Spatial Variation of Seismic B-Values of the Empirical Law of the Magnitude-Frequency Distribution from a Bayesian Approach Based On Spline (B-Spline) Function in the North Anatolian Fault Zone, North of Turkey

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Abstract. In this study, A Bayesian approach based on Spline (B-spline) function is used to estimate the spatial variations of the seismic b-values of the empirical law (G-R law) in the North Anatolian Fault Zone (NAFZ), North of Turkey. B-spline function method developed for estimation and interpolation of b-values. Spatial variations in b-values are known to reflect the stress field and can be used in earthquake hazard analysis. We proposed that b-values combined with seismicity and tectonic background. β=b*ln(10) function (the derivation of the G-R law) based on a Bayesian approach is used to estimate the b values and their standard deviations. A homogeneous instrumental catalog is used during the period 1900-2017. We divided into ten different seismic source regions based on epicenter distribution, tectonic, seismicity, faults in NAFZ. Three historical earthquakes (1343, *M*S = 7. 5, 1766, Ms=7.3, 1894, *M*S = 7. 0) are included in region 2 (Marmara Sea (Tekirdağ-Merkez-Kumburgaz-Çınarcık Basins)) where a large earthquake is expected in the near future because of a large earthquake hasn't been observed for the instrumental period. The spatial variations in ten different seismogenic regions are estimated in NAFZ. In accordance with estimates, b-values are changed between 0.52$\pm$0.07 and 0.86$\pm$0.13. The high b values are estimated the Southern Branch of NAFZ (Edremit Fault Zones, Yenice-Gönen, Mustafa Kemal Paşa, Ulubat Faults) region, so it is related low stress. The low b values are estimated between Tokat-Erzincan region, so it is related high stress. The maps of 2D and 3D spatial variations (2D contour maps, classed post maps (a group the data into discrete classes), image maps (raster maps based on grid files), 3D wireframe (three-dimensional representations of grid files) and 3D surface) are plotted to the b-values. The spatial variations b-values can be used earthquake hazard analysis for NAFZ.

1. Introduction

The North Anatolian Fault Zone (NAFZ) was one of the most important major continental strike-slip fault zones about 1200 km long from eastern Anatolia to the northern Aegean Sea. It forms the plate boundary between the Eurasian plate in the north and the Anatolian plate in the south (Barka, 1992; Sengör et al., 2005). It was one of the most active plate-bounding strike–slip faults in the world, slipping at a rate of 20–30 mm/yr.

In the last century, the entire NAFZ contained in a series of M>7 earthquakes and events started with the 1912 Ganos event west of the Sea of Marmara. The next major event occurred in Erzincan in 1939 year at the eastern Anatolia and then it was followed by a systematic westward propagation of earthquakes. This sequence ended with the M=7.4 İzmit and M=7.1 Düzce earthquakes at the east of the Sea of Marmara in 1999 year.

The NAFZ was estimated earthquake hazard parameters with Bayesian method for seismogenic source regions by Bayrak and Türker (2017). The Marmara Sea was estimated earthquake hazard parameters with Bayesian method. Region 9 in the NAFZ observed high seismicty and founded earthquake hazard level according to the determined parameters in region between Tokat and Erzincan. Marmara Region computed as 7.61, expected maximum earthquake size is 7.37 with 90 % occurrence probability in next 100 years.

A homogeneous catalog used between from 1900 to 2017 period. The NAFZ divided into ten different seismic source regions based on epicenter distribution, tectonic, geology, seismicity, faults (Figure 1). Three historical earthquakes (1343, M_S = 7. 5, 1766, Ms=7.3, 1894, M_S = 7. 0) are included in region 2 (Marmara Sea (Tekirdağ-Merkez-Kumburgaz-Çınarcık Basins)) where a large earthquake is expected in the near future because of a large earthquake hasn't been observed for the instrumental period.

The purpose of this study, A Bayesian approach based on Spline (B-spline) function used to estimate the spatial variations of the seismic b-values of the empirical law (G-R law) in the North Anatolian Fault Zone (NAFZ), North of Turkey.

