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# Approximation of Partition Curves for Electromagnetic Mill with Inertial Classifier - Case study

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**Abstract.** The article presents the impact of the separation parameters on the operation of the grinding and classification system using the electromagnetic mill and impingement-inertial classifier specially designed for this machine. Preliminary tests for the classifier have been carried out, and the effect of the separation efficiency on the value of the recycle flow that optimizes the operation of the grinding system is determined. Fourteen experiments were carried out to test and evaluate the classifier's work, with a variety of damping flaps acting on the airflow. The results provide an accurate assessment of the effectiveness and efficiency of the classifier. The approximation of the electromagnetic mill partition curves using the impingement-inertial classifier was performed by using Weibull distribution function.

## 1. Introduction

Actions related to the processing of raw materials require the appropriate fragmentation that takes place during crushing and grinding. Proper comminution is required by most of the minerals, e.g. coal as fuel, raw materials for the production of mineral binders or mineral fertilizers. The most energy-consuming comminution process is grinding. The demand on energy is determined by the type of milled material, the particle size of the feed, the milling product, and the construction and technical parameters of the mill and grinding system [1]. This indicator increases powerfully as the particle size decreases [2, 3], while the growth rate of this parameter is significantly higher for particles below 50 μm [4, 3]. Particle size distribution of the product is determined by the technological requirements of particular technologies, e.g. for copper ores below 75 μm [5], zinc ores below 10 μm [6], cement production below 90 μm [7], mineral fertilizers below 10 μm [8].

While required parameters of the comminution products can differ significantly (i.e. particles size or shape), a certain problem stands common, namely energetic efficiency of the process. This is because the most popular solutions still use ball mills, rod mills, autogenous grinding or semi-autogenous grinding mills. In such milling technologies most of the energy is consumed to rotate the grinding medium and the mill shell itself [3]. Additional disadvantage is a limited control of the product particles shape, which often results in low technological value of the obtained product. Due to a constant grinding time, the size of the product particles often cannot be controlled and in turn, it depends mostly on the feed particles mean size [9]. When enriching copper ores, the cost of milling is the most significant (about 50%) among other costs of ore processing [3]. Some metallic ores are being grinded to much finer particle size of d90 = 20 μm. Due to the fact that the process of comminution consumes large amounts of energy, the project of building a new raw material processing device for finely grained materials (especially for particle sizes below 10 μm) has been started. An important part of the design was the construction of a fully automated classification system. It will increase the expected efficiency of the equipment and will reduce investment costs and subsequent operating costs, particularly energy ones [10].

## 2. Design of classification system in electromagnetic mill

To control the grinding process more precisely and to increase its efficiency, a proper material transport and classification system needs to be designed. Preliminary studies with the gravitational material transport allowed to describe limitations of this idea and to outline requirements for the dry grinding and classification circuit with pneumatic transport. The concept of such transportation [11] is presented in Fig. 1. Vertical position of the working chamber is assumed, and the counter-flow pneumatic transport of the processed material is realized. The feed is loaded from the top by the auger conveyor, and the air for transport is supplied from the bottom of the working chamber. The feed is falling into the working chamber due to gravity force and is sustained in the proper position by the air stream flowing upwards. Particles which reach desired size during the comminution process start to move upwards with the air stream and are collected at the top end (the outlet) of the working chamber. Depending on the initial size of the particles and the material hardness, the required comminution time is different and influences the mill throughput.

