

Optimal Output of Distributed Generation Based On Complex Power Increment

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Abstract. In order to meet the growing demand for electricity and improve the cleanliness of power generation, new energy generation, represented by wind power generation, photovoltaic power generation, etc has been widely used. The new energy power generation access to distribution network in the form of distributed generation, consumed by local load. However, with the increase of the scale of distribution generation access to the network, the optimization of its power output is becoming more and more prominent, which needs further study. Classical optimization methods often use extended sensitivity method to obtain the relationship between different power generators, but ignore the coupling parameter between nodes makes the results are not accurate; heuristic algorithm also has defects such as slow calculation speed, uncertain outcomes. This article proposes a method called complex power increment, the essence of this method is the analysis of the power grid under steady power flow. After analyzing the results we can obtain the complex scaling function equation between the power supplies, the coefficient of the equation is based on the impedance parameter of the network, so the description of the relation of variables to the coefficients is more precise. Thus, the method can accurately describe the power increment relationship, and can obtain the power optimization scheme more accurately and quickly than the extended sensitivity method and heuristic method.

1. Introduction

Energy shortage and environmental problems are the bottleneck of the development of energy industry. In the electric power industry, the bottleneck behave as the uneven distribution of power and load, and the environmentally unfriendly power generation process. In order to balance the difference of power and load of China's eastern region and western region, help save energy and reduce emission, in the "13th Five-Year of power development plan" the National Energy Bureau stressed the priority layout of clean energy, to ensure the completion of the hard target that non fossil energy consumption reach to 15%[1]. Due to policy incentives and technological advances in power generation, the new energy generation represented by wind power generation, photovoltaic power generation and energy storage system has been introduced into distribution network in the form of distributed power supply[2]. The flexibility of power generation, the cleanness of process and other characteristics, lead to the scale of distributed power generation becoming more and more large, many problems have emerged, including grid connected mode, optimized configuration and optimized scheduling[3].

When the scale of distributed generation connected to the grid is small, administrators apply the management mode called "fit-and-forget"[4], with the continuous expansion of the scale of distributed



generation and the increase of adjustability of distributed generation, it is feasible and necessary to optimize the output planning. At present, the method of distributed generation optimization is divided into heuristic algorithm and classical mathematical optimization algorithm. The optimization target can be divided into two aspects: the highest comprehensive profitability and the highest profitability of Power Grid Corp. Document [5-8] uses heuristic algorithm to solve their own objective function, including genetic algorithm, particle swarm optimization algorithm, difference algorithm and so on. In the application of heuristic algorithm, each article has improved the algorithm for its own problems, but still cannot avoid the shortcomings such as slow speed of calculation and unstable results. In document [9, 10], the classical mathematical optimization method is adopted to optimize the objective function, including interior point method and two sequence programming method. The core of the classical method is the calculation of extended sensitivity. In classical mathematical optimization method, the calculation of the extended sensitivity ignores the coupling relation among the node variables, so the result is not accurate.

2. Analysis of extended sensitivity and complex power increment

With the progress of power generation technology, distributed power supply has a certain capacity of reactive power regulation [11, 12], in the limited power range, the power output can be adjusted freely. Based on this, the distributed power is considered as the power source with adjustable active and reactive power. There is an equation in the steady state power flow of power grid:

$$S_0 = \sum_{i=1}^l S_{Li} - \sum_{i=1}^q S_{DGi} + S_{loss} \quad (1)$$

Among them, S_0 is the injection power of the system node, and is regarded as the slack node in the text, i is the serial number of the load power, and the total quantity of the load is expressed by l , S_{DGi} is the distributed generation output power, the total quantity expressed by q , S_{loss} is the current loss power of the network. According to the equation, the key point is to analyze the incremental relationship between the network loss and the power supply in order to extract the incremental relationship between S_0 and S_{DGi} .

