

Hydrodynamic Characteristics and Strength Analysis of a Novel Dot-matrix Oscillating Wave Energy Converter

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Abstract. The paper analyzes hydrodynamic characteristics and the strength of a novel dot-matrix oscillating wave energy converter, which is in accordance with nowadays' research tendency: high power, high efficiency, high reliability and low cost. Based on three-dimensional potential flow theory, the paper establishes motion control equations of the wave energy converter unit and calculates wave loads and motions. On this basis, a three-dimensional finite element model of the device is built to check its strength. Through the analysis, it can be confirmed that the WEC is feasible and the research results could be a reference for wave energy's exploration and utilization.

1. Introduction

With the gradual exhaustion of the conventional energy, more and more serious pollution to human environment it causes, the contradiction between conventional energy and development of the economy worsens increasingly[1,2]. Thus, many countries begin to develop clean renewable energy. As a kind of the renewable energy with giant reserves, extensive attention is paid to wave energy[3].

Some countries started to develop wave energy converters earlier. More than 30 types of wave energy converter have been built in the recent 20 years[4]. At present, the research tendency in the world is as follows. On shore, persistent attention is paid to oscillating water column wave energy converters with high power, whereas off shore is paid to other types with ingenious design and high efficiency; under the support of government and other institutions, many wave energy converters enter the stage of full-scale experiment or commercial operation.

Compared with these countries, China started relatively late. Not only the type of wave energy converters is single, but also the technology is laggard[5]. Therefore, in the future, high power, high efficiency, high reliability and low cost are the research tendency of China.

Towards this tendency, the paper analyzes hydrodynamic characteristics and the strength of a novel dot-matrix oscillating wave energy converter (referred as the WEC, hereafter). The WEC is composed of the wave energy converter unit (referred as unit, hereafter), which can be assembled to the dot matrix based on demand.

The paper establishes the unit's motion control equations. Based on three-dimensional potential flow theory, the paper analyzes hydrodynamic characteristics of the unit, including wave loads and



motions. Then, a three-dimensional finite element model of the WEC is built. Based on the above result of hydrodynamic analysis, the paper checks its strength of all the 6 load combinations.

Through the above analysis, it can be confirmed that this novel WEC is feasible, in accordance with the research tendency of our country-- high power, high efficiency, high reliability and low cost. The research results could be a reference for wave energy's exploration and utilization.

2. Introduction of the novel Dot-matrix Oscillating Wave Energy Converter

The novel WEC is composed of the wave energy converter unit, which is the main part, shown in Figure. 1[6]. The unit can be assembled to the dot matrix by square tubes based on demand. By this way high power can be realized. The paper studies on a 3×3 array, shown in Figure 2[6]. The design parameters of each component are shown in the Figures.

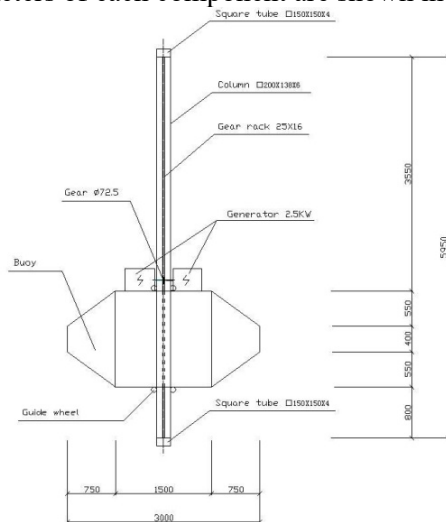


Figure 1. The wave energy converter unit.

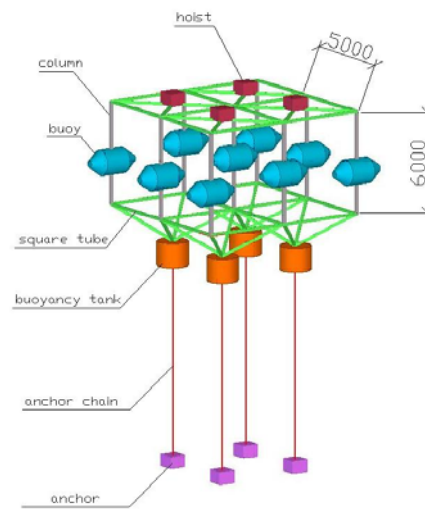


Figure 2. The 3×3 array.

