Using Copulas in the Estimation of the Economic Project Value in the Mining Industry, Including Geological Variability

Zbigniew Krysa[1], Katarzyna Pactwa[1], Justyna Wozniak[1], Michal Dudek[1]

[1] Poland, Wrocław, 50-315, na Grobli 15, Wrocław University of Science and Technology, Faculty of Geoingineering, Mining and Geology, Industrial and Geoeconomics Division

michal.dudek@pwr.edu.pl

Abstract. Geological variability is one of the main factors that has an influence on the viability of mining investment projects and on the technical risk of geology projects. In the current scenario, analyses of economic viability of new extraction fields have been performed for the KGHM Polska Miedź S.A. underground copper mine at Fore Sudetic Monocline with the assumption of constant averaged content of useful elements. Research presented in this article is aimed at verifying the value of production from copper and silver ore for the same economic background with the use of variable cash flows resulting from the local variability of useful elements. Furthermore, the ore economic model is investigated for a significant difference in model value estimated with the use of linear correlation between useful elements content and the height of mine face, and the approach in which model parameters correlation is based upon the copula best matched information capacity criterion. The use of copula allows the simulation to take into account the multi variable dependencies at the same time, thereby giving a better reflection of the dependency structure, which linear correlation does not take into account. Calculation results of the economic model used for deposit value estimation indicate that the correlation between copper and silver estimated with the use of copula generates higher variation of possible project value, as compared to modelling correlation based upon linear correlation. Average deposit value remains unchanged.

1. Introduction

Input parameters used in deposit value modelling include inter alia product price and production costs. In previous research, the relationship between the value of individual useful elements was constant or correlated linearly. The research presented in this article aims at investigating whether by using a copula function to combine variable product prices one can obtain a value of a sample fragment of a polymetallic deposit that will be significantly different from the price obtained when the price proportions will be constant or correlated linearly. In order to verify the existence of the said phenomenon, deposit value modelling will be performed for different price levels and copula types. The copula theory is one of the methods used to express the linear non-linearity between the variables.

The constructed model served as a basis in an attempt to apply copula equations as tools for financial analysis. The investigations covered the influence of using copulas on the results of estimations of a polymetallic deposit value.

Generally, copulas are a tool used inter alia in dependence structure modelling and are referred to as joint cumulative distribution function. Originally, the function was used in financial markets to model dependences between variables describing financial market processes.

Application of the copulas theory in mining and geology is discussed inter alia in Kopacz [1] who uses the copulas theory (empirical copulas) in an analysis of hard coal deposit model to solve a problem of non-linear variables and correlation coefficients for geological parameters. Similar methods were used in [2] to find a multivariable dependence between variable oil prices and the prices of selected metals listed on stock markets, such as copper, gold and silver.

In another study Krysa [3] shows the suitability of using the Gumbel copula (a function which combines the marginal distribution of coal prices and energy into a multivariate distribution using non-linear dependencies between them) in estimating market risk for a hard coal mine. The authors of the above-mentioned articles, as well as others, for instance Ghorbel [4], point to evident usefulness of copulas as a tool for correlating variables in any structure, in the absence of requirements regarding data normality or linearity.

Kopacz et al. [5] used inter alia empirical copulas to construct and evaluate an integrated model of the development of hard coal mining in Poland. The analysis included broadly understood aspects of sustainable development, i.e. the impact of economics, environment and society in a situation of uncertain development of underground hard coal mining.

The selection of a proper copula allows modelling multidimensional dependencies between explanatory variables. The review of current literature on using copulas does not indicate that this tool has been previously used to estimate the economic value of a mining project in the case of a polymetallic deposit with regard to geological variability. Such an attempt is offered in this article.

2. Geological structure of the analysed deposit

Polish copper ore deposit is located in the middle part of the Fore Sudetic monocline. The Fore Sudetic monocline, together with the Fore Sudetic block and the Żary pericline, form the Fore Sudetic zone. The Fore Sudetic block is separated from the side of the Fore Sudetic monocline by the middle Odra river zone of faulting [6]. The views on the genesis of the copper ore deposit evolved over many years [7]. Oszczepalski and Rydzewski [8] distinguish three basic genetic types of the deposit: syngenetic, syngenetic-diagenetic, and late diagenetic-epigenetic, while Piestrzyński [9], in his synthesis distinguishes only two types, contrasting a syngenetic deposit model to a diagenetic one. By analysing the following criteria: geological and tectonic conditions, form and structure of the deposit, hypotheses on the origin of metals and sulphur in the deposit, the role of organic matter and the age of the minerals, Piestrzyński concludes that the deposit is diagenetic.