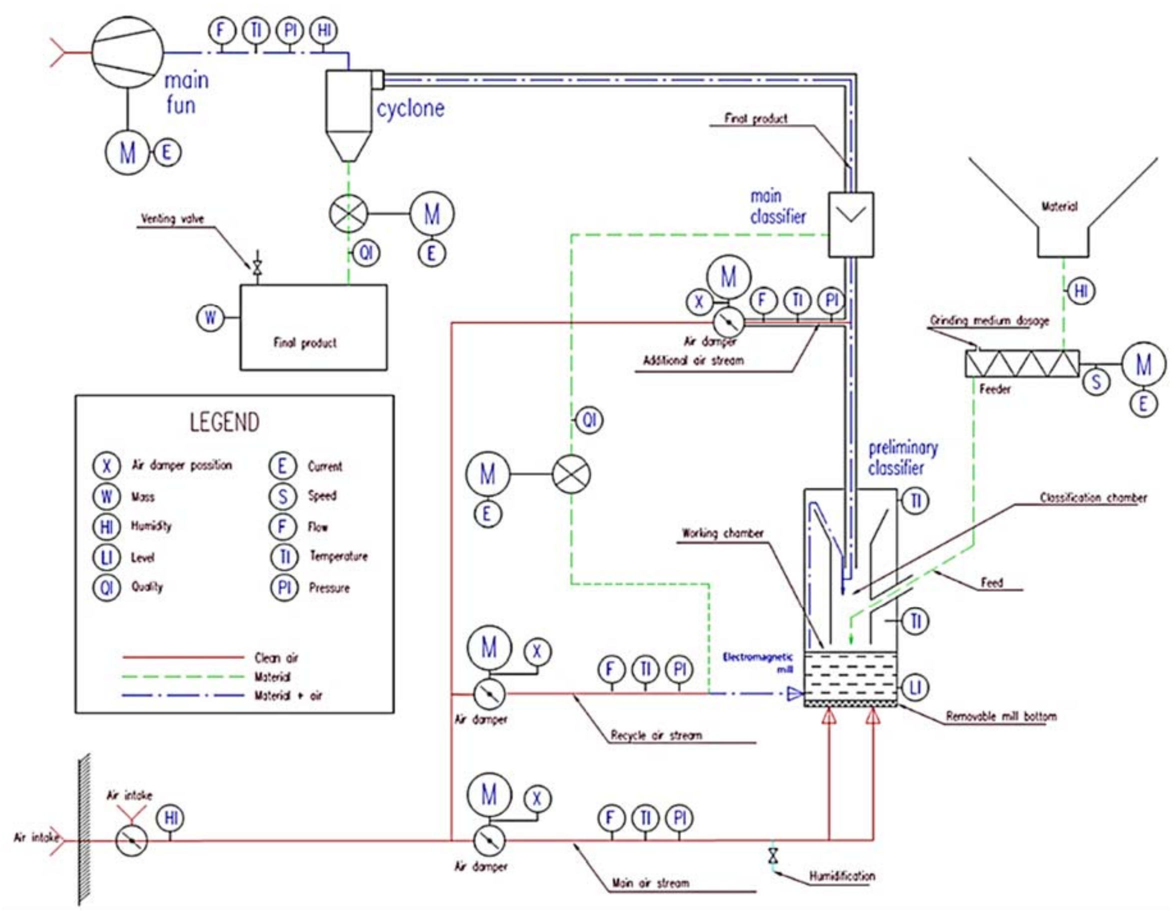


Figure 1. Grinding and classification circuit diagram

The mill is integrated with a preliminary classifier, which is positioned above the working chamber [12]. Internal recycle of the oversized particles is enforced within the classifier, which in turns allows controlling the cut-off size of the grinding product more precisely.

Oversized particles leave the working chamber prematurely and they return with the stream of the fresh feed to be grinded one more time. To improve the grinding process and control the cut-off size even more precisely, the main classifier (inertial separator) is included in the circuit. The most important parameter of the separation is the air speed throughout the main classifier.

Thus, the additional air stream is supplied at its input. The mill output stream collected at the output of the preliminary classifier is directed into the separator input at its bottom. The final product (fine particles) leaves the separator as an overflow stream and it is separated from the transport air in a cyclone [13]. Coarse particles are fed back to the working chamber with the external recycle air stream through the bottom of the mill to provide regrinding. The vertical arrangement of the working chamber and counter-flow material transport causes instability of the whole system [14]. Without the proper air flow through the working chamber all the processed material would fall to the bottom of the mill or would suck out of the mill without grinding. The key issue determining the quality of the grinding process and its efficiency is then to keep the proper fulfilment of the working chamber [13, 15]. The fulfilment level depends on actual throughput of the mill, fresh feed and recycle ratio, volume of the grinding media and on transport air streams ratio. Maintaining the fulfilment allows to control the grinding time, what is the most important component of the final product quality index.

## 3. Material and methods

The following sections of the paper describe experiments and measurements, which were performed using the circuit developed at the Silesian University of Technology in Gliwice, Poland [13]. These experiments aimed at obtaining static models for the upper level control algorithm, which determines set points for the direct control to ensure required quality of the product.