3. Hydrodynamic Characteristics Analysis

3.1 Calculation Method

The paper establishes the unit's motion control equations, using three-dimensional potential flow theory to calculate wave loads towards the buoy and its motion responses [7-9]. The rectangular coordinate system OXYZ is selected, with its origin at the buoy's center of gravity when static balancing. The XOY plane is parallel to the static water and the Z-axis is vertically up.

The general velocity potential includes the contribution of the incident wave, the contribution of the floating body disturbing the flow field and the contribution of the floating body's motion disturbing the flow field, shown as follows:

$$\phi = \phi_R + \phi_D, \quad (1)$$

$$\phi_R = i\omega \sum_{j=1}^6 \eta_j \phi_j, \quad (2)$$

$$\phi_D = \phi_0 + \phi_\gamma, \quad (3)$$

where ϕ is the complex form of the general velocity potential, η_j denotes the complex amplitude of floating body's displacement of six degrees of freedom, ϕ_j is the unit radiation potential, ϕ_0 is the incident potential, ϕ_γ is the diffraction potential.

The general velocity potential satisfies Laplace equation and four kinds of boundary conditions: the free surface condition, the bottom condition, the wet surface condition and the infinite point condition.

The incident potential of the incident wave is:

$$\phi_0 = \frac{gA}{i\omega} \frac{ch[k_0(z+h)]}{ch(k_0h)} e^{ik_0(x \cos \beta + y \sin \beta)} \quad (k_0 h = \frac{\omega^2}{g}), \quad (4)$$

where A is the amplitude of the incident wave, k_0 is the wave number, ω is the frequency of the incident wave, β describes the direction of the incoming wave.

The definite conditions of the radiation potential $\phi_j (j=1,2,\dots,6)$ and the diffraction potential ϕ_7 are:

$$\nabla^2 \phi_j = 0 \quad (\text{in the flow field}) (j=1,2,\dots,7), \quad (5)$$

$$\frac{\partial \phi_j}{\partial z} - v \phi_j = 0 \quad (z=0) (v = \frac{\omega^2}{g}; j=1,2,\dots,7), \quad (6)$$

$$\frac{\partial \phi_j}{\partial z} = 0 \quad (z=-h) \quad \text{OR} \quad \nabla_{\phi_j} \rightarrow 0 \quad (z \rightarrow -\infty; j=1,2,\dots,7), \quad (7)$$

$$\frac{\partial \phi_j}{\partial n} = n_j \quad (j=1,2,\dots,6), \quad \frac{\partial \phi_7}{\partial n} = -\frac{\partial \phi_0}{\partial n} \quad (\text{on the } s_H), \quad (8)$$

and
$$\phi_j = O\left(\frac{1}{\sqrt{\rho}} e^{ik_0 \rho}\right) \quad (j=1,2,\dots,7), \quad (9)$$

where s_H denotes the average wet surface of the floating body.

In order to solve $\phi_j (j=1,2,\dots,7)$, the paper introduces Green function, which satisfies Laplace equation, the free surface condition, the bottom condition and the infinite point condition. The Green function of infinite depth with no speed in three-dimensional frequency domain is:

$$G(p,q) = \frac{1}{r_{pq}} + \frac{1}{r_{pq}^-} + 2vP.V. \int_0^\infty \frac{1}{k-v} e^{k(z+\xi)} \times J_0(kR) dk + b e^{vz} J_0(vR). \quad (10)$$

Applying Green formula and introducing a proper internal solution, the distributed source model is established to solve $\phi_j (j=1,2,\dots,7)$:

$$\phi_j(p) = \iint_{s_H} \sigma_j(q) G(p,q) ds_q, \quad (11)$$

where $\sigma_j(q)$ is the density of the distributed source.