Several theories on the origin of metals exist. Their genesis has been described as: hydrothermal solutions seawater, hydrothermal solutions with seawater, or the Zechstein evaporates. Other theories taken into consideration include the connection between mineral content and thermal conditions in base rocks of the Carboniferous period (Speczik) or the build-up of deposit erosion and manifestations of polymetallic mineralization, as well as the Rotliegend desert sediments on the concentrations in seawater. Disagreements about the genesis of the deposit and its metal content indicate that the copper ore deposit was formed as a result of a complicated, multistage process, with many sources of mineralization (inter alia [10]).

Discussions of mineralization in the copper ore deposit on the Fore Sudetic monocline primarily focus on copper content, since it was the identification of copper that resulted in a decision to mine the ore. Basic copper minerals which occur in the copper deposit on the Fore Sudetic monocline include [11, 12, 13]: chalcocite, bornite, digenite, chalcopyrite, anilite, covellin, djurleite, and tennantite.

Balanced copper grades, as defined in the Regulation of the Minister of the Environment of 20 June 2005 (Dz. U. No 116 of 29 June 2005), occur in sandstone, shale and carbonates. The concentration of copper in sandstone gradually increases, as the distance to shale layer decreases. Highest copper grades are observed in shale ore, then they decrease again in carbonate ore as the distance increases.

The deposit in the Fore Sudetic monocline has been identified as a polymetallic deposit. One of the economically important metals which occur in the deposit along with copper is silver. This mineral is concentrated in shale (average of 0.022%), and its highest quantities are found in the Lubin and Sieroszowice mine areas. Silver forms own minerals, i.e. stromeyerite, mckinstryit, eugenit, chlorargyrite. It also occurs as native silver, in the form of impurities in ore minerals (mostly in chalcocite, the occurrence of copper and silver is confirmed by the correlation coefficient between 0.37 and 0.57, depending on the lithology). Two phases of silver mineralization are observed in carbonates (primary and secondary) and three mineralization phases are observed in shale: originally dispersed silver, secondary sulphides and silver amalgams. The two last phases account for 25% of the total silver content; horizontal distribution of silver is similar to the distribution of copper [14, 15, 16].

3. Economic model of the planned mining exploitation

The object of the analysis in this article is a fragment of copper ore deposit planned for exploitation, located on the Fore Sudetic monocline, at a depth of approx. 1200-1400 m. The deposit is developed by driving horizontal underground excavations from the already existing Polkowice-Sieroszowice and Rudna divisions as well as by driving shafts. The economic analysis of the profitability of mining production is offered in [17], in which the planned exploitation costs were estimated on the basis of contemporary values for divisions with a similar exploitation technology. The values of the parameters used in the model were taken from the documents provided by KGHM – the operator of the deposit. Tables 1 and 2 show baseline values of the geological and technical parameters, as well as the economic parameters indexed for current prices. The calculation of the deposit value for the presented parameters is described with the following algorithm:

1) EBIT for a period is designated as a difference between the revenue from sales of metals and the production cost:

$$EBIT = \sum_{i=1}^{n} M_J \cdot (p_j - c_j) \tag{1}$$

where:
M_j – the quantity of metal produced
p_J – metal sales price
c_J – unit cost of metal production

2) The quantity of metal produced M_j results from the volume of ore production W, metal content in the deposit α_{zj}, dilution assigned to each metal u_{rj} and yield from ore processing ε_{pj}, smelting ε_{hj} and metallurgy ε_{mj}.

$$M_J = W \cdot 0.01 \left(\alpha_{zj} (1 - 0.01 u_{rj}) \right) \cdot \varepsilon_{pj} \cdot \varepsilon_{hj} \cdot \varepsilon_{mj} \tag{2}$$