Literature reports on a few examples of the electromagnetic mill applications [13, 16], where the working chamber is in a fixed leaning position. The feed is supplied at the inlet of the chamber (situated higher), what causes the processed material to slide through the chamber by gravity. The main disadvantage of such solution is almost complete lack of the sliding speed control and a constant (or uncontrolled) grinding time. Fixed position limits also throughput control. There are applications with adjusted chamber angle, but still the gravity and the friction determines functionality of such milling system. To control the grinding process more precisely and to increase its effectiveness a proper material transport and classification system needs to be designed. Preliminary studies with the gravitational material transport allowed to describe limitations of this idea and to outline requirements for the dry grinding and classification circuit with pneumatic transport. Each classifier has a specific distribution capacity that operates at a specific boundary or otherwise, at the specified sorted grain size. Sorted grain is the diameter of the particle expressed e.g. in µm, for which the probability of being found in the lower or upper fraction is exactly 50%.

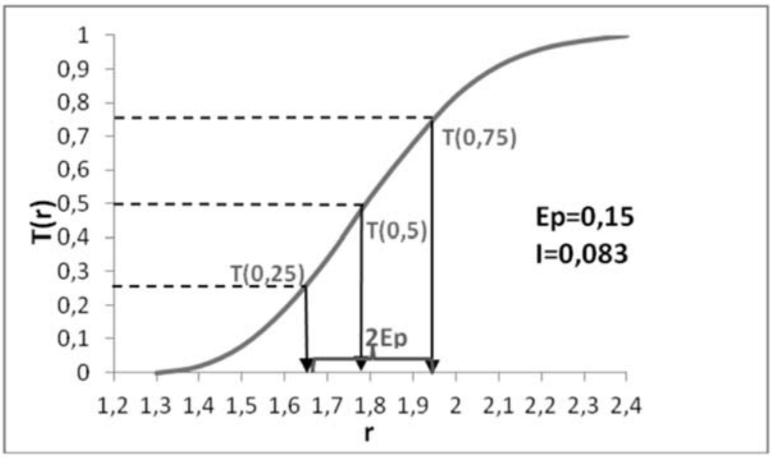


**Figure 2.** A model partition curve [17]

The classification capacity of a given classifier is a function of the degree of separation depending on the diameter of the particle. Most often, it is presented in the form of a curve in a system in which the particle size is marked on the abscissa, and the degree of separation in [%] is on the ordinate. The partition curve is not a particle size distribution curves, only a set of points representing the extent to which an infinitely narrow particle size fractions of medium particle size d (in Figure 2 denoted as r – separation feature) has been split into upper and lower products [17].

Performing a series of experiments in the air-inertial classifier with the changing parameters of the device (listed in Table 1) has allowed analysis and evaluation of the separation process of grained material. Based on experimental partition curves, it is not possible to estimate the effects of the investigated device for narrow particle size fractions.

**Table 1.** Experiment details

| **Experiment** | **Size fraction** | **Feed conveyor [%]** | **Opening of main air flap $K_G$ [%]** | **Opening of recycle air flap $K_R$ [%]** | **Opening of additional air flap $K_D$ [%]** |
|---|---|---|---|---|---|
| 1 | 1–2 | 85 | 100 | 100 | 0 |
| 2 | 1–2 | 85 | 100 | 100 | 0 |
| 3 | 1–2 | 85 | 100 | 100 | 0 |
| 4 | 1–2 | 85 | 100 | 100 | 0 |
| 5 | 1–2 | 85 | 100 | 50 | 0 |
| 6 | 1–2 | 85 | 100 | 25 | 0 |
| 7 | 1–2 | 85 | 50 | 100 | 0 |
| 8 | 1–2 | 85 | 25 | 100 | 0 |
| 9 | 1–2 | 85 | 100 | 100 | 5 |
| 10 | 1–2 | 85 | 100 | 100 | 10 |
| 11 | 1–2 | 85 | 100 | 100 | 20 |
| 12 | 1–2 | 85 | 100 | 100 | 30 |
| 13 | 1–2 | 50 | 100 | 100 | 0 |
| 14 | 1–2 | 100 | 100 | 100 | 0 |

## 4. Results of investigations

For this purpose, approximation of the partition curves has been done, whose final effect in the form of functional formulas, as well as the evaluation of their fit to the empirical data, are presented in Table 2.

Weibull distribution function in the analysis of issues of the broadly understood mineral engineering has been applied since the 1930s. Many years of practice have shown that this is a very good tool for describing particle size distribution curves and beneficiation curves [18, 19, 20, 21, 22].