2.1. Grid loss analysis of extended sensitivity

The grid loss analysis of extended sensitivity is essentially to find out the relation among node power and grid loss power, node voltage value and phase angle respectively. After dividing, the incremental relationship between grid loss and power supply can be obtained. The concrete deduction process is as follows.

$$P_{loss} = \sum_{i=1}^n U_i * \sum_{j=1}^n U_j * (G_{ij} * \cos q_{ij} + B_{ij} * \sin q_{ij}) \quad (2)$$

$$Q_{loss} = \sum_{i=1}^n U_i * \sum_{j=1}^n U_j * (G_{ij} * \sin \theta_{ij} - B_{ij} * \cos \theta_{ij}) \quad (3)$$

$$P_i = U_i * \sum_{j=1}^n U_j * (G_{ij} * \cos \theta_{ij} + B_{ij} * \sin \theta_{ij}) \quad (4)$$

$$Q_i = U_i * \sum_{j=1}^n U_j * (G_{ij} * \sin \theta_{ij} - B_{ij} * \cos \theta_{ij}) \quad (5)$$

According to formula (2) ~ (5), the formula is deduced as follows:

$$\frac{dP_{loss}}{dP_i} = \sum_{m=1}^{n-1} \frac{dP_{loss}}{dU_m} * \frac{1}{\frac{dP_i}{dU_m}} + \sum_{m=1}^{n-1} \frac{dP_{loss}}{d\theta_m} * \frac{1}{\frac{dP_i}{d\theta_m}} \quad (6)$$

$$\frac{dP_{loss}}{dQ_i} = \sum_{m=1}^{n-1} \frac{dP_{loss}}{dU_m} * \frac{1}{dQ_i/dU_m} + \sum_{m=1}^{n-1} \frac{dP_{loss}}{d\theta_m} * \frac{1}{dQ_i/d\theta_m} \quad (7)$$

$$\frac{dQ_{loss}}{dP_i} = \sum_{m=1}^{n-1} \frac{dQ_{loss}}{dU_m} * \frac{1}{dP_i/dU_m} + \sum_{m=1}^{n-1} \frac{dQ_{loss}}{d\theta_m} * \frac{1}{dP_i/d\theta_m} \quad (8)$$

$$\frac{dQ_{loss}}{dQ_i} = \sum_{m=1}^{n-1} \frac{dQ_{loss}}{dU_m} * \frac{1}{dQ_i/dU_m} + \sum_{m=1}^{n-1} \frac{dQ_{loss}}{d\theta_m} * \frac{1}{dQ_i/d\theta_m} \quad (9)$$

In the process of calculating the extended sensitivity, the coupling relation between the voltage value and the phase angle of different nodes is neglected, which results in inaccurate results.

2.2. Grid loss analysis of complex power increment

The essence of grid loss analysis based on the complex power increment is tracing the branch loss according to the source of current, then the complex proportion function of S_{loss} and S_{DGi} can be obtained. Because the complex coefficient is composed of the impedance parameters of the grid, the incremental relation between the grid loss and the power supply is obtained by the derivation of the formula. The concrete deduction process is as follows:

$$S_{loss,i} = (U_{i1} - U_{i2}) * [(-Y_{i1,i2}) * (U_{i1} - U_{i2})]^* \quad (10)$$

The load under the steady state flow is equivalent to the ground impedance, a new admittance matrix is formed by adding the earth impedance to the node admittance matrix, expressed by YN . The nodal voltage equation $I_s = YN * U \Rightarrow U = YN^{-1} * I_s$ is substituted into the formula (10) to obtain the formula:

$$S_{loss,i} = (U_{i1} - U_{i2}) * [(-Y_{i1,i2}) * \sum_{j=0}^q (YN^{-1}_{i1,j} - YN^{-1}_{i2,j}) * I_{js}]^* \quad (11)$$

Divide the formula (11) by the distributed generation output $S_{DGi} = U_i^* I_{is}$, and then get the function:

$$S_{loss,i} = \sum_{j=0}^q \frac{U_{i1} - U_{i2}}{U_j} * [(-Y_{i1,i2}) * (YN^{-1}_{i1,j} - YN^{-1}_{i2,j})]^* * S_j \quad (12)$$

In the above formula, S_j for all power generation in the grid, including system injection and distributed generation, to simplify the representation, use G_{ij} to express the coefficients in front of S_j , subscript i is the serial number of the branch, and j is the serial number of the power after simplification, the function is:

$$S_{loss,i} = \sum_{j=0}^q G_{ij} * S_j \quad (13)$$

It can be seen from the formula G_{ij} that the complex proportionality factor only depends on the impedance parameter of the grid under the steady state power flow, and can be regarded as a fixed value. Therefore, the grid loss description based on the complex power increment is more accurate than the extended sensitivity method.