Unit cost of metal production was determined as follows:

$$c_j = \frac{c_{wj} + c_{pj} + 0.01\alpha_j \cdot \varepsilon_{pj} \cdot \varepsilon_{hj} \cdot \varepsilon_{mj} \cdot c_{mj} - \sum_{i=1}^{n} \alpha_t \cdot \varepsilon_{pt} \cdot \varepsilon_{ht} \cdot \varepsilon_{mt} (p_t - c_t) \, 10^{-3}}{0.01\alpha_j \cdot \varepsilon_{pj} \cdot \varepsilon_{hj} \cdot \varepsilon_{mj}}$$

$$-c'_j - \sum_{j=1}^{m_u} r_{uj} \cdot p_{uj} \tag{3}$$

where c_{wj}, c_{pj}, c_{mj}, are unit costs of mining production, processing and smelting, respectively. The t index is related to the technological parameters for accompanying metals. Unit price of an accompanying metal was designated with symbol p_{uj}, while symbol r_{uj} is a ratio between accompanying metal quantity

and annual primary metal production. Knowing the projected annual mining production in successive periods and assuming general economic parameters for the project, such as:

- total capital expenditure incurred: 650 million USD
- WACC: 10%
- mine liquidation value -172.3 million USD
- tax rate: 0.28

the value of the deposit can be calculated using classic discounting cash flow method (DCF). Thus, the deposit value calculated on the basis of the assumed constant parameters in the baseline scenario amounted to 131.42 million EUR.

Table 1. Main geological and technical parameters of the model.

Parameter	Value	Parameter	Value
Cu content in industrial resources	2.33%	Ore dilution for copper and silver	8%
Ag content in industrial resources, in g/Mg	79.53	Copper yield in processing	0.89
target mining production, in million Mg	4.35	Copper yield in smelting	0.97
Gold production, in kg	700	Copper yield in metallurgy	0.98
Lead production, in Mg	21000	Silver yield in processing	0.85
Production of nickel sulphate, Mg	2,000	Silver yield in smelting	0.95
Production of copper sulphate, in Mg	6,700	Silver yield in metallurgy	0.98
Production of sulphuric acid, Mg	620,000	Silver yield in additional processes	0.99
Production of technical selenium, in Mg	80,000		

Table 2. Main economic parameters of the model.

Product price	Value (USD/Mg)	Cost	Value (USD/Mg)
Copper	5600	Extraction of minerals	47.65
Silver	360[a]	Processing	10.63
Gold	25 000[a]	Smelting and metallurgy	750
Lead	1,750	Silver production (outside the main technological path)	350[a]
Nickel sulphate	5,000	Production of gold	18 100[a]
Copper sulphate	1,500	Lead production	1,350
Sulfuric acid	18.5	Nickel sulphate production	5,000
Technical selenium	6.55	Copper sulphate production	3,250
		Sulfuric acid production	95
		Technical selenium production	0

