

# Inversion effects on wind and surface pressure in atmospheric front propagation simulation with a hyperbolic model

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**Abstract.** In this paper the effects of an inversion layer in a stratified atmosphere on the surface wind speed and pressure are investigated with models based on the compressible Navier-Stokes equations in two dimensions. Artificial compressibility is introduced into the models in order to make the governing equations hyperbolic. For comparison with available simulation data, the physical processes under study are assumed to be adiabatic. Plain orography is considered in surface pressure simulations with a finite-difference version of the model, while surface wind speed effects are estimated in artificial cold front propagation over a hill with a finite-element version of the model. The front surface is described in both models by an equation for advection of a scalar substance, which is solved with a third-order semi-Lagrangian procedure. The results of simulations show various meteorological effects in agreement with observations and in accordance with a theory proposed by Charba [3].

## 1. Introduction

A front in the atmosphere is a phenomenon of gravitational flows that take place in a variety of forms: breeze fronts, storm flows etc [1,8]. Phenomena of great importance are cold atmospheric fronts propagating near the surface with high speeds. These fronts may be retarded and changed in shape under the influence of the underlying surface and stratification of the atmosphere. The retardation of a frontal system on the windward side of a mountain is a commonly observed phenomenon [11, 12].

Atmospheric gravity currents occupy a wide range of length scales from several meters to thousands of kilometers. These currents can be subdivided into classes varying from micro- to macro-scales. Meso-scale flows lie in an interval approximately from two to two thousand kilometers [9]. The flows of interest in the present study are meso-scale currents. These flows are relatively shallow: they belong to the atmospheric boundary layer and range only a few kilometers from the surface in the lower atmosphere.

Stratification and inversion effects on surface pressure in the propagation of an atmospheric gravity current (cold front) over flat terrain are estimated in the present study with a non-hydrostatic finite-difference and finite-element models of atmospheric dynamics. Artificial compressibility is introduced into the models in order to make the equations hyperbolic.

The property of hyperbolicity is important for the correct formulation of proper boundary conditions. It also allows efficient implementation of the computational algorithms and methods used in the simulation. An important step forward was recently made by Peshkov and Romenski [2] in formulating a hyperbolic model for viscous flows.

In this paper the effects of introducing an inversion layer into a stratified atmosphere on the surface wind speed and pressure are investigated with models based on the compressible Navier-Stokes equations in two dimensions.

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assumed to be adiabatic. Plain orography is considered in surface pressure simulations with a finite-difference version of the model, while surface wind speed effects are estimated for artificial cold front propagation over a hill with a finite-element version of the model. The front surface is described in both models by an equation for advection of a scalar substance, which is solved with a third-order semi-Lagrangian procedure. The results of simulations show various meteorological effects in agreement with observations and confirm a theory proposed by Charba [3].

The results of simulations of surface pressure under stable stratification with an inversion layer are presented. The model is based on spatial discretizations that conserve some important quantities of the phenomena under study like momentum and scalars. Also, an efficient procedure is used to calculate the advection of scalars. Also, in the present paper, a preliminary investigation is carried out to simulate cold front propagation over a steep hill in two dimensions with a finite-element model. A 2D finite-element model based on triangular elements is used to simulate the of cold front propagation over an idealized orographic obstacle.

## 2. 2D model equations

The Navier-Stokes equations for a compressible air flow are used here for the calculation of gravity flows in a stratified atmosphere. An exact formulation of the equations is given in [15]. A more detailed description of the model can be found, for instance, in [17]. In the present study, a two-dimensional finite-difference and finite-element versions of the model are employed [17]. The time discretization is similar to that proposed in [13]. It is also described in paper [16].

