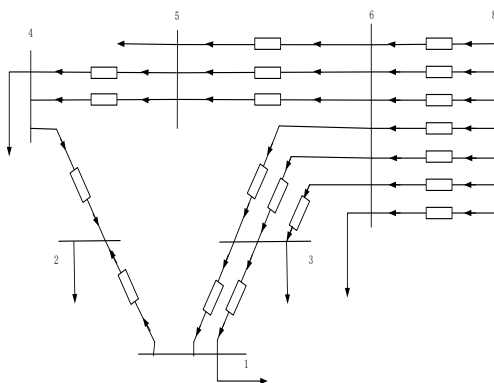


Figure 8. Transmission impedances by s 8

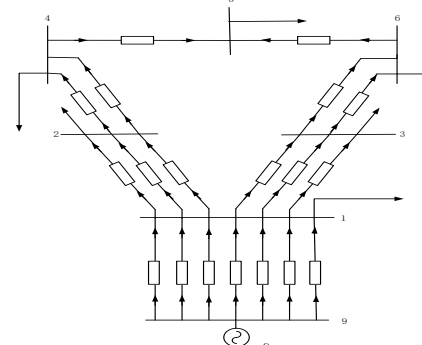


Figure 9. Transmission impedances by s 9

As shown figures (Figure 6, Figure 7, Figure 8 and Figure 9) above, we can get transmission impedances between nodes under the influence of each power source. For the original network, the transmission impedance between two nodes is equal to the parallel result of transmission impedances between the two nodes under the influence of 3 power sources. We take the absolute value of

transmission impedance as electric distance. In order to compare with the sensitivity method, the electrical distances between the PQ nodes are only listed in Table 1 and Table 2.

Table 1. The electrical distances of transmission impedance method

Electrical distance	node1	node2	node2	node3	node4	node5	node6
node1	0	0.0856	0.0936	0.6951	0.1886	1.3268	0
node2	0.0856	0	0.3294	0.1641	0.7306	0.3303	0.0856
node3	0.0936	0.3294	0	0.2908	0.3010	0.1744	0.0936
node4	0.6951	0.1641	0.2908	0	0.0725	0.2538	0.6951
node5	0.1886	0.7306	0.3010	0.0725	0	0.1015	0.1886
node6	1.3268	0.3303	0.1744	0.2538	0.1015	0	1.3268

Table 2. The electrical distances of sensitivity method

Electrical distance	node1	node2	node2	node3	node4	node5	node6
node1	0	1.2070	1.2873	2.9487	3.4520	3.0799	0
node2	1.2070	0	2.3910	1.9812	2.9118	3.4616	1.2070
node3	1.2873	2.3910	0	3.4604	3.2376	2.0962	1.2873
node4	2.9487	1.9812	3.4604	0	1.1508	2.5794	2.9487
node5	3.4520	2.9118	3.2376	1.1508	0	1.5639	3.4520
node6	3.0799	3.4616	2.0962	2.5794	1.5639	0	3.0799

Cluster according to the shortest distance and here are the dendrograms:

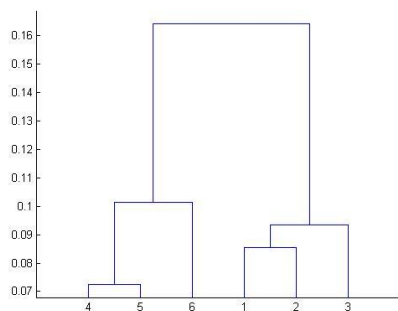


Figure 10. Dendrogram of new method

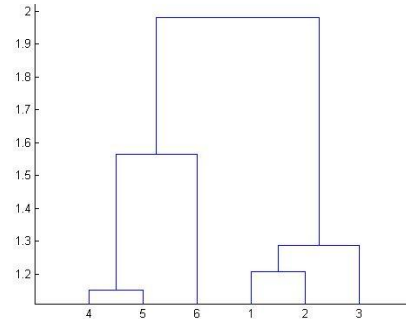
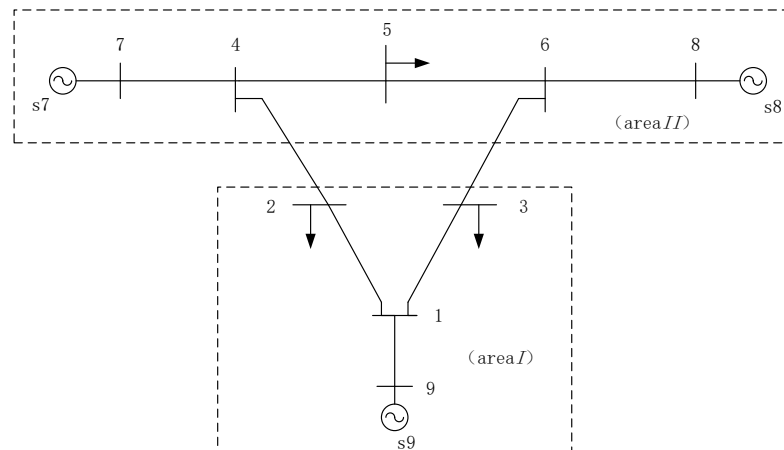
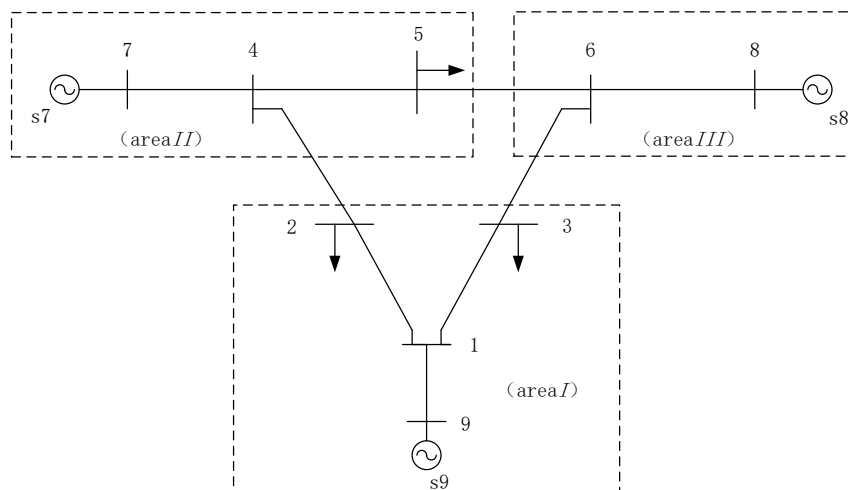