[a] in USD/kg

    The deposit value calculated in the above model is based on constant values of the explanatory parameters. Wirth additionally introduces an analysis of deposit value assuming two scenarios: a pessimistic one and an optimistic one. The pessimistic scenario assumes that the values of the explanatory variables in the model will suffer a 20% negative change, while the optimistic scenario assumes a 20% positive change. Instead of the scenario analysis, the Monte Carlo simulation may be used, which does not require the knowledge of all parameters to calculate deposit value, also on the basis of the NPV (net present value) method. The input parameters (explanatory variables) were simulated with triangular distribution, in order to preserve the variability range proposed in the scenario analysis, while the minimum value in the distribution of a given variable corresponded to the values from the pessimistic scenario. The modal value set at the level from the baseline scenario and the maximum value was based on the optimistic scenario. As a consequence, due to the independence of random variables in the model, the variability range decreases as compared with the scenario analysis, which assumed a constant change of the model's parameters.
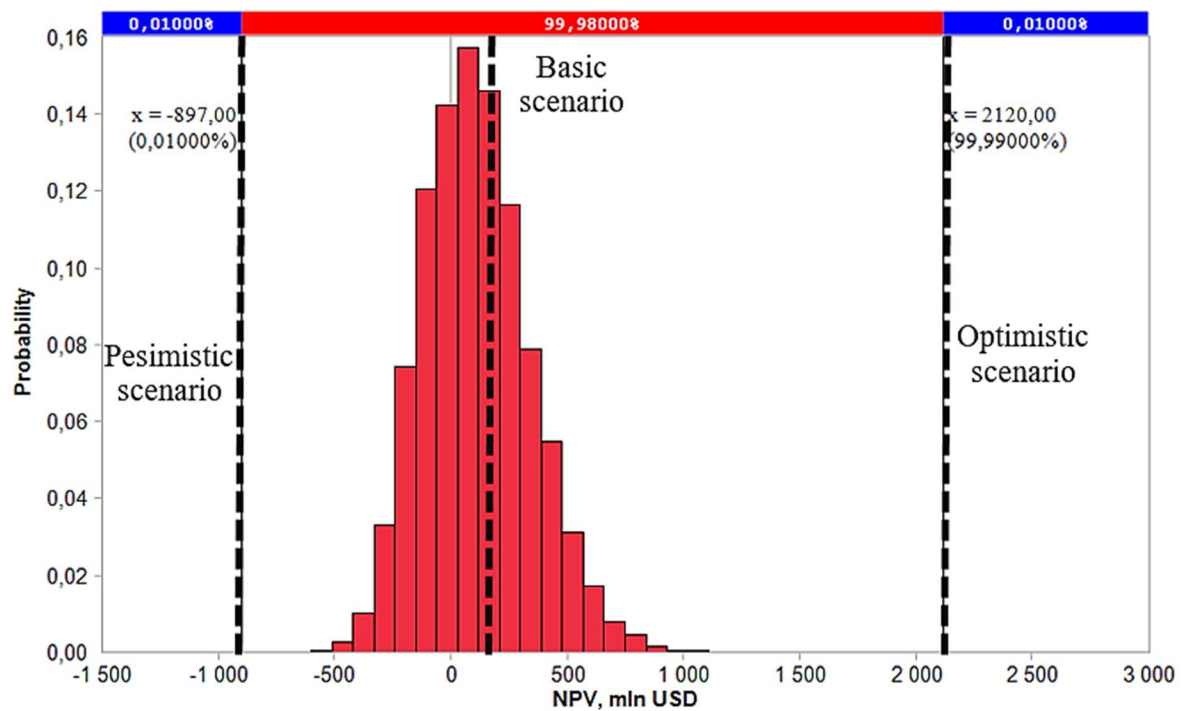
Figure 1. Variability range of the deposit value in a simulation based on parameter variability estimated on the basis of scenario analysis

Table 3. Scenario analysis vs Monte Carlo simulation based on independent random variables.

Scenario	Value (10^6 USD)	Simulation	Value (10^6 USD)
Baseline	131	Mean	110
Pessimistic	-897	Min.	-590
Optimistic	2.120	Max.	1100
		Quantile 10%	-1.66
		Quantile 90%	412

The dashed lines on the NPV graph denote the deposit value obtained by analysing the pessimistic, baseline and optimistic scenarios. Variability range for the deposit value is significantly lower after simulations are considered. Importantly, the above simulation is based on an assumption that the relationship between individual variability generators for the model's parameters is not taken into account. This fact means that also the content of copper and silver was modelled on the basis of independent random variables.
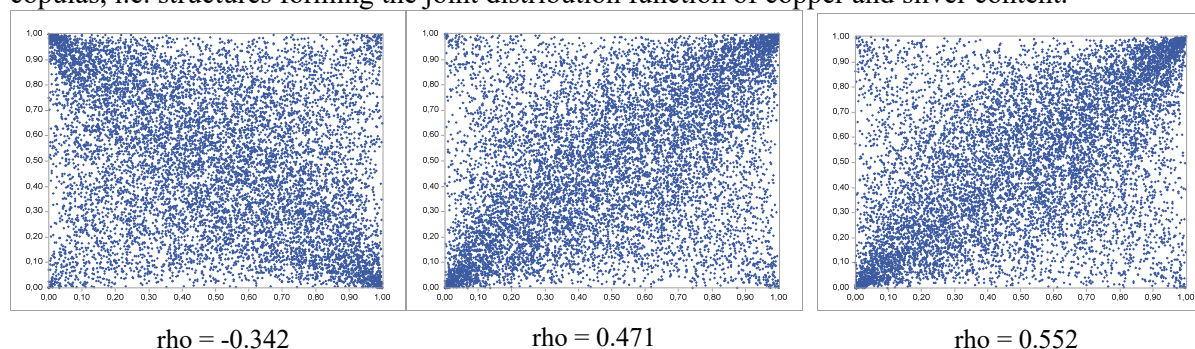
Geological information about copper and silver grades in the selected profiles from mining divisions in the areas neighbouring with the exploitation area served to construct a second model of deposit value. The parameters simulated on the basis of linear correlation included the content of silver and copper in the ore. The analysis covered 70 samples from 3 geological profiles in Rudna mine. Table 4 shows the results of the calculations and the input parameters for the deposit value model.