If it is necessary to create a model of partition curves, Normal distribution function can be successfully applied, as confirmed in [19]. Nevertheless, in most of the analyzed cases (about 85%) very good results of mapping experimental data were obtained by approximating partition curves with the Weibull distribution function. Match fit was rated to 98.3% in average.

The density function and the distribution of the random variable D with the Weibull distribution function are given by the formulas (1) and (2) respectively [24]:

$$f(d) = \begin{cases} 0 \; for \; d \leq 0 \\ \frac{\nu}{b^{\nu}} \cdot d^{\nu-1} \cdot e^{-\left(\frac{d}{b}\right)^{\nu}} \; for \; d > 0 \end{cases} \tag{1}$$

$$\Phi(d) = \begin{cases} 0 \; for \; d \leq 0 \\ 1 - e^{-\left(\frac{d}{b}\right)^{\nu}} \text{ for d>0} \end{cases} \tag{2}$$

where: D - particle size,

$v$>0 - shape parameter,
b>0 - scale parameter.

**Table 2.** Summary of approximation functions of the partition curves

| No. | Formulas for approximation functions | $R^2$ |
|---|---|---|
| Experiment 1 | $\Phi(d) = \begin{cases} 14.21d & for \quad d\in[0, 0.0355] \\ \frac{1}{1+0.9936e^{-0.3202d}} & for\ d\in(0.0355, 0.8150] \\ 2.34d-134 & for \quad d\in(0.8150, 1] \end{cases}$ | 0.9975 |
| Experiment 2 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2614}\right)^{1,6620}}$ | 0.9785 |
| Experiment 3 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2467}\right)^{1,7798}}$ | 0.9877 |
| Experiment 4 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2075}\right)^{1,3167}}$ | 0.9899 |
| Experiment 5 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2080}\right)^{1,2116}}$ | 0.9167 |
| Experiment 6 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2634}\right)^{2,0292}}$ | 0.9529 |
| Experiment 7 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2024}\right)^{2,4724}}$ | 0.9967 |
| Experiment 8 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,2053}\right)^{2,4424}}$ | 0.9961 |
| Experiment 9 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,1852}\right)^{2,4420}}$ | 0.9958 |
| Experiment 10 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,1629}\right)^{2,1514}}$ | 0.9964 |
| Experiment 11 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,164424}\right)^{1,1513}}$ | 0.9321 |
| Experiment 12 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,1619}\right)^{1,4372}}$ | 0.9933 |
| Experiment 13 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,1815}\right)^{1,8753}}$ | 0.9953 |
| Experiment 14 | $\Phi(d) = 1 - e^{-\left(\frac{d}{0,1756}\right)^{2,7969}}$ | 0.9931 |

As was already mentioned, not for all the empirical partition curves, the models obtained by the Weibull distribution function resulted in the intended effects, which were verified by the determinant R2. In this situation, it was necessary to determine the theoretical partition curves by other methods for Experiments 1 and 11. Based on the results of Experiment no 1, conditional distribution Φ(d) was constructed for random variable D for three particle size fractions: 0-0.355; 0.0355-0.8150, 0.8150-1 [mm]. Then followed by linear functions and distributions of logistic distribution approximations have been done. The distribution equation is represented by the formula (3). It has been estimated that the error value of conditional fit matching for the investigated case is small [24, 25].

$$\Phi(d) = \frac{1}{1+b\cdot e^{-c\cdot d}} \tag{3}$$

where: b, c – parameters.

When examining the results of experiment 11, it was revealed that some sort of disturbance occurred during the classification process, which is confirmed by the shape of the partition curve (Fig. 3).

The occurrence of certain anomalies during the experiment made it difficult to implement approximation with traditional methods, as Figure 3 shows, the value of τ (0.3575) = 0.4783 is significantly different from the results in the nearest neighborhood. It has been found that it is necessary to perform the interpolation of the τ (0.3575) value by the ordinary kriging method based on the remaining experimental data, which has translated into an increase in the accuracy of the modeling of the partition curve of the case in question.

Kriging in the processing of mineral raw materials is a novel technique in comparison to other employed methods – it was firstly introduced in 2010. The exact description of the methodology used during interpolation can be found in [26, 22]. This technique was used to determine the new value of τ (0.3575) = 0.8603 and thus set the changes of partition curve, differing from the original only by the value of one outlying point.