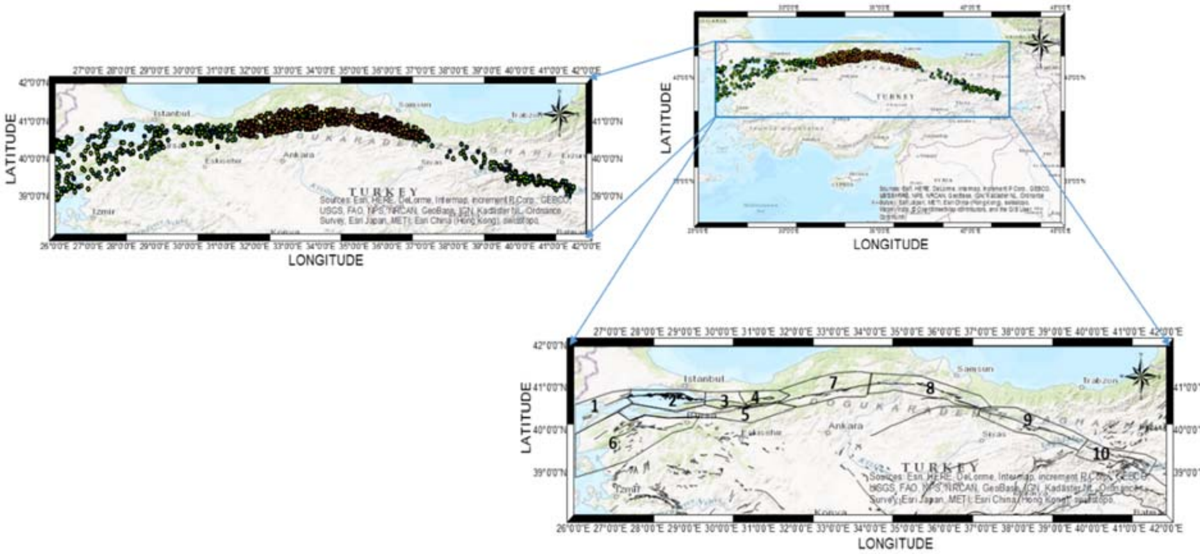

Figure 1. The NAFZ observed epicentre distribution of earthquakes between 1900-2017 period. The earthquakes are divided into 10 different seismic source regions with epicentre distribution of formed earthquakes in the instrumental period, focal mechanism solutions, and existing tectonic structures (Bottom figures and maps plotted for 1/750.000 scales). The NAFZ is demonstrated on Turkey map (Top figure and map plotted for 1/750.000 scale).

B-spline function method developed for estimation and interpolation of b-values. Spatial variations in b-values are known to reflect the stress field and can be used in earthquake hazard analysis. We proposed that b-values combined with seismicity and tectonic background. β=b*ln(10) function (the derivation of the G-R law) based on a Bayesian approach is used to estimate the b values and their standard deviations.

2. Method

In this study, we use free-knot spline functions to estimate and interpolate b-values in space. Since many parameters are required for the spline function, we maximize the log-likelihood function, which measures data fitting to obtain parameter estimations. On the other hand, we adopt the objective Bayesian method to avoid unnecessary fluctuation of the estimated spline surface.

A Bayesian approach with smoothness prior was suggested by Ogata et al. (1991) as an alternative approach to obtain the b-value, and it performs significantly better than the maximum likelihood estimate (MLE) approach. The smoothing spline is a method of fitting a smooth curve to a set of noisy observations using a spline function.

Ever since Gutenberg and Richter (1944) observed the empirical law of the magnitude–frequency distribution:

$$\log_{10} N(M) = a - bM \tag{1}$$

where N is the cumulative number of earthquakes with magnitude ≥M, the coefficient b has been playing a central role in earthquake hazard models.