Summary of all network branch loss form the whole loss:

$$S_{loss} = \sum_{i=1}^m S_{loss,i} = \sum_{i=1}^m \sum_{j=0}^q G_{ij} * S_j = \sum_{j=0}^q G_{loss,j} * S_j \quad (14)$$

Spread the above formula according to the real part and the imaginary part, derivation of active power and reactive power of distributed power generation gives us the grid loss increment based on complex power increment.

2.3. An example of increment analysis of grid loss

In order to demonstrate the accuracy of the grid loss increment obtained by two methods, the 14-node distribution grid model with distributed generation is selected to perform analysis. The number of nodes in the model is $n=14$, and the node of system injection is selected as slack node, its number is 14. There are 4 nodes that are distributed generation access points, and their numbers are 1, 6, 7 and 9. The grid branch number is $m=13$. In order to maintain the safe and stable operation of the power grid, the amount of distributed generation output will not exceed 15% of the total power of the grid. It is assumed that the output of the distributed generation is 50% active power and 40% reactive power. According to the formulas of the last two sections, calculate the incremental derivative of grid loss, then, calculate the incremental value of the grid loss when the output of distributed generation is increased by 1%. For the sake of simplicity, only the derivative of the active power of the grid loss to the active power of the distributed generation is listed in the following table 1:

Table 1. Three methods comparing.

Increment value	Extended sensitivity method	Complex power increment method	Numerical method
dP_{loss}/dP_1	-0.003548856696838	-0.018556638568433	-0.030976916460812
dP_{loss}/dP_6	-0.001335291930379	-0.033946516255844	-0.059520110509137
dP_{loss}/dP_7	0.051617733291405	-0.030641780835950	-0.053696161612660
dP_{loss}/dP_9	0.002129094978441	-0.019949855624131	-0.032213682314985

After comparing the data, it can be found that the derivatives obtained by applying the complex power increment method are closer to the numerical results than those obtained by the extended sensitivity method. This shows that the method of complex power increment can make the optimization process more rapid and accurate in the optimization of power output.

3. Interior point optimization method

The complex power increment method described in the previous section is a new method for obtaining the direction of steady state power flow, this method is more accurate and faster than the extended sensitivity method, but it still needs to be combined with specific optimization methods when solving specific problems. The interior point method is a superior and widely used continuous optimization method for solving the optimization problem of distributed generation [13]. In the optimization process, the interior point method combined with the complex power increment does not need to consider the equality constraint. Due to the analysis of complex power increment is based on steady-state power flow, the power flow balance is always kept in the optimization iteration process. Thus, compared to the interior point optimal power flow, which considered the power flow balance constraints, will be simpler and faster. Compared with the heuristic algorithm, the defect of the classical mathematical optimization method is easy to fall into the local optimal solution and make the result of optimization inaccurate [14]. But considering that the capacity of the distributed generation will not exceed 15% of the total power generation capacity, the feasible region of the function variable is smaller and the possibility of falling into the local optimal solution is greatly reduced. Therefore, the optimization scheme based on the complex power increment is simpler and faster than the existing optimization methods.