The variability of copper and silver content in the second deposit value model was dependent on the linear correlation coefficient between these parameters. Two-dimensional copulas were the generators of dependencies between the content of copper and silver for the given correlation coefficients. In the copula which reconstructed the dependency structure of copper content and silver content, triangular distributions served as marginal distributions.

Table 4. Linear correlation between copper content, silver content,
and the height of mine face.

Profile	Ag-Cu	m-Cu	m-Ag
1	0.471	-0.594	-0.188
2	0.552	-0.503	-0.527
3	-0.342	-0.511	0.531

The parameters for these distributions consisted of minimum value, modal value and maximum value, in accordance with the values taken from the pessimistic, baseline and optimistic scenarios, respectively. The best copula from the perspective of a statistical match was selected on the basis of the Akaike, Schwartz, and Hannah-Quinn information capacity criteria. Verification covered the following five types of two-dimensional copulas available in the ModelRisk application: the two-dimensional copula, the T copula, the Frank copula, the Gumbel copula and the Clayton copula. The results of the verification indicate that the relationship between copper content and silver content for the range of data from the geological profiles is best expressed by the T copula. Fig. 2 shows examples of generated T copulas, i.e. structures forming the joint distribution function of copper and silver content.

rho = -0.342   rho = 0.471   rho = 0.552

Figure 2. The T copula for various linear correlation coefficients between copper price and silver price, used in the deposit value model

The deposit value was estimated again for the selected method of modelling the relationship between copper content and silver content. In order to evaluate only the influence of the method of designing the dependency structure, all remaining technical and economic parameters were constant, as in the baseline scenario. 10 000 iterations of the simulation model were performed. The results presented in table 5 indicate that the useful element variability structure (not the mineral content itself) in the model based on the best matched copula does not have a significant influence on the deposit value, since the differences of mean pricing do not exceed 2%.

Table 5. Estimation of deposit value using a variability model based on the T copula.

ρ	Mean	Min	Max	0.1	0.9
-0.342	130	-215	470	-17	280
0.471	130	-220	480	-44	310
0.552	132	-220	470	-49	310

Further calculations were performed for the copula, whose structure of dependencies between the variables was not defined by the theoretical copula with parameters estimated on the basis of actual data, as in the previous case, but instead was based on a structure directly adjusted on the basis of input data. This step allowed avoiding the inadequate fitting between the actual variability distribution of copper and silver and the theoretical distribution, which in the case of copula is a marginal distribution. The baseline copper content in the model scenario was higher than the mean content obtained from

geological profile estimations, and hence the expected value of the estimated marginal distribution in the stochastic model was increased to match the value from the scenario analysis. Thus, the marginal distribution will properly transfer the variability structure. Analogical steps were performed for silver content.
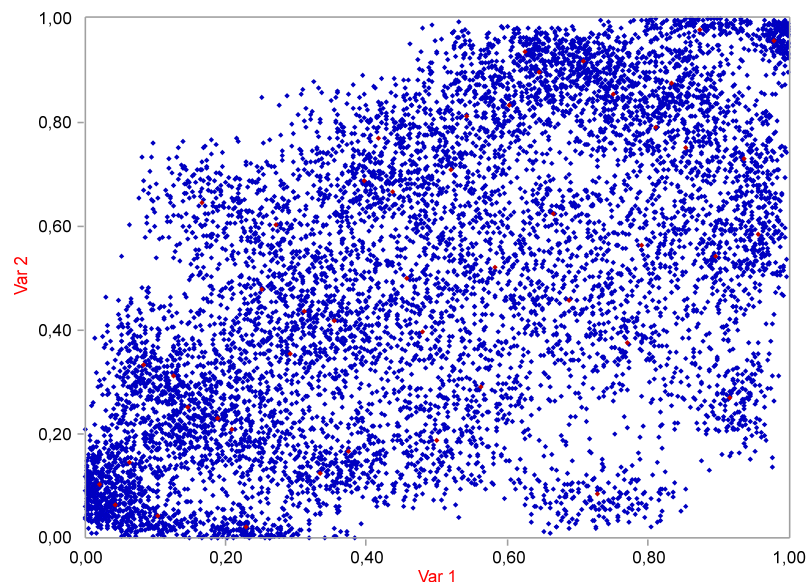

Figure 3. Empirical copula for the dependency between copper content and silver content in the geological profile.