For the simulations a small-scale non-hydrostatic model is used, which is essentially a form of the Navier-Stokes equations in the Boussinesq approximation. A version of the equations relevant for a compressible fluid is used here. Since sound waves in the atmosphere have small effects on gravity flows like fronts, a compressible version of the Navier-Stokes equations is taken here to make the equations hyperbolic, and also for pure technical reasons. The equations are discretized with a splitting method in time, and the compressible form of the equations is suitable for an efficient splitting. Sound waves are then eliminated by the use of an Asselin-type filter [4]. The time discretization of the governing equations may be written as follows:

$$\begin{aligned} \delta_t U + \frac{\partial}{\partial x} \bar{P}^t + \frac{\partial}{\partial \xi} (G^{13} \bar{P}^t) &= -ADVU, \\ \delta_t W + \frac{1}{G^{1/2}} + \frac{\partial \bar{P}^t}{\partial \xi} + \frac{g \bar{P}^t}{Cs^2} &= BUOY - ADVW, \\ \frac{1}{Cs^2} \delta_t P + \frac{\partial}{\partial x} \bar{U}^t + \frac{1}{G^{1/2}} \frac{\partial \bar{W}^t}{\partial \xi} \\ + \frac{\partial}{\partial \xi} (G^{13} \bar{U}^t) &= PFT, \\ \bar{f}^t &= \frac{1+\alpha}{2} f^{t+\Delta t} + \frac{1-\alpha}{2} f^{t-\Delta t}, \\ \delta_t f &= \frac{f^{t+\Delta t} - f^{t-\Delta t}}{2\Delta t} \end{aligned}$$

The problem examined in this study has very small physical diffusion, and the governing equations can be considered as almost inviscid and adiabatic. Special operators of space discretization are used to provide conservation of momentum and scalars in the finite-difference model (for a detailed description see [15,17]). Triangular elements are used in the finite-element model [16]. In both model versions a suitable scheme for the advection of a scalar, e.g. temperature, is implemented.

For fully hyperbolic problems explicit schemes have been widely used in the past and will probably continue to be used in the future. Traditionally, problems for which the governing equations are hyperbolic have traditionally been solved by methods of characteristics. Sometimes the conventional methods of characteristics are slower than competitive finite difference methods. However, some finite difference schemes use a form of the governing equations such that knowledge of the characteristic locations can be exploited in an efficient way [10].

In this paper, the advection of a scalar like temperature is treated with an efficient semi-Lagrangian finite difference scheme of third-order [10] that is used as a reasonable compromise between cost and accuracy. A detailed description of the scheme can be found in [4].

### 3. FDM version : smooth surface pressure simulation

The results of a series of calculations are presented here to simulate the stratification and inversion effects on the surface pressure in the propagation of an atmospheric gravity current (cold front) over flat terrain. The model parameters are taken from paper [11]. The calculation domain is 25x25 km. In contrast to [11], where an atmospheric front is generated by a volume of cold air, in the present study the front is initially given in the form of a step- function of 400 m in height.

Figure 1 shows the initial location of the front. Figure 2 shows the results of calculations of the surface pressure at the point of 12 km under neutral stratification. In Figures 3 and 4 the time evolution of the surface pressure is given at the same spatial point in an atmosphere under stable stratification and in an atmosphere under stable stratification with an inversion layer.

Under stable stratification the front moves faster and shows an abrupt pressure jump at the point of observation. The introduction of an inversion layer into the atmosphere increases the pressure further. In contrast to the slow evolution of surface pressure under neutral stratification, there is a considerable pressure jump in a stable atmosphere. This effect is increased by the introduction of an inversion layer. This phenomenon was explained by Charba [3]. The results of calculations in the present paper are in good agreement with Charba's theory.

