Probabilistic model of bridge vehicle loads in port area based on in-situ load testing

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Abstract: Vehicle load is an important factor affecting the safety and usability of bridges. An statistical analysis is carried out in this paper to investigate the vehicle load data of Tianjin Haibin highway in Tianjin port of China, which are collected by the Weigh-in-Motion (WIM) system. Following this, the effect of the vehicle load on test bridge is calculated, and then compared with the calculation result according to HL-93(AASHTO LRFD). Results show that the overall vehicle load follows a distribution with a weighted sum of four normal distributions. The maximum vehicle load during the design reference period follows a type I extremum distribution. The vehicle load effect also follows a weighted sum of four normal distributions, and the standard value of the vehicle load is recommended as 1.8 times that of the calculated value according to HL-93.
Key words: bridge; vehicle load model; weigh-in-motion system; probability distribution

1. Introduction

With the high-speed development of economy in recent years, road transportation is accordingly subjected to an increasing pressure. Especially in some port areas, overloading is also very serious. In general, the bridge structure is designed according to the design code, whereas the design load may not satisfies the actual traffic condition. Therefore, it is necessary to investigate and statistically analyze the current vehicle load so as to reflect the real stress condition suffered by in-service bridges.

Numerous researchers have studied the actual load of existing Bridges[1-3]. O'Connor and Eichinger[4-5] proposed a prediction method to simulate load effect values based on the data from Weigh-in-Motion system (WIM). Obrien et al[6] used the asymptotic extremum theory to predict the extreme value of vehicle load in different traffic situations. Gindy Mayrai[7] made a statistical analysis on the data collected by WIM systems on a bridge in New Jersey, where the maximum vehicle load and the vehicle load effect on the bridge are obtained and analysed. However, few studies studied the randomness of correlation parameters of the actual vehicle load.

This paper aims to investigate the the actual vehicle load condition of Tianjin Haibin highway in Tianjin port of China. The paper is organized as follows. First, the WIM system in Tianjin Haibin highway adjacent to Tianjin Port of China is established. Following that, the actual vehicle parameters based on the actual monitoring data is calculated. Then, the vehicle load model is established, and the load effect of the vehicle during the design reference period is deduced. Finally, several conclusions and findings are summarized. Details are showing in the following.

2. Vehicle load model

2.1 Installation of WIM system

Tianjin Binhai highway is located in the Haihe River Estuary. It is a main expressway connecting the northern and southern directions of Tianjin Binhai New area, of which four highway bridges span are 30meters, 40meters, 50meters and 48meters, respectively. In June 2015, the WIM system was installed on the bridge, as shown in figure 1. Traffic loads were monitored for 7 consecutive days from June 17th to 23rd. The contents include speed, length, weight, number of axles and axle load.

Figure 1. Install of the WIM system.

2.2 Statistical parameters

Statistical parameters were analyzed to reflect the time variability of vehicle load, as shown in Table 1.

Table 1. Vehicle load statistic parameters

date	Number of samples	average（kN）	median（kN）	Maximum(kN)	coefficient of variation
6/17/2015	30342	85.0	36.9	1489.2	1.53
6/18/2015	27869	171.9	80.7	1491.7	1.26
6/19/2015	27640	149.8	42.4	1491.0	1.38
6/20/2015	27585	171.2	42.0	1495.7	1.38
6/21/2015	31606	105.2	21.4	1476.9	1.79
6/22/2015	28450	147.4	33.5	1476.3	1.49
6/23/2015	28264	109.0	21.0	1489.2	1.79

As table 1 shows, the mean value and the coefficient of variation of the vehicle load per day varies little, which indicates that the vehicle load is almost stable in the short term. On the other hand, the variation coefficient of the vehicle load is larger than 1.0, which indicates a great dispersion in vehicle load.

2.3 Vehicle load probability model

After the introduction of probabilistic limit state design method to structural design, variable actions are usually determined by probabilistic statistical analysis. Therefore, probability distribution models are the basis of various representative values of variable actions. The above analysis shows that the vehicle load in this highway is relatively stable, so the model of the vehicle load can be described accurately by the probabilistic model of this test data. Besides, the probability distribution of the sample should be tested to accurately realize the statistical regularity of vehicles loads.