Applying Green formula to solve the wet surface condition, the paper derives the integral equation of $\sigma_j(q)$ is:

$$2\pi\sigma_j(q) + \iint_{s_H} \sigma_j(q) \frac{\partial G(p,q)}{\partial n_p} ds_q = \begin{cases} -\frac{\partial \phi_0}{n(p)} & (p \text{ is on the } s_H) \\ n_j(p) & \end{cases} \quad (12)$$

Then, the Hess-Smith method is adopted to solve $\sigma_j(q)$, which changes the above integral equation to the system of linear algebraic equations, by making s_H discrete.

Through the above calculation, the paper derives the general velocity potential ϕ . Utteriorly, the dynamic pressure P to the floating body can be obtained by Bernoulli equation:

$$p(x,y,z,t) = -\rho \frac{\partial \Phi}{\partial t} = \text{Re}[i\rho\omega(\phi_0 + \phi_7 + \sum_{j=1}^6 v_j \phi_j) e^{-i\omega t}]. \quad (13)$$

Thus, the fluid dynamic forces (F_1, F_2, F_3) and the dynamic moments (F_4, F_5, F_6) to the floating body are:

$$F_k = \iint_{s_H} p n_k ds = \text{Re}[(f_{0k} + f_{7k} + \sum_{j=1}^6 T_{kj} v_j) e^{i\omega t}], \quad (14)$$

where $\bar{n} = (n_1, n_2, n_3)$ is the unit normal vector of S_H , pointing to the inner of the floating body, \bar{r} is the position vector, $\bar{r} \times \bar{n} = (n_4, n_5, n_6)$.

The sum of the incident forces/moments f_{0k} and the diffraction forces/moments f_{7k} is the wave excitation forces/moments to the floating body in the regular wave, where f_{0k} is the main part, called Froude-Kriloff forces/moments. Furthermore, the radiation forces/moments are made up of two parts: one is directly proportional to the floating body's acceleration, where the proportional coefficient A_{ij} is called the added mass, and the other is directly proportional to the floating body's velocity, where the proportional coefficient B_{ij} is called the wave making damping coefficient.

Besides the fluid dynamic forces/moments, the static force/moment—the restoring force/moment also exists, which is the contribution of $-\rho g z$ in Bernoulli equation.

Towards the buoy of this new WEC, the paper presents the system of motion control equations:

$$\sum_{j=1}^6 [(M_{ij} + A_{ij})\ddot{\eta}_j + B_{ij}\dot{\eta}_j + C_{ij}\eta_j] = F_i e^{i\omega t} + F'_i e^{i\omega t} \quad (i = 1, 2, \dots, 6), \quad (15)$$

where M_{ij} is the mass matrix, C_{ij} is the coefficient of the restoring force complicated with the gravity, F_i is the wave excitation forces/moments, F'_i is the resistances/moments by the column, F'_3 is the meshing force.

Because the paper has a try to analyze hydrodynamic characteristics of the unit preliminarily, F'_i is not considered in this paper. In the situation, the result of wave excitation forces/moments F_i is as the same as the real situation, whereas the result of motions is different. The next work of this paper is to do physical model test to measure F'_i . Combined with theoretical analysis, we can obtain F'_i . Then, adding it to the model, the real result of motions can be derived.

3.2 Transfer Function

From the above, the paper can derive wave forces/moments and motions of six degrees of freedom in the regular wave, defined response variables, referred as $R(\omega, \beta, t)$. The reported responses are normalized with respect to the incident wave amplitudes. With a transfer function $H(\omega, \beta)$ the corresponding time dependent response variable $R(\omega, \beta, t)$ can be expressed as:

$$R(\omega, \beta, t) = A |H(\omega, \beta)| \cos(\omega t + \phi), \quad (16)$$

where A is the amplitude of the incoming wave, ω is the frequency of the incoming wave, β describes the direction of the incoming wave, t denotes time and ϕ describes the phase angle between the incident wave and the time varying response. $|H(\omega, \beta)|$ is the amplitude of the transfer function, referred as RAO. The transfer function and the phase angle may be expressed as:

$$H = H_{\text{Re}} + iH_{\text{Im}} \quad \text{and} \quad \phi = \tan^{-1} \frac{H_{\text{Im}}}{H_{\text{Re}}}. \quad (17)$$

Known elements of the incident wave to solve each response variable, the transfer function can be derived based on Eq. (20), including RAO and ϕ . Then, it is convenient to solve the floating body's loads and motions in various regular waves.