Figure 11. Dendrogram of sensitivity method

Comparing Figure 10 with Figure 11, it can be seen that the clustering processes of transmission impedance method and sensitivity method are consistent, so the partition results are always same no matter the system is divided into two or three areas, which fully demonstrates the feasibility of transmission impedance method applied for electrical distance calculation and subarea division. According to the merging principle of PV nodes [15], the generators of 7, 8 and 9 are divided into different regions, and the final partition results are shown in Figures 12 and 13:

**Figure 12.** partition results of 2**Figure 13.** partition results of 3

4.2. Results after adjusting power flow

Increase the output of generator 7, reduce the output of generator 8, and adjust the loads properly. The results of electrical distance calculated by transmission impedance method and sensitivity method are listed in Table 3 and Table 4.

Table 3. The electrical distances of transmission impedance method after adjusting power flow

Electrical distance	node1	node2	node2	node3	node4	node5	node6
node1	0	0.0856	0.0936	0.5744	0.1591	0.9293	0
node2	0.0856	0	0.2411	0.1641	0.5182	0.3153	0.0856
node3	0.0936	0.2411	0	0.2823	0.2956	0.1744	0.0936
node4	0.5744	0.1641	0.2823	0	0.0725	0.2765	0.5744
node5	0.1591	0.5182	0.2956	0.0725	0	0.1015	0.1591
node6	0.9293	0.3153	0.1744	0.2765	0.1015	0	0.9293

Table 4. The electrical distances of sensitivity method after adjusting power flow

Electrical distance	node1	node2	node2	node3	node4	node5	node6
node1	0	1.2285	1.3013	3.1047	3.4780	2.9864	0

node2	1.2285	0	2.4252	3.4874	2.9275	2.0034	1.2285
node3	1.3013	2.4252	0	2.1091	3.2513	3.4902	1.3013
node4	3.1047	3.4874	2.1091	0	1.5539	2.5672	3.1047
node5	3.4780	2.9275	3.2513	1.5539	0	1.1405	3.4780
node6	2.9864	2.0034	3.4902	2.5672	1.1405	0	2.9864

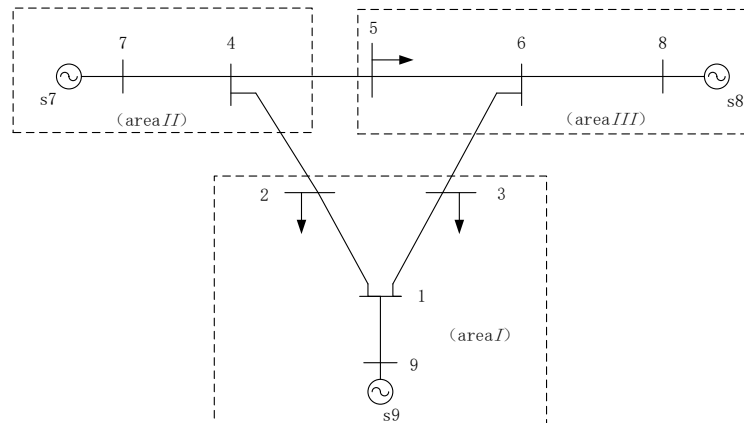


Figure 14. 3 partitions of sensitivity method after adjusting power flow

After clustering according to the shortest distance, it can be found that after the power flow adjustment, when the partition number is 2, the partitioning results of two methods all remain unchanged. When the partition number is 3, the partitioning result of the transmission impedance method is unchanged, while the result of sensitivity method is different where node 5 belongs to area *III*, as shown in Figure 14. Although the partitioning results are not identical before and after the adjustment, the partition schemes all satisfy the evaluation of the partition results [16].

Because the partition results of two methods are different for node 5, now we can identify which partition scheme is more accurate according to reactive power compensation ability to node 5 of generators in area *II* and *III* [17]. Increasing the reactive power of load 5 by 10%, the reactive power changes of generator 7 and 8 are obtained: $\Delta Q_{G7} = 0.001$, $\Delta Q_{G8} = 0.0004$. It can be seen that generator 7 has more reactive power compensation to load 5, so the electrical coupling relationship between them is stronger, and node 5 should be divided into area *II*. It proves that the subarea division based on the transmission impedance method can adapt to the dynamic change of the system better.

5. Conclusion

The widely used sensitivity method of calculating electric distance is inaccuracy and lack of physical meaning. This paper presents a new method for calculating electrical distance, namely transmission impedance method. It forms power supply paths based on power flow tracing, then establishes generalized branches to calculate transmission impedances. By describing the power transmission relationship of the branch and drawing block diagrams in both forward and reverse directions, it can be found that the numerators of feedback parts of two block diagrams are all the transmission impedances. To ensure the distance is scalar, the absolute value of transmission impedance is defined as electrical distance. Dividing network according to the electric distance and comparing with the results of sensitivity method, it proves that the new method can adapt to the dynamic change of system better and reach a reasonable subarea division scheme.

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