The vehicle load follows an obvious multiple-peaked distribution, which can be seen as the result of the combination of different types of vehicle loads. Assuming the overall probability density function of the i car loads is $f_i(x)$, n types of vehicle loads are combined together, and then the probability density function of the combined vehicle load can be expressed as

$$f_X(x) = \sum_{i=1}^{n} p_i f_i(x)$$

(1)

where p_i is the proportion of the i vehicle load, $\sum\limits_{i=1}^{n} p_i = 1$.

Then the probability distribution function of the combined distribution is determined as

$$F_X(x) = \sum_{i=1}^{n} p_i F_i(x) \tag{2}$$

In general, the vehicle load has its own statistical law. Thus, the probability density function $f_i(x)$ of the different vehicle load in equation (2) is different, which is determined by fitting the measured results of the vehicle load. In this paper, all $f_i(x)$ are taken as the same probability density function, and follow normal distribution, but the statistical parameters are different. Therefore, the probability model of all vehicle loads is simulated using a weight sum of multiple normal distributions. The probability density function is calculated as

$$f_X(x) = \sum_{i=1}^{n} p_i \frac{1}{\sigma_i} \varphi\left(\frac{x-\mu_i}{\sigma_i}\right) f_i(x) \qquad \sum_{i=1}^{n} p_i = 1 \tag{3}$$

where $\varphi(\square)$ is the standard normal probability density function , μ_i is the mean of the vehicle load i; σ_i is the standard deviation of the vehicle load i.

The probability distribution equation is taken as

$$F_X(x) = \sum_{i=1}^{n} p_i \Phi\left(\frac{x-\mu_i}{\sigma_i}\right)_i \qquad \sum_{i=1}^{n} p_i = 1 \tag{4}$$

Using the measured data of the road section, the probability density function and the probability distribution function can be obtained by applying maximum likelihood estimation method and estimating the *(3n-1)* unknown parameters of $\mu_1 \sim \mu_n, \sigma_1 \sim \sigma_n, \quad p_1 \sim p_{n-1}$ in equation (4).

The analysis shows that the overall vehicles can be described by the sum of four normal distribution weights, i.e., *n=4* in all above equation. Using the measured data, 11 unknown parameters in equation (4) are calculated and the results are shown in equation (5):

$$F_X(x) = 0.438\Phi\left(\frac{x-2.353}{0.391}\right) + 0.205\Phi\left(\frac{x-3.659}{0.876}\right)$$
$$+ 0.241\Phi\left(\frac{x-12.547}{8.338}\right) + 0.116\Phi\left(\frac{x-52.077}{17.173}\right) \tag{5}$$
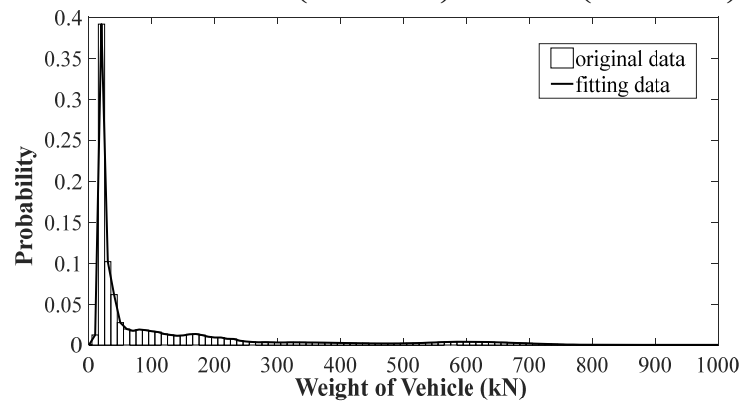

Figure 2.   Comparison of the measured data and fitting distribution of vehicle load.

Figure 2 shows the comparison between the measured data and the fitting results. It can be seen from figure 2 that the probability distribution function can describe the probability model of the overall vehicle load. Note that the well description of statistical characteristics of the heavy truck load also depends on the amount and quality of the collected heavy truck load data.