3.3 Building the Model

Because the paper analyzes hydrodynamic characteristics of the buoy based on three-dimensional potential flow theory, including wave loads and motions, a panel model of the buoy is built.

Basic components of the panel model are plane units of quadrilateral or triangle, composed the floating body's wet surface. There is the source and sink strength on the plane units uniformly, expressing the distribution of the flow field's velocity potential.

The global coordinate system is selected, with its origin at the buoy's geometric center and the X-axis along the buoy's axis. Known the total weight of the buoy 4.71kN, barycentric coordinates are (0,0,0.3). The buoy's hydrodynamic model is shown in Figure 3. In addition, the draft 0.31m in the condition of static water can be derived.

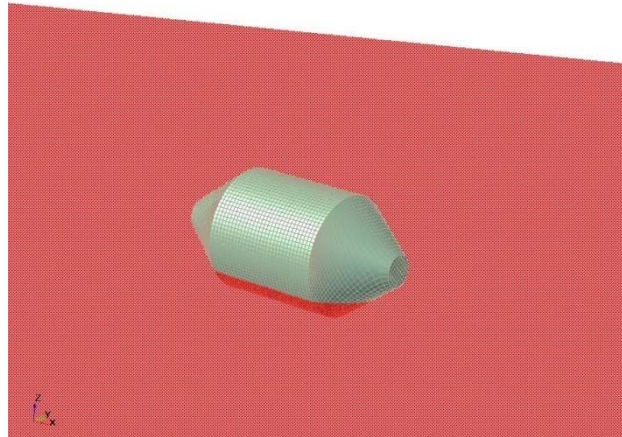
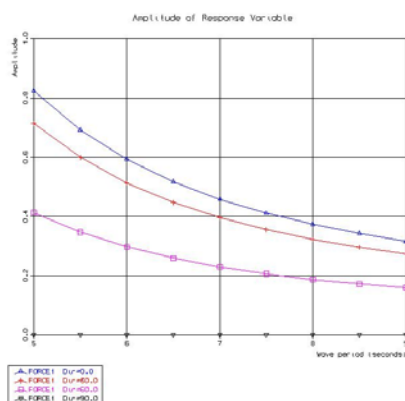


Figure 3. The buoy's hydrodynamic model.

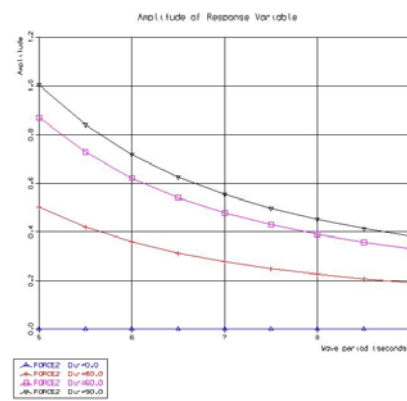
3.4 Calculation Conclusion

Known that the depth of buoy's sea area is 20m, the acceleration of gravity is 9.8 m/s^2 and the density of sea water is 1025 kg/m^3 . Considering the symmetry of the model, the paper selects the wave direction $0^\circ \sim 90^\circ$, with an interval of 30° . Based on the statistical data of the buoy's sea area, the paper selects the wave period $5 \text{ s} \sim 9 \text{ s}$, with an interval of 0.5 s .