In this paper, the objective function takes the profit of distributed generation as the main factor, meanwhile, considers the profit of the grid and system power supply. Currently, distributed generation owners are often not grid companies, therefore, the maximum profit of power supply is the primary

purpose of its operation considerations. But the distributed power supply will affect the stability and profitability of the whole power grid, and its influence will increase with the capacity of the network increases. After the distributed power supply access to the grid, in order to guarantee the operation of them has a positive impact on the profit of power grid, when planning the output of the distributed power supply, the profit of the grid should be taken into account. Similarly, the system power is responsible for most of the active power load and almost the whole reactive power load. The increase of output of distributed generation access to the network will reduce the power supply of the system source and reduce the power factor, thus reduces the profitability of the traditional power plants. Therefore, in the optimization process, the impact of the profit on the system power (slack node) should also be taken into account. To sum up, the objective function, that is, the comprehensive profit per unit time (1hour) under steady flow, is:

$$obj : F = f_1 + k * f_2 + \lambda * f_3 \quad (15)$$

$$f_1 = \sum_{i=1}^q (D_i - C_i) * P_i \quad (16)$$

$$f_2 = \sum_{i=1}^l D_{Li} * P_{Li} - \sum_{i=1}^q D_{bi} * P_i - D_{b0} * P_0 \quad (17)$$

$$f_3 = D_0 * P_0 - (a_2 * P_0^2 + a_1 * P_0 + a_0) \quad (18)$$

In the objective function, D_i is the power on line price, that is, the power supplier selling price. In this paper, the price of wind power is 0.52 ¥/kWh, and the photoelectric power price is 1 ¥/kWh. D_{bi} is the purchase price of Power Grid Corp, due to policy factors, distributed power generation has central and local financial subsidies, therefore, in addition to the system power supply, there is a big gap between D_i and D_{bi} . Taking D_{bi} as 0.3 ¥/kWh. D_{Li} is the selling price of Power Grid Corp, which takes 0.6 ¥/kWh, and C_i is the comprehensive cost of distributed generation, according to the data in the literature [15], the cost of photovoltaic power generation is 0.2 ¥/kWh, and the cost of wind power generation is 0.15 ¥/kWh. a_1 , a_2 and a_3 are the cost coefficient of the system power generation. To simplify calculation, they are 0.008, 0.08 and 0, k and λ are the effects of grid profit and system power profit on the objective function, taking 0.5 and 0.25. In the function, P_i is the active power output of the power supply, the unit is kW, l is the number of the load, and q is the number of the distributed power supply.

Before to optimize the objective function, considering the price of different nodes are not the same, in function f_2 , the cost calculation node and the charging calculation node are different, therefore, the cost should be recalculated according to the conservation principle of fee flow [16]. The cost is expressed by C , and the active power is expressed by P , the conservation formula of nodes and branches are:

$$\sum C_i^{in} * P_i^{in} = \sum C_i^{out} * P_i^{out} \quad (19)$$

$$C_l^{in} * P_l^{in} = C_l^{out} * P_l^{out} \quad (20)$$

For fairness, the active power flow from the same node is assumed the same price, the function f_2 can be written in the following form:

$$f_2 = \sum_{i=1}^l \sum_{j=0}^q (D_{L,i} - D_{Lb,i,j}) * P_{L,i,j} \quad (21)$$

In this way, the variables in the object function are independent variables except P_0 , then replace it by the formula (1).

For the constraints of the function, we consider the upper and lower limits of the distributed power supply, the upper and lower limits of the network node voltage, and the upper and lower limits of the system power supply. Due to the distributed power supply capacity is small, the line power limit is not considered in this paper, considering that the power factor of the distributed power supply should not be too small, the reactive power limit shall not exceed 20% of the active power limit when the power limit is setted. The specific restrictions are as follows:

$$\begin{aligned} P_{i,\min} &\leq P_i \leq P_{i,\max} \\ Q_{i,\min} &\leq Q_i \leq Q_{i,\max} \\ U_{i,\min} &\leq U_i \leq U_{i,\max} \\ P_{0\min} &\leq P_0 \leq P_{0\max} \end{aligned} \quad (22)$$

When the constraints are taken into consideration, the initial values of the variables of the function are set within the feasible region, then the optimization results in the current situation can be obtained after iteration.