3. Vehicle Load Effect

Vehicle load effect on bridges is very complicated. In principle, it should be determined by in-situ measurement. However, it is impossible to make a direct measurement due to the restrictions of objective conditions. So it can only be determined by the structural simulation calculation on the basis of the measured data of the vehicle load.

3.1 Calculation of vehicle load effect

In the practical engineering, vehicles passe through bridges in the form of a platoon. Thus, the density of motorcade has an important influence on the structure. The operation condition of motorcade can be divided as the normal traffic state and the dense traffic state. The largest difference between the two states is the difference in the space of the vehicles.

The probability distribution of vehicle spacing is established based on the measured data, which is usually the information of vehicles in normal traffic state. Equation (6) is a probability model of vehicle spacing based on WIM system in a heavy duty road[8]，which has the representation in the analysis of the vehicle spacing under normal traffic state.

$$f_X(x) = \frac{1}{\sqrt{2\pi} \times 0.935 x} \exp\left[-\frac{(\ln x - 4.343)^2}{2 \times 0.935^2} \right] \tag{6}$$

In the dense traffic state, the vehicle runs slowly or keeps quiescent. In this situation, the vehicle spacing is independent of the traffic volume. Therefore, the model of vehicle spacing can be determined by the vehicle distance equation (7).

$$f(x) = \frac{1}{\sqrt{2\pi} \times 0.279707 x} \exp\left[-\frac{\left(\ln x - 1.561165\right)^2}{2 \times 0.279707^2} \right] \tag{7}$$

The vehicle spacing can be randomly simulated as the probability density function of vehicle spacing is determined. The Monte Carlo method is used to produce the random vehicle spacing. A random motorcade model is established by analyzing the data for June 17th to June 23rd in 2015. Take the motorcade model into the influence line of the all kinds of bridges, and then calculate the vehicle load effect including the mid-span moment, the negative moment at support, the shearing force and so on. In order to facilitate the analysis, the ratio of the calculated results to the computation results according to the HL-93(AASHTO LRFD) is considered as the object of statistical analysis, i.e., take the dimensionless parameter $K=S/S_K$ as the basic statistical object of vehicle load effect. S is the vehicle load effect calculated according to the measured date, and S_K is the vehicle load effect calculated according to HL-93.

3.2 Statistical Analysis of Vehicle Load Effect

As stated above, the vehicle load effect is calculated according to the measured date. A large number of samples with different running states, different effects and all spans are obtained. Then the calculated samples are statistically analyzed. The calculation results are shown in Table 2. Note that, the situation without a car on the bridges is common under the general operating conditions, so the vehicle load effect is zero. In this paper, only the digital features of the non-zero elements are discussed.

Table 2. Statistical results of general running state effects

	sample number	Effect	maximum value	mean	coefficient of variation
General Running State effects	18122104	Bending moment	2.497	0.118	1.580
		Shear force	2.433	0.108	1.504
intensive running state effects	3320899	Bending moment	3.462	0.642	0.807
		Shear force	3.438	0.590	0.823

As Table 3 and 4 show, this calculation result has a total of 42886006 vehicle load effect under the normal traffic state and dense traffic state. The sample quantity is huge and can well reflect the real operation condition.

3.3 Probability Distribution of Vehicle Load Effect
It is necessary to establish the probability distribution model of the vehicle load effect after the statistical sample of the vehicle load effect is determined. The above analysis shows that the shear effect is not dominant, which is consistent with the basic situation of the bridge design. Therefore, simulating the probability distribution of the vehicle load effect is based on the calculation results of the bending moment effect. For convenience, the bending moment ratio of the calculated results according to the measured data to the computation results according to the HL-93 is considered.

The actual analysis shows that the vehicle load effect in the road follows multi-modal distribution, and the probability model is simulated using a weight sum of multiple normal distributions. The probability distribution of the vehicle load effect for all spans under normal traffic state and dense traffic state are shown in equation (8) and (9).