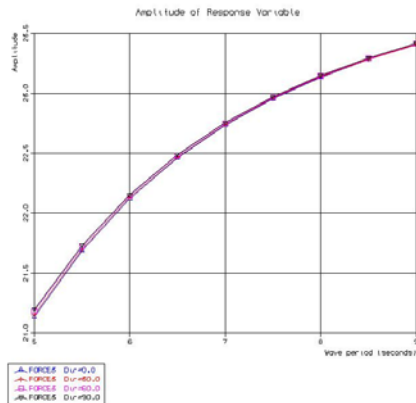
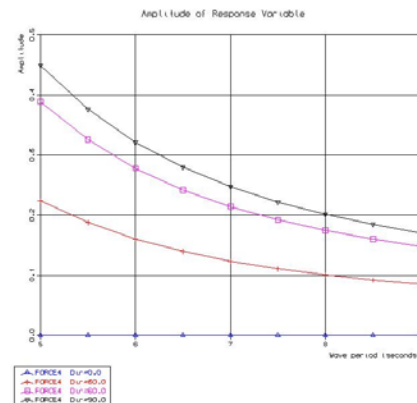
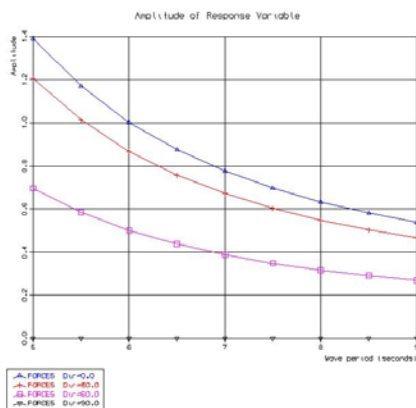
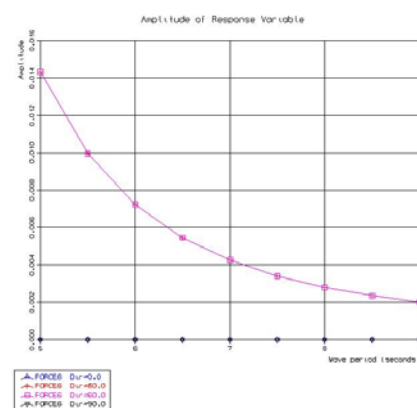
Through calculating, the solution of wave excitation forces/moments RAO to the buoy can be derived, shown in Figure 4.



(a) F_1

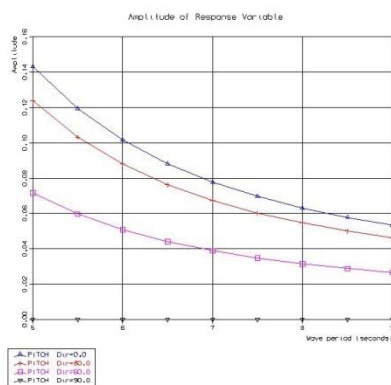
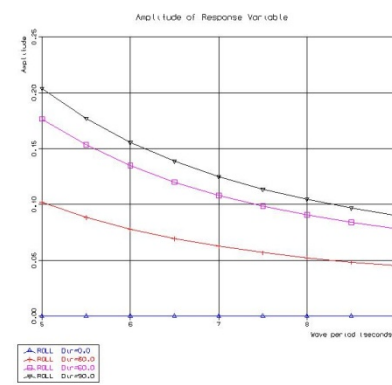


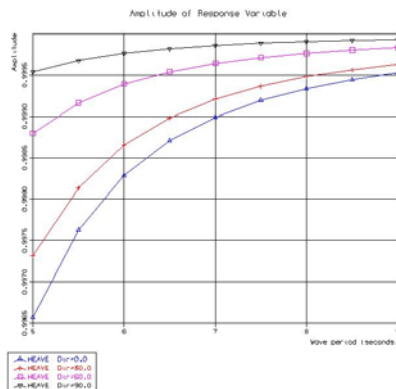
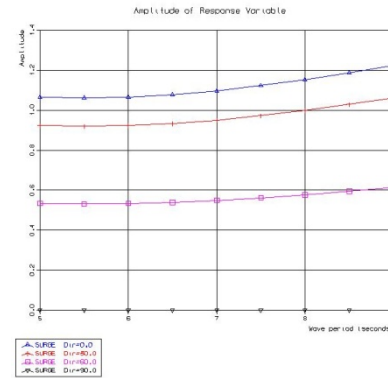
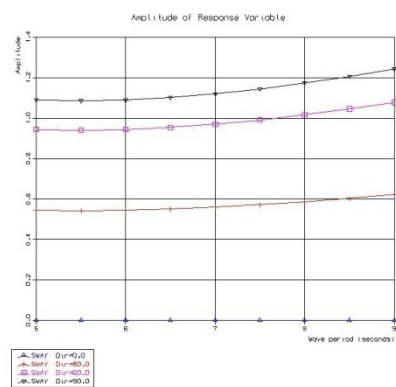
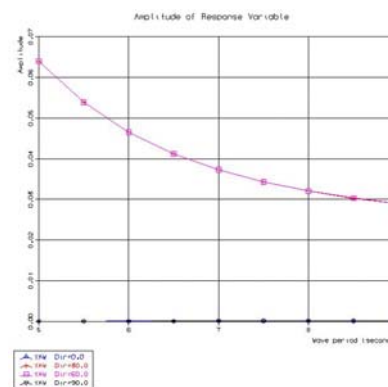
(b) F_2

(c) F_3 (d) F_4 (e) F_5 (f) F_6 **Figure 4.** Wave excitation forces/moments RAO.