The Gutenberg–Ritcher's law indicates the probability density function of the earthquakes with magnitude M as below:

$$f(M) = \beta \mathrm{e}^{-\beta(M-M_c)}, M > M_c. \tag{2}$$

Since we assume that b is dependent on the location of the space, the current likelihood function can be rewritten as:

$$L(\theta) = \prod_{i=1}^{n} \beta_\theta(x_i, y_i) \exp\{-\beta_\theta(x_i, y_i)(M_i - M_c)\} \tag{3}$$

The parameterization of the function b(x,y):

$$\beta_\theta(x, y) = \mathrm{e}^{\phi_\theta(x,y)} \tag{4}$$

2-D B-spline function:

$$\phi_\theta(x, y) = \sum_{l=0}^{L+3} \sum_{m=0}^{M+3} c_{l,m} F_l(x) G_m(y) \tag{5}$$

B-spline (DeBoor 1978):

$$N_j^1 = \begin{cases} 1 & (\xi_j \le x < \xi_{j+1}) \\ 0 & \text{otherwise} \end{cases}$$

$$N_j^k(x) = \frac{\xi_{j+k} - x}{\xi_{j+k} - \xi_{j+1}} N_{j+1}^{k-1}(x) + \frac{x - \xi_j}{\xi_{j+k-1} - \xi_j} N_j^{k-1}(x) \tag{6}$$

The local variation of a surface:

$$\varphi_1(\theta|w_1) = \int_v w_1\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y$$

$$\varphi_2(\theta|w_2) = \int_v w_2\left[\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\varphi}{\partial y^2}\right)^2\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

The penalized log-likelihood function (Ogata et al. 1991):

$$\begin{aligned} Q(\theta|w_1, w_2) &= \log L(\theta) - \varphi_1(\theta|w_1) - \varphi_2(\theta|w_2) \\ &= \sum_{i=1}^{n}\left[\phi_\theta(x_i, y_i) - \mathrm{e}^{\phi_\theta(x_i,y_i)}(M_i - M_c)\right] \\ &\quad - \varphi_1(\theta|w_1) - \varphi_2(\theta|w_2) \end{aligned} \tag{8}$$

The data set:

$$\{(x_i,\ y_i,\ M_i),\ \mathrm{i} = 1,2,\ldots n\} \tag{9}$$

In order to obtain the optimal weights, we introduce the Bayesian Likelihood (Akaike 1980; Good 1965) into method. Bayesian Likelihood:

$$L(w_1, w_2, c_p) = \int L(\theta)\pi(c^r, c_p|w_1, w_2)\mathrm{d}c^r \tag{10}$$

To obtain w1, w2, and cp, we need to maximize the Bayesian Likelihood. Log Bayesian Likelihood approach (Ogata et al. (1991):

$$\begin{aligned} \log L(w_1, w_2, \hat{c}_p) &= Q(\hat{\theta}|w_1, w_2) + \frac{1}{2}\log\{\det\Sigma_r\} \\ &\quad - \frac{1}{2}\log\{\det \boldsymbol{H}_r\}, \end{aligned} \tag{11}$$

3. Results and Discussions

The spatial variations in ten different seismogenic regions are estimated in NAFZ, table 1. In accordance with estimates, b-values are changed between 0.52±0.07 and 0.86±0.13. The high b values are estimated the Southern Branch of NAFZ (Edremit Fault Zones, Yenice-Gönen, Mustafa Kemal Paşa, Ulubat Faults) region, so it is related low stress. The low b values are estimated between Tokat-Erzincan regions, so it is related high stress. For example: spatial variations of b-values in region 3 and region 9 plotted 2D and 3D maps (Figure 2 and 3).