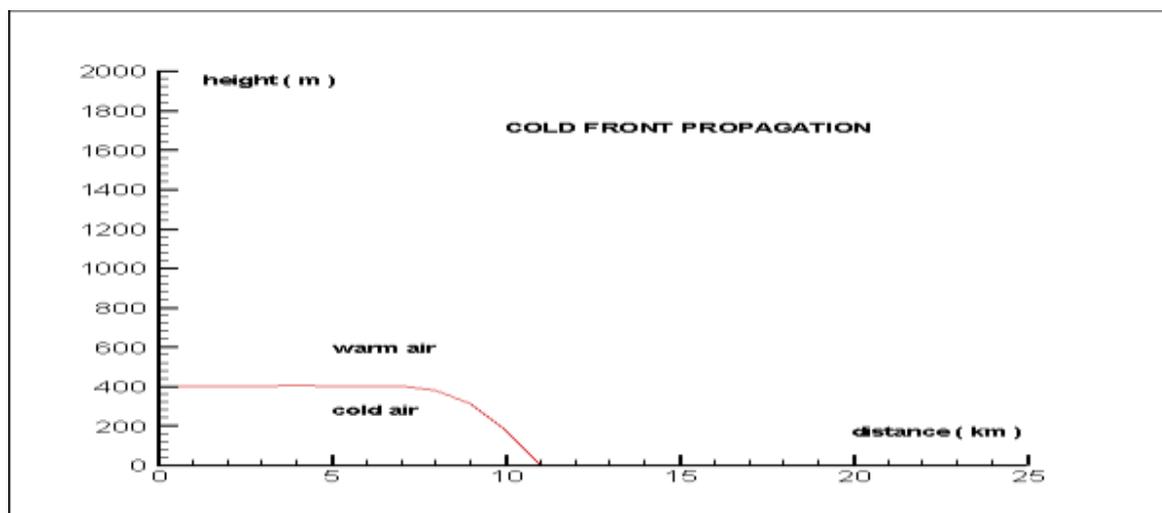
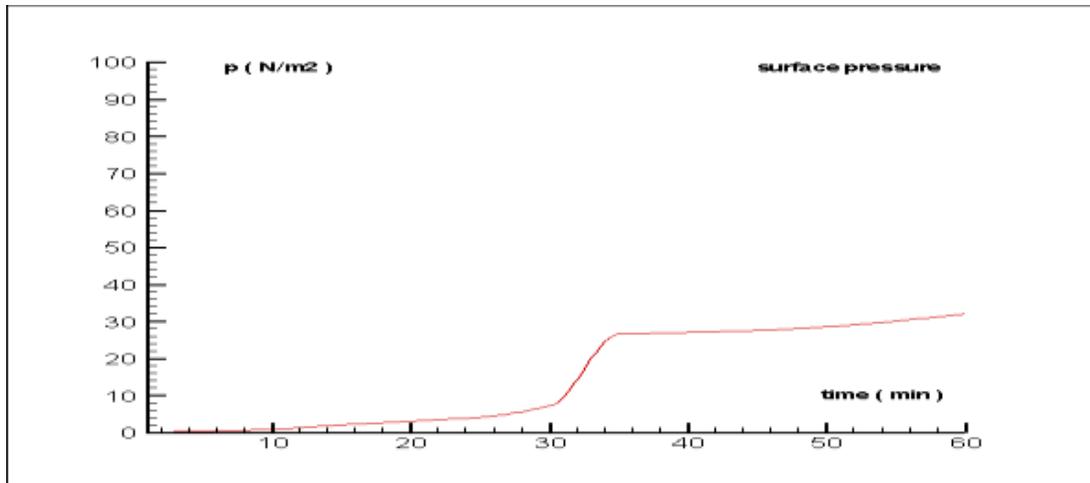
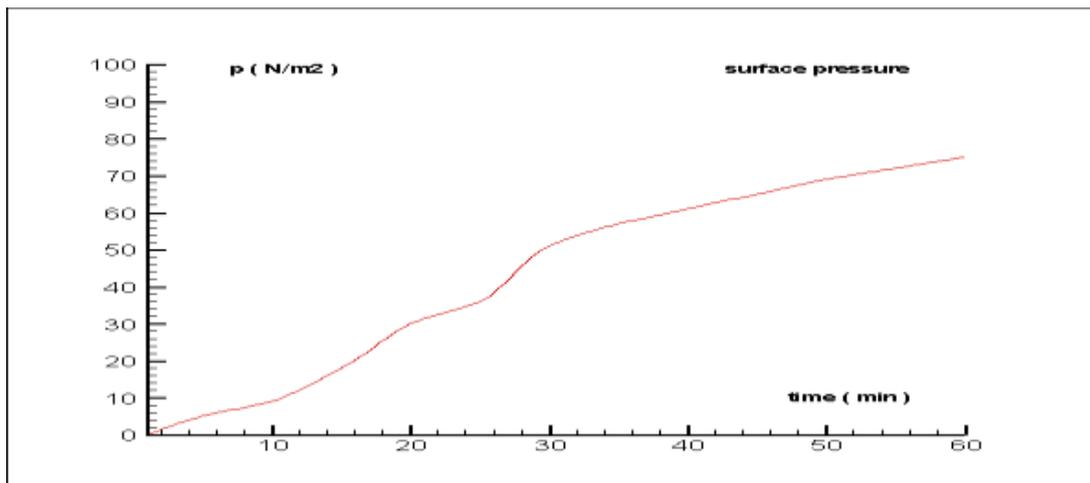