Bases on the wave statistical data of buoy's sea area, the design wave height $H=2.5\text{m}$ is chosen. Through conversing, the maximum amplitude of the wave excitation force F_3 is 29.36kN , when the incident wave direction is 90° and the wave period is 9s . Correspondingly, the maximum amplitude of wave excitation force F_1 , F_2 is $1.52 \times 10^{-5}\text{kN}$ and 0.48kN respectively.

Through calculating, the solution of buoy's motions RAO can be derived, shown in Figure 5. Through conversing, the maximum amplitude of the buoy's heave motion η_3 is 1.25m , in the situation $H=2.5\text{m}$, $\beta = 90^\circ$, $T=9\text{s}$ (T denotes the wave period).

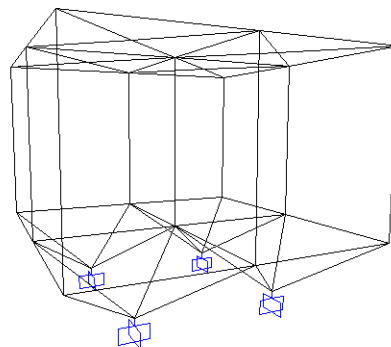
(a) η_1 pitch(b) η_2 roll

(c) η_3 heave(d) η_4 surge(e) η_5 sway(f) η_6 yaw**Figure 5.** Buoy's motions RAO.

4. Strength Check of the New Dot-matrix Oscillating Wave Energy Converter

4.1 Building a Three-dimensional Finite Element Model

Because the WEC can lift following the flood and ebb tide to make sure that the draft of the column is constant, the WEC can be seen static, relative to the initial sea level. The three-dimensional finite element model of the WEC is shown in Figure 6.

**Figure 6.** The three-dimensional finite element model of the WEC.

4.2 Load Combination

The paper uses F_3 to approximate the meshing force F_3' , when F_1 , F_2 are equal to F_1' , F_2' . Known that the amplitude of F_1 is 1.52×10^{-5} kN, the amplitude of F_2 is 0.48 kN, the amplitude of F_3 is 29.36 kN.

In the situation $H=2.5\text{m}$, $T=9\text{s}$, $d=20\text{m}$ (d denotes the water depth), the wave length is solved $L=105.14\text{m}$, according to:

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right). \quad (18)$$

Considering the wave length $L=105.14\text{m}$ and the array's size, there are 6 load combinations, shown in Figure 7.