4. Example analysis

In order to demonstrate the analysis process of power output optimization, the distribution network model adopted in this paper is the model used in the third section, second chapter. Nodes 7 and 9 are wind turbines, nodes 1 and 6 are photovoltaic generators, their output limit is: active power 1MW and reactive power 0.2MVar. The system generation output limit is: active power 60MW and reactive power 12MVar, the per-unit value of voltage limit of each node is 0.95~1.05. The extended sensitivity algorithm and the complex power increment algorithm are used respectively to optimize the results:

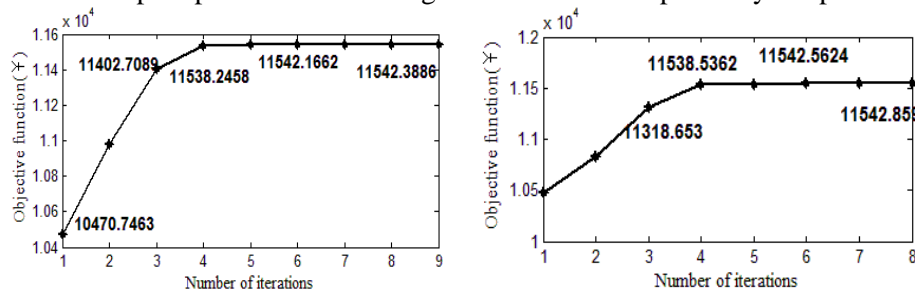


Figure 1. Comparison of optimization results.

Figure 1 shows: the diagram on the left is the result of the extended sensitivity method, the right one is the result of the complex power increment method, after comparison, it is found that the complex power increment optimization results are more accurate and more efficient. For further explanation, the direction of flow adjustment obtained by the method of complex power increment is more accurate, in the adjustment process, if we observe the power supply output adjustment of the two methods, we will find that the adjustment process of the active power is approximately the same, but there are differences in the adjustment of reactive power, and even the opposite direction of adjustment. The difference of reactive power adjustment will directly affect the overall grid loss, in the process of adjustment, the grid loss of the two methods are compared, listed in the figure 2:

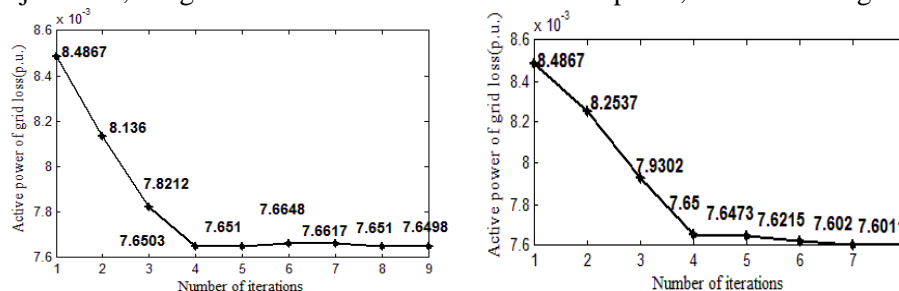


Figure 2. Comparison of grid loss active power adjustment.

In the optimization process, if we apply the method of complex power increment, the grid loss will decrease, if we apply the extended sensitivity method, the network loss will fluctuates. The comparison of the three sets of graphs shows that the optimization method based on the complex power increment is superior to the optimization method based on the extended sensitivity in the optimization speed and the accuracy of results.

5. Conclusion

First of all, the deficiency of sensitivity algorithm is pointed out in this paper, a new method based on complex power increment is proposed, which can obtain the derivative of variables more accurately.

Secondly, a power relation equation is established under steady-state power flow, according to the nodal voltage equation, we can replace and decompose the equation, then form a complex scaling function between the power supplies. This complex coefficient is composed of distribution network impedance, and can be regarded as fixed value in steady state power flow. Therefore, the power relation in steady state flow can be represented by linear equation.

Last, based on the idea of complex power increment, the coupling relationship between the parameters of the nodes is taken into account, therefore, the results are more accurate. The linearity of the equation makes the process more rapid, and this is also the reason why the method is superior to the extended sensitivity algorithm.

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