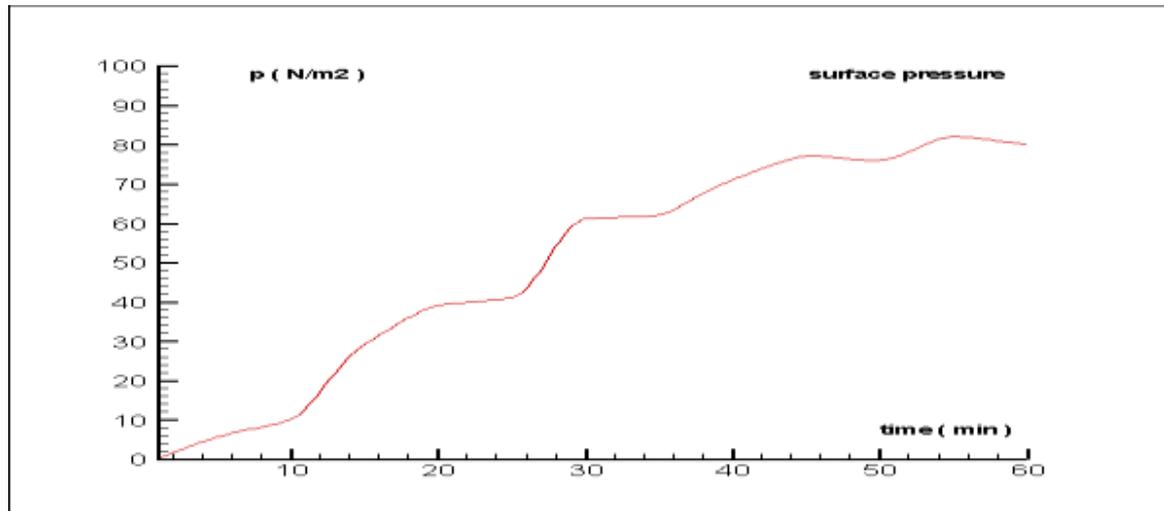
Figure 1. Gravity current over flat orography.



**Figure 2.** Surface pressure at 12 km. Neutral stratification.



**Figure 3.** Surface pressure at 12 km. Stable stratification.



**Figure 4.** Surface pressure at 12 km. Stable stratification with inversion.

#### 4. FEM version : steep surface wind speed simulation

In order to make the study more complete, in this section the results of paper [4] related to the effects of inversion on surface wind speeds in cold front motion over a steep obstacle are shown. Specifically, the effects of introducing an inversion layer into a stably stratified atmosphere are studied with a finite-element model described in paper [4]. The model parameters are taken as in paper [12]. In this paper, a cold front propagating over an axially-symmetric obstacle in the form of a bell-shaped hill is simulated with a finite-difference model. The height of the hill is 600 m. The calculation domain is 25x2 km. In contrast to [12], where the front is generated by a volume of cold air, in the present study the front is initially given in the form of a step-function of 400 m in height.

An inversion layer of 300 m in height is placed over the isolated hill described above. The atmosphere has the standard stable stratification of 3.5K/100 m, whereas the stratification of the inversion layer is 1K/100 m. The results of a series of calculations of cold front propagation over the hill and a plain are shown in Table 1. The calculated values of windward and leeward speeds are given in the last columns.

It can be seen from the table that the introduction of the inversion layer produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography. This is in agreement with the results of papers [11,12], where the front was generated by a volume of cold air. As in [11,12], the introduction of an elevated inversion layer causes a considerable intensification in the motion of vertical air and increases the meso-scale heat flux. This results in a more intensive entrainment.