In general running state

$$F_X(x) = 0.168\Phi\left(\frac{x-0.022}{0.005}\right) + 0.295\Phi\left(\frac{x-0.041}{0.016}\right)$$
$$+0.192\Phi\left(\frac{x-0.125}{0.080}\right) + 0.345\Phi\left(\frac{x-0.166}{0.216}\right) \tag{8}$$

In intensive running state

$$F_X(x) = 0.089\Phi\left(\frac{x-0.108}{0.028}\right) + 0.184\Phi\left(\frac{x-0.205}{0.069}\right)$$
$$+0.177\Phi\left(\frac{x-0.356}{0.127}\right) + 0.550\Phi\left(\frac{x-0.888}{0.466}\right) \tag{9}$$

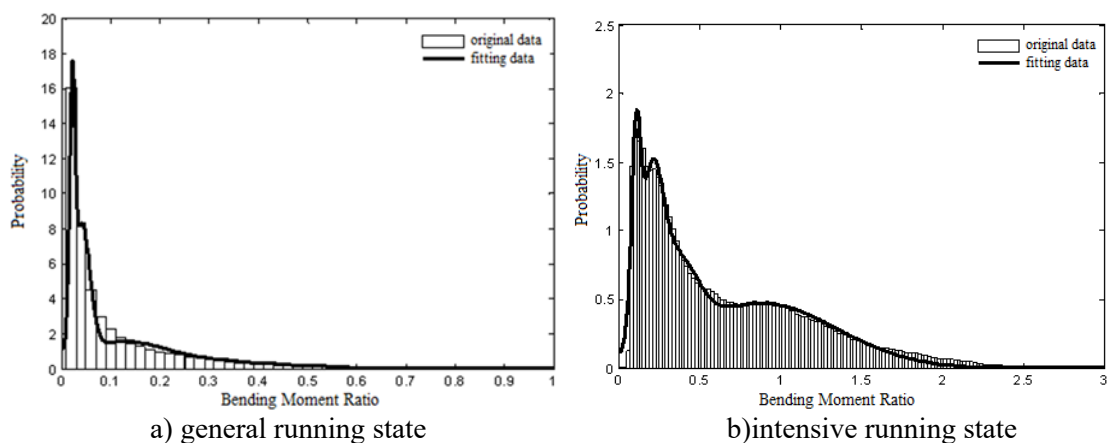a) general running state   b) intensive running state

Figure3.   Comparison of the measured data and fitting distribution of vehicle load effect.

Figure 3 shows the comparison of vehicle load effect between the measured data and the fitting results. It can be seen from figure 3 that the probability distribution function can describe the probability model of the vehicle load effect in two operation conditions.

3.4 Standard value of vehicle load effect

For the comprehensive analysis of the traffic lane load standard value of the road, the maximum value of the measured data, the 95th percentiles of the section distribution and the 95th percentiles of maximum distribution in the design reference period are calculated, as shown in Table 3. S_K is the effect value calculated according to HL-93.

Table 3 Representative values of vehicle loads with different spans

running state	95th percentiles of the current situation	95th percentiles of the reference period
general running state	$0.51[S_k]$	$0.86[S_k]$
intensive running state	$1.68[S_k]$	$2.43[S_k]$

As Table 3 shows, the 95th percentiles of the reference period under the normal traffic state is 0.86 [S_k]. By contrast, the 95th percentiles of the dense traffic state is 1.68 [S_k]. Considering the fact that the sample of the measured period does not fully represent the actual vehicle load, it is suggested that the load standard should be appropriately improved on the basis of the 95th percentiles value of the maximum distribution under the normal traffic state to reduce the uncertainty caused by the limitations of the sample. The value of the load standard is recommended as $1.8[S_k]$. Obviously, 1.8 times of the HL-93 load standard involves the 95th percentiles of the reference period under the normal traffic state, and also involves the 95th percentiles of the section distribution under the dense traffic state.

4. Conclusion
This paper uses the data of Tianjin Haibin highway from June 17th to June 23rd in 2015 to study the statistical law of vehicle load and load effect. The main conclusions are drawn as follows:

(1) The total weight of the vehicle can be described in the form of multiple normal distribution. The maximum value distribution of the reference period follows the extreme I distribution.

(2) The load effects under the normal traffic state and dense traffic state can be described by the form of weighted sum of multiple normal distributions，and the maximum distribution of the reference period is subjected to the extreme I distribution. According to the statistical analysis of the actual data, the vehicle load effect criterion of this road is 1.8 times of the vehicle load effect calculated by HL-93.

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