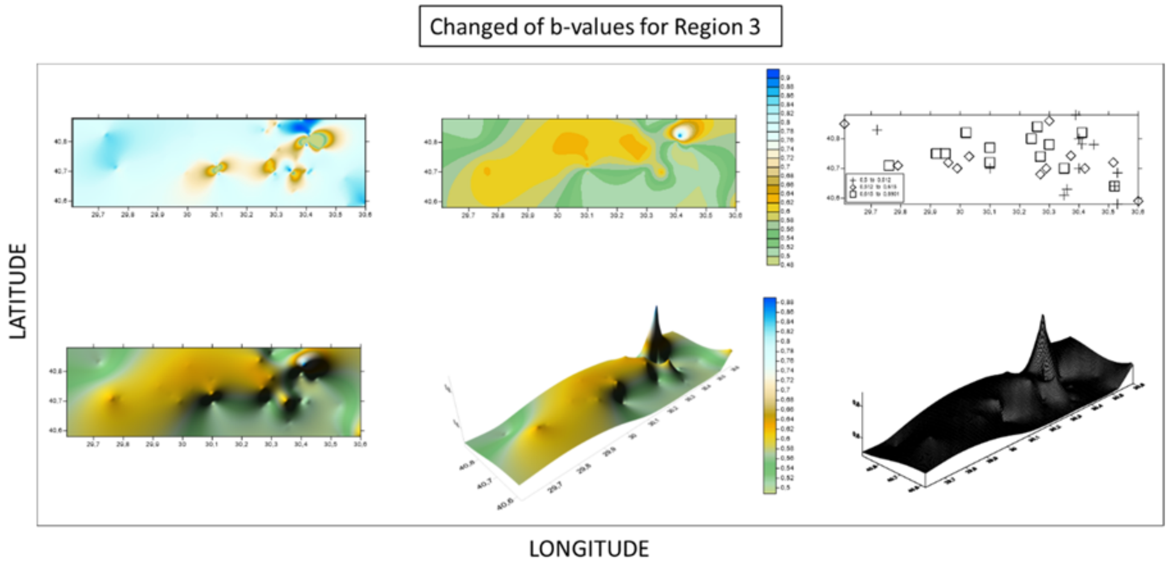

Figure 2. Spatial variations of b-values mapped for Region 3

The maps of 2D and 3D spatial variations (2D contour maps, classed post maps (a group the data into discrete classes), image maps (raster maps based on grid files), shaded maps, 3D wireframe (three-dimensional representations of grid files) and 3D surface) are plotted to the b-values (Figure 4 and 5). This study shows that spatial variations of the b-values can be used the study of earthquake hazard models.