Also in this case, the deposit value estimation model was constructed with value simulation using the empirical copula of the dependency between copper price and silver price. Figure 4 is a comparison of the results of scenarios based on the best matched theoretical T copula and on the empirical copula.
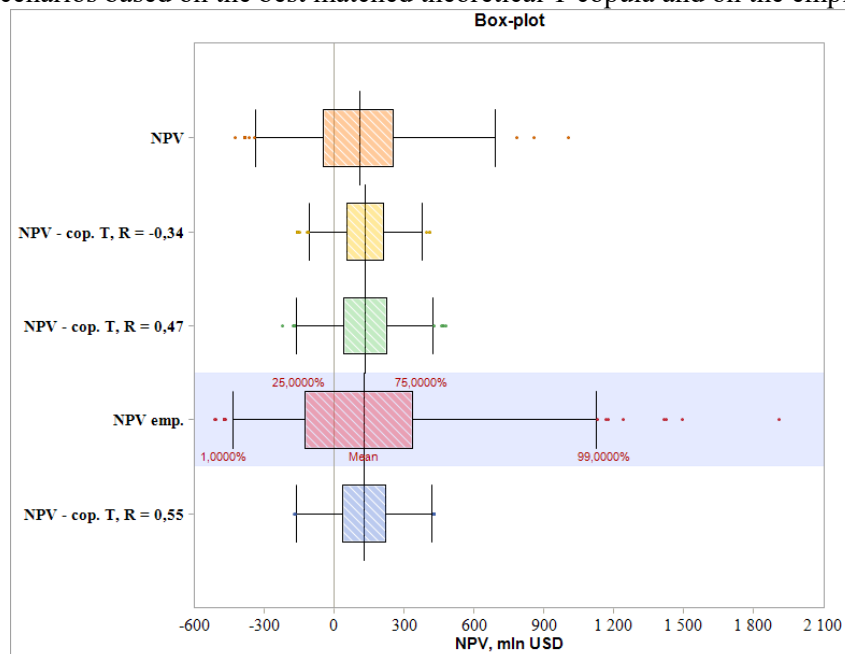

Figure 4. Comparison of the estimated deposit value in the case of determining the dependency between useful elements content with the T copula model, with the empirical copula model and without dependencies.

The results indicate that a properly selected dependency model has a significant influence on the modelling of deposit value and as a result, on the modelling of risk in mining projects. Regardless the assumed correlation coefficient value, models in which the dependency was simulated on the basis of the T copula, the deposit value remained close to the output value. Adopting an empirical copula model based on adjusted lognormal distributions for copper and silver prices resulted in generating a significantly wider range of project values. In the case of the empirical copula, quantile range $Q_3 - Q_1$ was 460 million USD, while in the case of the T copula it was more than twice smaller, at 226 million USD. In the variant, where correlation was not involved in the content of copper and silver, quantile range was 296 million USD. However, mean deposit value remained stable regardless of the implemented model of dependencies between copper content and silver content. Hence, the selection of a copula model did not have a significant impact on the change in mean deposit value. Rather, it influenced width range, which in the deposit value may be situated if mining production is started.

4. Conclusion

The obtained results allow a conclusion that estimating deposit value with the use of parameter dependency models based on copulas provides results different from those in the case of simple scenario analysis. Using a properly selected copula may yield different and more precise results. Copulas estimate random variables on the basis of quantiles rather than moments, and hence a large number of results has the greatest impact on the precision of estimations. Being limited to key factors, i.e. the content of useful minerals, this article offers only a narrow representation of the possibilities which result from using copulas in the analyses of mining projects viability. Copula-based models can be easily applied to other dimensions, creating dependencies between many parameters within a single copula. A more precise estimation of dependencies between explanatory variables in models with multidimensional correlations translates into a more precise estimation of deposit value.