**Table 1.** Calculated windward and leeward speeds of cold front propagation over a hill and a plain. Stable stratification.

obstacle height (m)	initial front height (m)	stratification ( K / 100 m)	windward speed (m/s)	leeward speed (m/s)
600	400	no inversion	4.9	2.7
600	400	Inversion	4.4	2.2
0	400	no inversion	5.1	5.1
0	400	Inversion	4.6	4.6

## 5. Conclusions

The results of the simulations described above have been obtained with two types of models: a 2D non-hydrostatic finite-difference meteorological model and a 2D finite-element model. Both models are discretized versions of a non-hydrostatic model formulated as the Navier-Stokes equations in the Boussinesq approximation written in a compressible form.

The 2D finite-element model is based on triangular elements. In this paper an application of the model was made to simulating cold front propagation over an idealized hill-type obstacle in a stratified atmosphere with an inversion layer over an isolated hill. The study was performed under stable stratification in and beyond the inversion layer. It has been shown that the introduction of the inversion produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography.

The 2D finite-difference model is based on spatial discretizations that conserve some important quantities of the phenomena under study, atmospheric gravity currents. Also, an efficient procedure is used in both versions of the model to calculate the advection of scalars.

The change in stratification from neutral to stable in the propagation of the cold atmospheric front has shown a time evolution of the surface pressure that is in good agreement with the available observational data [11]. Also, in contrast to the slow evolution of the surface pressure under neutral stratification, there is a considerable pressure jump in a stable atmosphere. This effect is increased by the introduction of an inversion layer. This phenomenon was explained by Charba [3]. The results of calculations in the present paper are in good agreement with Charba's theory.

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## References

- [1] Schultz D M 2005 A Review of Cold Fronts with Prefrontal Troughs and Wind Shifts *Mon. Wea. Rev.* **133** 2449–72
- [2] Peshkov I and Romenski E 2016 A hyperbolic model for viscous newtonian flows *Continuum Mechanics and Thermodynamics* **28** 85–104
- [3] Charba J 1974 Application of a gravity current model to analysis of squall-line gust fronts *Mon. Wea. Rev.* **102** 140–56
- [4] Yudin M S 2016 A numerical study of gravity waves in the atmosphere: smooth and steep orography effects *IOP Conference Series: Earth and Environmental Science* **48** DOI <http://dx.doi.org/10.1088/1755-1315/48/1/012024>
- [5] Marchuk G I 1974 *Numerical Solution of Atmospheric and Oceanic Problems* (Leningrad: Gidrometeoizdat) p 303
- [6] Marchuk G I 1982 *Mathematical Models in Environmental Problems* (Moscow: Nauka) p 319
- [7] Penenko V V and Aloyan A E 1985 *Models and Methods for Environmental Protection Problems* (Novosibirsk: Nauka) p 256
- [8] Pielke R A 2002 *Mesoscale Meteorological Modeling* (San Diego: Academic Press) p 676
- [9] Davies H C 1984 On the Orographic Retardation of a Cold Front *Beitr. Phys. Atmos.* **57** 409–18
- [10] Ritchie H 1987 Semi-Lagrangian Advection on a Gaussian Grid *Mon. Wea. Rev.* **115** 136–46
- [11] Bischoff-Gauss I and Gross G 1989 Numerical Studies on Cold Fronts. Part 1: Gravity Flows in a Neutral and Stratified Atmosphere *Meteorol. Atmos. Phys.* **40** 150–58
- [12] Bischoff-Gauss I, Gross G and Wippermann F 1989 Numerical Studies on Cold Fronts. Part 2: Orographic Effects on Gravity Flows *Meteorol. Atmos. Phys.* **40** 159–69
- [13] Ikawa M 1988 Comparison of Some Schemes for Non-Hydrostatic Models with Orography *J. Meteor. Soc. Japan* **66** 753–76

- [14] Schumann U 1987 Influence of Mesoscale Orography on Idealized Cold Fronts *J. Atmos. Sci.* **44** 3423–41
- [15] Yudin M S 2011 Propagation of a Gravity Current in the Atmosphere over a Valley *Bull. Novos. Comput. Center* **14** 65-70
- [16] Yudin M S and Wilderotter K 2006 Simulating Atmospheric Flows in the Vicinity of a Water Basin *Computational Technologies* **11** 128–34
- [17] Yudin M S 2012 Comparison of FDM and FEM Models for a 2D Gravity Current in the Atmosphere over a Valley *Bull. Novos. Comput. Center* **13** 95–101