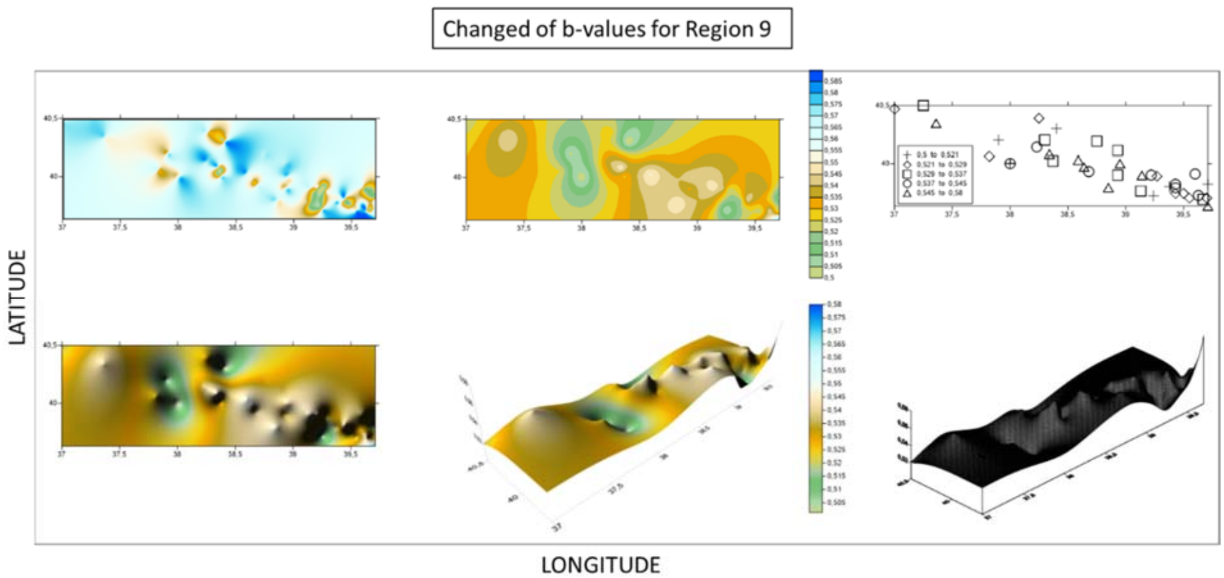

Figure 3. Spatial variations of b-values mapped for Region 9

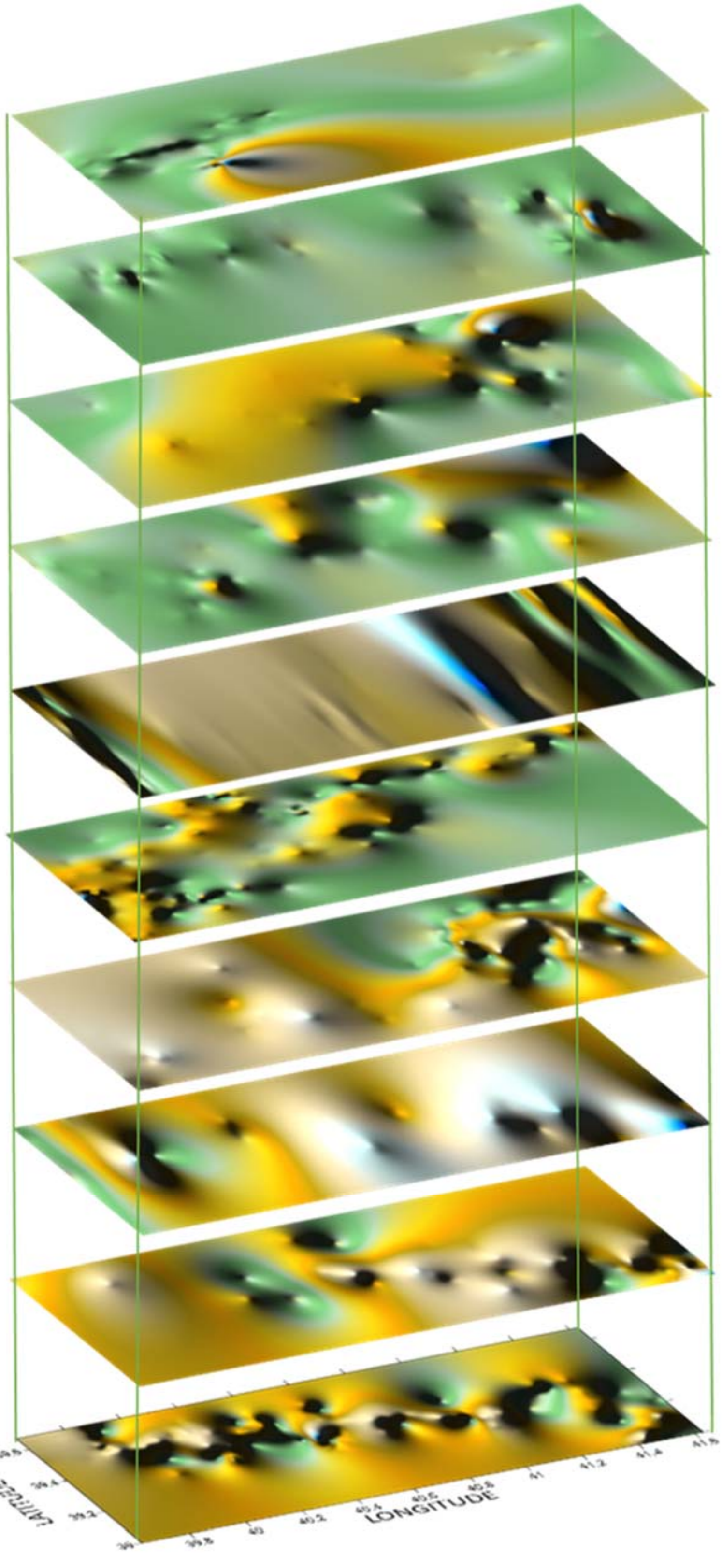

Figure 4. 2D Image maps of spatial variations of b-values for 10 different seismogenic source regions