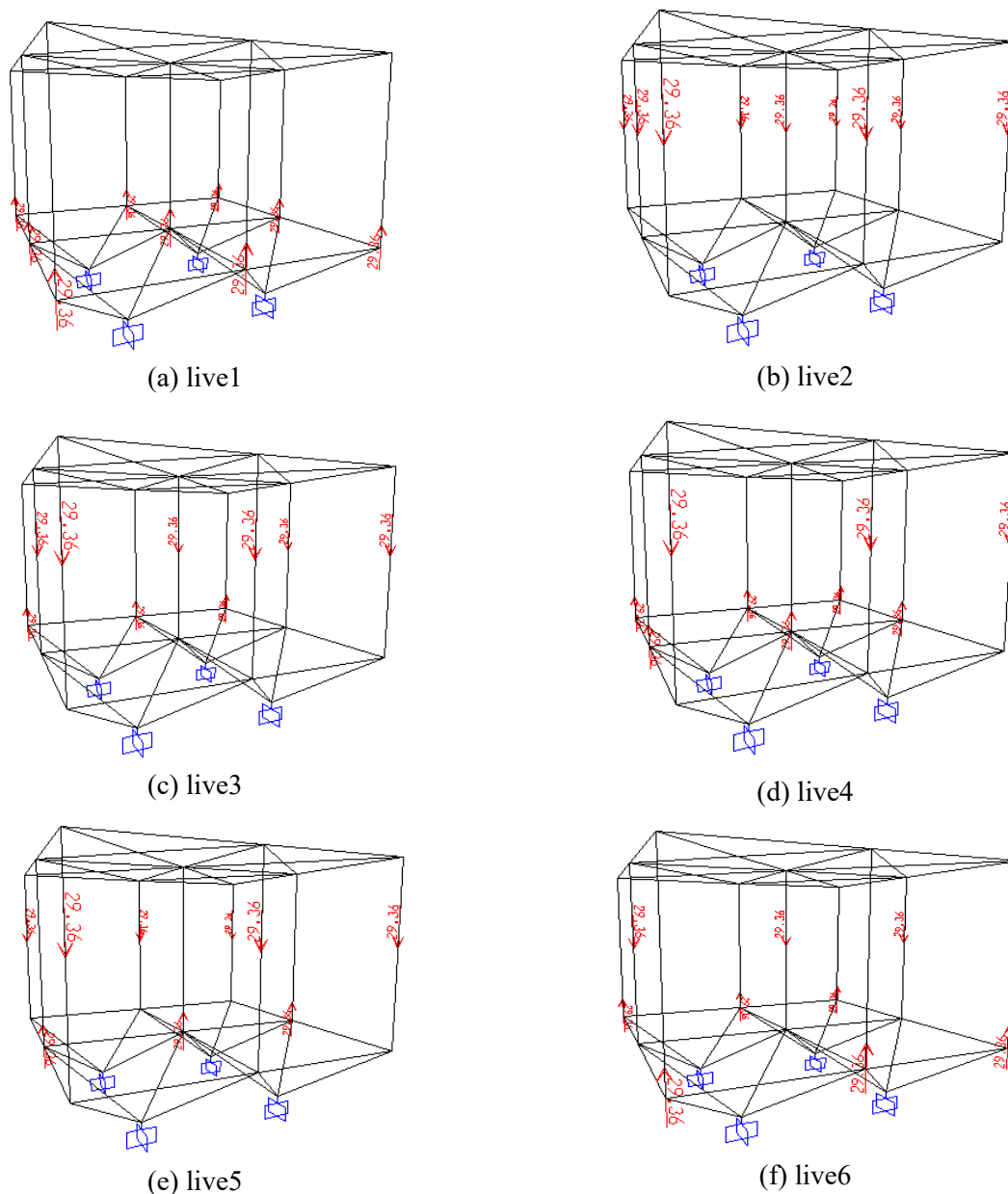


Figure 7. The 6 load combinations.

4.3 calculation conclusion

Towards the above 6 load combinations, the paper checks the strength of the WEC. The result is shown in Figure 8.