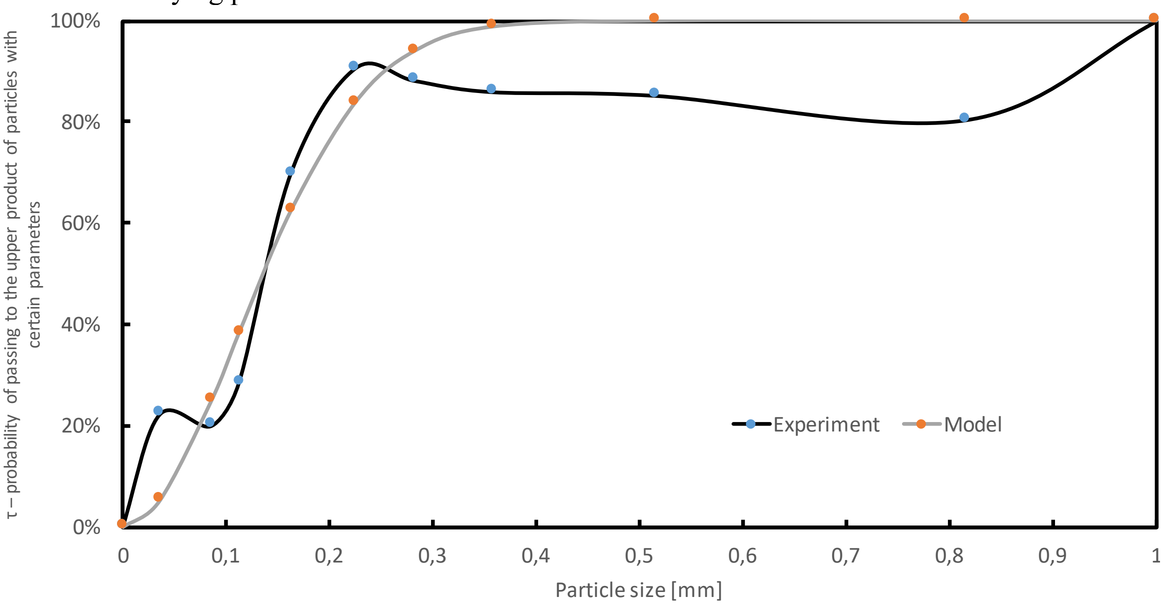


**Figure 3.** Partition curve determined based on empirical data for experiment 11 (Source: personal elaboration).

After the kriging, it was possible to match a satisfactory model based on the Weibull distribution function to the improved partition curve of the material graded in experiment 11. In this case, the determination factor was more than 93%, which proves to be a very good reproduction of the actual data.

## 5. Conclusions

Raw material comminution processes are crucial in different technological systems, as main operations, to obtain a final product, or to prepare for further beneficiation. Obtaining the appropriate particle characteristics of the comminuted product is a significant technological and economic concern; therefore, controlling the course of the process is a necessary part of any processing system for any raw material. This issue is particularly important in case of electromagnetic mills from which small granular material is obtained in narrow particle size fractions. The electromagnetic mills used to date have had a limited quality control system of final product, which has contributed too many problems.

In the analyzed case, the electromagnetic mill is equipped with an internal classification system, which main purpose is to stabilize the particle size distribution of the particulate product. The main factor affecting grading effects in the air-inertial classifier is the flow rate of the air stream regulated by the appropriate configuration of the air damping flaps. In order to investigate the accuracy of the separation of the narrow particle size fractions 0-1 mm, a series of experimental studies was carried out in order to determine the empirical distribution curves. However, they do not allow accurate analysis of the effects of work of the air-inertial classifier. For this purpose, it was necessary to create functional dependency models τ (d) which allowed the evaluation of the separation process and the selection of optimal adjustment points for direct control and ensuring the required product quality.

Model algorithms for the analyzed cases were based on classical mathematical statistics methods. The approximation of the distribution curves was based on the Weibull distribution function, which is widely used in the processing of mineral resources. In most cases, the match results were very good, confirming the determination coefficient values. However, due to the nature of the partition curves, finding their approximating functions often results in some problems. This is also confirmed by experiments (Number 1 and 11) for which non-classical statistical methods were required to obtain a satisfactory match between distribution curve model and empirical data. In these cases, a conditional cumulative distribution function based on a linear function and a logistic conditional cumulative distribution, as well as an innovative kriging method, was used to eliminate the rising point on the partition curve. Constructed models of partition curves are characterized by a slight approximation error its value varies between 0.25 - 8.33%.

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