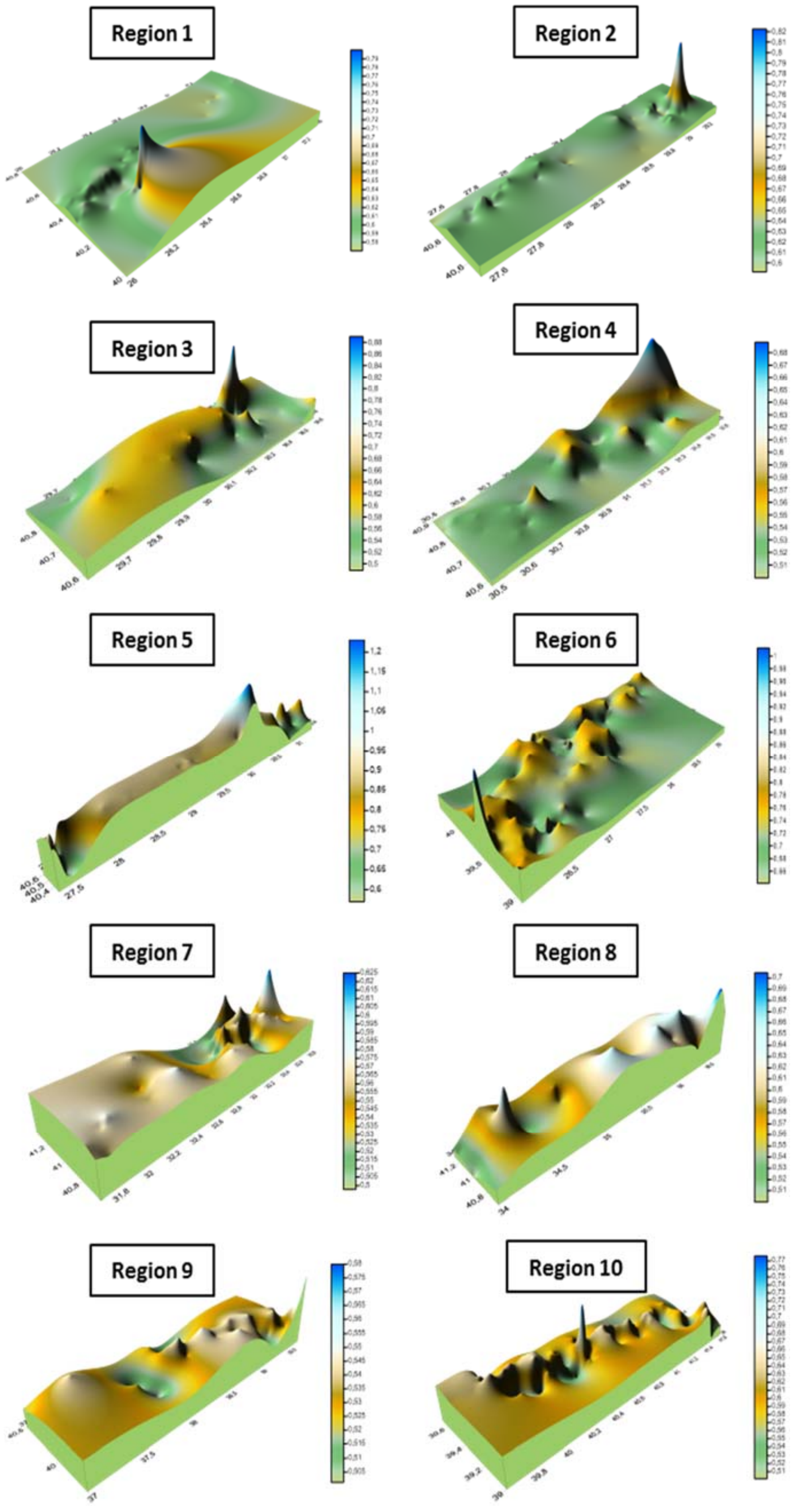

Figure 5. 3D surface maps of spatial variations of b-values for 10 different seismogenic source regions

Table 1. Ten different seismic source region divided into in the NAFZ.

Region	Region Name	N	M_{max}^{obs}
1	Between Saroz Gulf-Ganos	51	7.3
2	Marmara Sea (Tekirdağ-Merkez-Kumburgaz-Çınarcık Basins)	50	7.5
3	İzmit Gulf (Gölcük-Tepetarla-Arifiye-Dokurcun Basins)	37	7.8
4	Hendek Fault, Karadere-Düzce Basins	42	7.2
5	The Southern Branch of NAFZ (Edremit Fay Zonları, Yenice-Gönen, Mustafa Kemal Paşa, Ulubat Faults)	40	7.0
6	The Southern of Marmara Sea(Taşkesti Baseni, Geyve-İznik, Mekece, Gemlik Faults)	118	7.2
7	Between Düzce-Tosya	56	7.3
8	Between Tosya-Erbaa	21	7.1
9	Between Tokat-Erzincan	40	7.9
10	The east of Erzincan	91	6.8

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