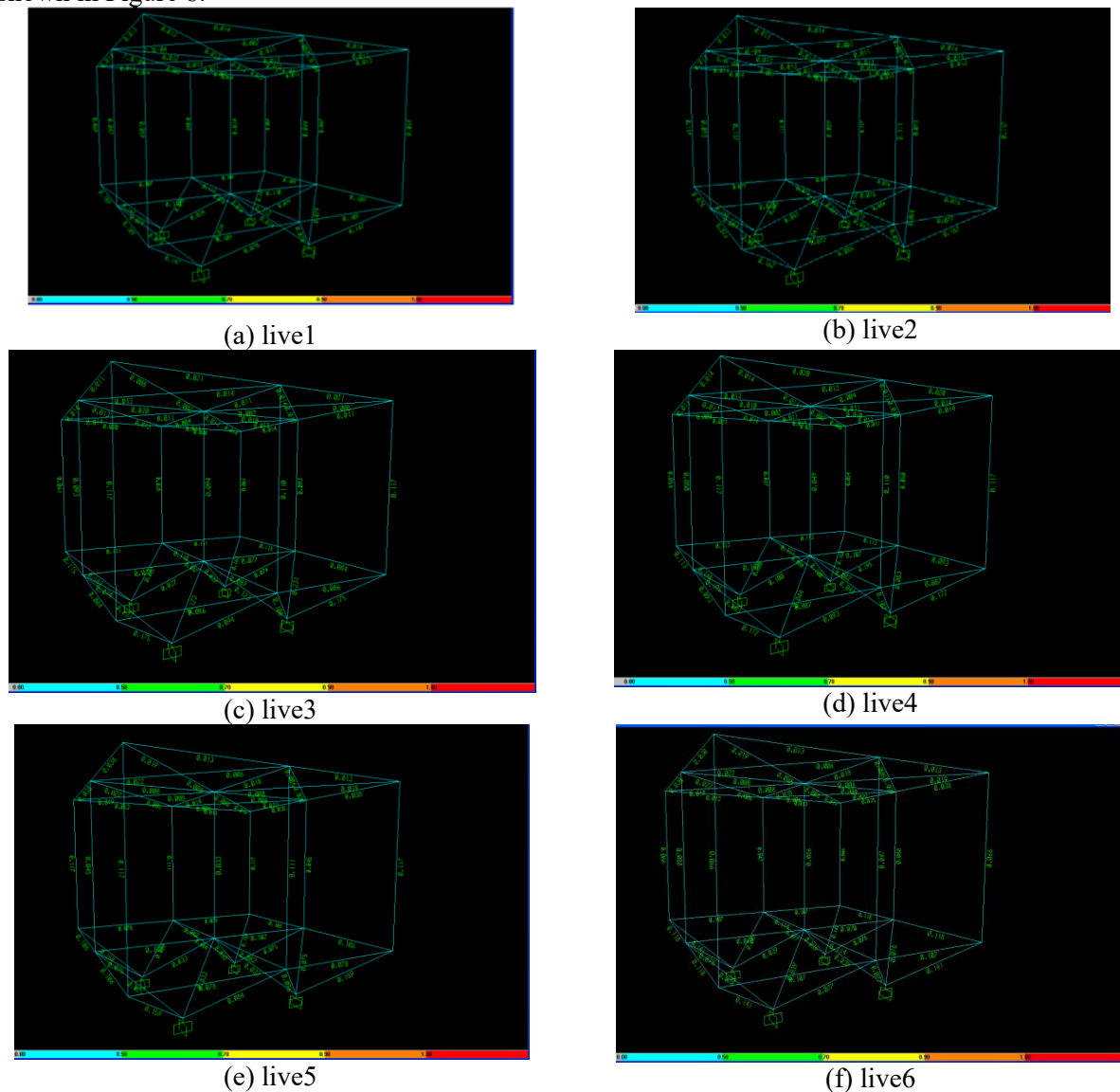


Figure 8. The strength of the WEC.

From Figure. 8, we can see that in the situation of all the 6 load combinations the strength of the new WEC satisfies the requirement. Thus, the design scheme is feasible.

5. Conclusions

Based on Chinese research tendency of wave energy converters-- high power, high efficiency, high reliability and low cost, the paper analyzes hydrodynamic characteristics and the strength of a novel dot-matrix oscillating wave energy converter

The paper establishes the unit's motion control equations. Based on three-dimensional potential flow theory, the paper analyzes hydrodynamic characteristics of the unit. The calculation method is introduced in detail and the transfer function is utilized. Based on that, the paper builds the three-dimensional hydrodynamic model, obtaining the hydrodynamic coefficients-- wave loads and its motions.

On the basis of hydrodynamic analysis, a three-dimensional finite element model of the WEC is built to check the strength. Under the 6 load combinations, it can be confirmed that the strength of this novel WEC satisfies the design requirement.

From the above analysis, the conclusion that the design scheme of this new WEC is feasible and in accordance with Chinese research tendency of wave energy converters in the future, can be derived. The research results could be a reference for wave energy's exploration and utilization.

The full-scale experiment has been done, confirming that the WEC is efficient and reliable. Thus, the next work of the paper is to analyze experiment data and optimize the converter.

In order to analyze hydrodynamic characteristics of the unit more accurately, the other next work is to do physical model test of the unit to measure F_i' . Combined with theoretical analysis, we can obtain F_i' . Then, adding it to the model, the real result of motions can be derived.

Acknowledgements

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