References

[1] Kopacz M., 2016: The influence of seam thickness, bulk density and interlayers in a coal deposit on the value of mining investment projects in the simulation method, (Ocena wpływu miąższości, gęstości przestrzennej oraz przerostów w pokładzie węgla na wartość górniczych projektów inwestycyjnych w metodzie symulacyjnej), *Przegląd Górniczy* nr 5, 2016, 63-74.

[2] Juan C. Reboredo, Andrea Ugolini, 2016: The impact of downward/upward oil price movements on metal prices, *Resources Policy* 49 (2016) 129-141.

[3] Krysa Z., Modeling market risk for a hard coal mine on the basis of the volatility of coal and energy prices, (Modelowanie ryzyka rynkowego kopalni węgla kamiennego na podstawie zmienności cen węgla i energii), *Przegląd Górniczy* 2014, 48-51.

[4] Ahmed Ghorbel, Wajdi Hamma & Anis Jarboui, 2016: Dependence between oil and commodities markets using time-varying Archimedean copulas and effectiveness of hedging strategies, *Journal of Applied Statistics* ISSN: 0266-4763 (Print) 1360-0532 (Online).

[5] Kopacz M., Kryzia D., Kryzia K., Assessment of sustainable development of hard coal mining industry in Poland with use of bootstrap sampling and copula-based Monte Carlo simulation. *Journal of Cleaner Production* 159 (2017), 359-373.

[6] Tomaszewski J. B.: Geological structure in the region of Lubin and Sieroszowice (Lower Silesia), (Budowa geologiczna okolic Lubina i Sieroszowic [Dolny Śląsk]), *Geologia Sudetica* vol. XIII, nr 2, 1978, str. 85-132

[7] Piestrzyński A.: The deposit's genesis, (Geneza złoża), in: *Monografia KGHM Polska Miedź S.A.*, 1996.

[8] Oszczepalski S., Rydzewski A.: Metallogenic atlas of the zechstein copper-bearing series in

Poland, *Polish Geological Institute*, Warsaw, 1997.

[9]   Piestrzyński A.: A historical perspective on the development of theories related to the genesis of the copper ore deposit in the Fore Sudetic monocline – discussion, (Historyczny rozwój poglądów na genezę złoża rud miedzi na monoklinie przedsudeckiej – dyskusja), *Biuletyn Państwowego Instytutu Geologicznego*, 423: 69-76, 2007a

[10] Speczik St.: The Kupferschiefer mineralization of Central Europe: New aspects and major areas of future research, *Ore Geology Reviews* 9 (1995) 411-426

[11] Piestrzyński A.: Mineralisation, (Okruszcowanie), in: *Monografia KGHM Polska Miedź S.A.*, 1996, 200-237.

[12] Kucha H., Mayer W.: Geochemistry, (Geochemia), in: *Monografia KGHM Polska Miedź S.A.*, 1996, 237-251.

[13] Kucha H.: Ore mineralogy and geochemistry of the ore body in the Lubin-Sieroszowice deposit, (Mineralogia kruszcowa i geochemia ciała rudnego złoża Lubin-Sieroszowice), *Biuletyn Państwowego Instytutu Geologicznego* 423: 77-94, 2007.

[14] Kucha H.: Geochemistry of the Kupferschiefer, Poland, *Geologische Rundschau* 79/2, Stuttgart 1990, 387-399.

[15] Kucha H., Sawłowicz Z.: Geochemical and mineralogical characteristics of the elements co-occurring with copper in the area of the Lubin Zachodni mine, (Charakterystyka geochemiczna i mineralogiczna pierwiastków towarzyszących miedzi na obszarze kopalni Lubin Zachodni), *Geologia*, t.6, z.3, 1980, 53-69.

[16] Konstantynowicz-Zielińska J.: Metallic minerals co-occurring with copper mineralisation in shales and interpretation of their correlation coefficients, (Kruszce towarzyszące mineralizacji miedziowej łupków oraz interpretacja ich współczynników korelacji), *Rudy Metale*, R 38, nr 10, 1993, 243-246.

[17] Wirth H., 2011, Multi-factor valuation of deposits and their resources on the example of the non-ferrous metals industry, (Wieloczynnikowa wycena złóż i ich zasobów na przykładzie przemysłu metali nieżelaznych), *Studia Rozprawy Monografie* (171), PAN IGSMiE, Kraków.