# A water ecosystem model based on complex dynamics and its application

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**Abstract.** This paper presents a complex dynamical model to give the evaluation and prediction to water scarcity, which contains four subsystems: population subsystem, agricultural subsystem, industrial subsystem and environment subsystem, that are coupled each other and affected by the inner parameters. Then, the supply water coming from environment subsystem, and the demand water depending on the other three subsystems are evaluated and predicted by Vensim. Correspondingly, we choose the water deficit calculated from the difference between the quantities of supply and demand water to measure the ability of a region to provide clean water. Focusing on the region of the South Africa, where the water is moderately overloaded, we employ the dynamical water ecosystem model to evaluate and predict the water scarcity. It is obtained that there are three main factors that influence water deficit: rainfall, recycle sewage and water demand per capita by parametric sensitivity analysis. Simulation results demonstrate that the year, at which the quantity of supply water can't meet the demand, is delayed by changing the three factors in a positive direction. We expect our work to provide insight into the challenging problem of harnessing the global water scarcity problem.

## 1. Introduction

Water is the source of life that binds all living begins on earth. With the shortage of water resources being increasingly serious since 1960s [1], a large number of researches has payed more attention to mitigate it. Dating back to thousand years ago, human beings began to apply surface water and ground water to develop agriculture and industry. However, ancient people did more consumption rather than reservation, leading to the deterioration of water shortages. Now, it is well know that developing water source vigorously, establishing water-saving society and allocation water resources rationally are the main approaches to improve the carrying capacity of the water resources. To quantify the sustainability of water resources utilization, we combine economy, population, society, natural resources and environment together and then establish a dynamic complex system[2], which can be studied by System Dynamics proposed by Jay.W.Forrester in 1956[3]. It fuses systematology, cybernetics and information theory with each other to solve the analogical problems of large systems.

There are 3 milestones in the application of System Dynamics. The first is the two world model (WORLD II: World Dynamics and WORLD III: The Limits to Growth and Toward Global Equilibrium) and Club of Rome became world famous. The second is researches of Economic Long Wave in America and other foreign countries[4-6]. The third one, which is the most useful, is referred to System Dynamics and devised the water model of the world, as well as evaluating water availability[7-10].

China has also reached some achievements on approaching water issues through *System Dynamics*. In 1995, Bifeng Shen took Beijing as an example, presenting System Dynamics and Input-Output models, integrating society, economy, water resources and environment into a group, and predicted the imbalance between water supply and demand from a microcosmic prospective [11].

In this paper, we establish a water ecosystem model based on System Dynamics. The model contains many parameters to express the evaluation measure and predict the water scarcity in the future. The goal is to propose several indexes to measure the ability of water supply. The paper is organized as follows. Firstly, a water ecosystem model is described with the method of System Dynamics which is practical in terms of ease of application and ability to cope with extreme circumstances. In this model, the whole system is divided into four subsystems: population subsystem, agricultural subsystem, industrial subsystem and environment subsystem. Then, the difference between the quantity of the water supply and demand are given as the measure criteria of water scarcity. Finally, we apply of the water ecosystem model to evaluate and predict water scarcity in South Africa and give sensitivity analysis to discuss the main reasons.

**2. The Water Ecosystem Model**
System Dynamics can deal with the systems with high order, nonlinearity, multi-feedback and complex time-change. For the sustainable utilization of water resources can be seen a complex system, we introduce a dynamic evaluating and forecasting model to simulate the water Scarcity, which is divided into four subsystems: agricultural subsystem, industrial subsystem and environment subsystem respectively.

*2.1. Population Subsystem*
Two main factors are concerned in this subsystem: population size predication and water consumption per capita. A region's whole water usage per year is the result of multiplying population and water consumption of each person.
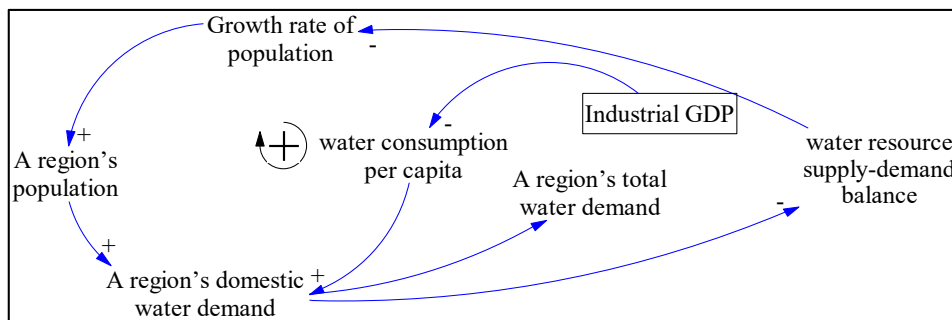


**Figure 1. Relationship of inner factors in Population Subsystem**

To predict population growth, we use the model *Logistic Population Growth Model*[12], which is expressed by:

$$\frac{dP(t)}{dt} = r\left[1 - \frac{P(t)}{P_m}\right] \cdot P(t), \tag{1}$$

where $P(t)$ is the population size at time *t*; $P_m$ denotes the people's maximum carrying capacity of one region; *r* is rate of population growth.
To get the figure of water consumption of the selected region, we give the following equation:

$$u_1(t) = P(t) \cdot \lambda, \tag{2}$$

where $u_1(t)$ is the gross water consumption of the region; $\lambda$ denotes annual water use per capita in one year. Thus, $u_1(t)$ can be obtained the following differential equation:

$$\frac{du_1(t)}{dt} = r\left[1 - \frac{u_1(t)}{\lambda P_m}\right] \cdot u_1(t),$$

(3)

As for the water cannot be recycled, it is gathered in sewage treatment plant for secondary utilization.

### 2.2. Agricultural Subsystem

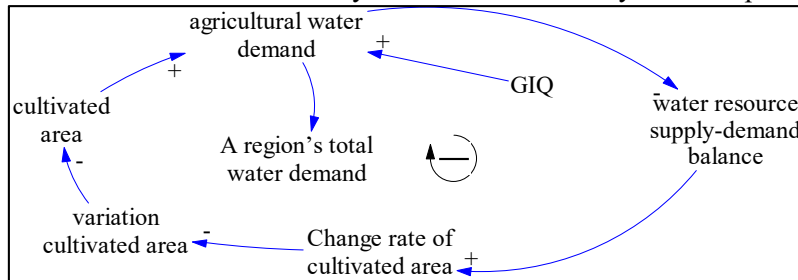Agricultural water demand is classified by whether it can be recycled. We plot a graph as follows:



**Figure 2. Relationship of inner factors in Agricultural Subsystem**

In this system, it contains values of total cultivated area and GIQ (irrigation water per cultivated area) and is shown as:

$$u_2(t) = a(t) \cdot S,$$

(4)

where $u_2(t)$ denotes the total agricultural water; $S$ is GIQ; $a(t)$ stands for total cultivated area at time $t$, which satisfies:

$$a(t) = a(0) \cdot (1 + r_1)^t,$$

(5)

where $a(0)$ is an initial value of $a(t)$; $r_1$ is the relative change ratio of cultivated land.

### 2.3. Industrial Subsystem

In detailed, industrial water demand is classified by whether it can be recycled. We plot a graph as follows:
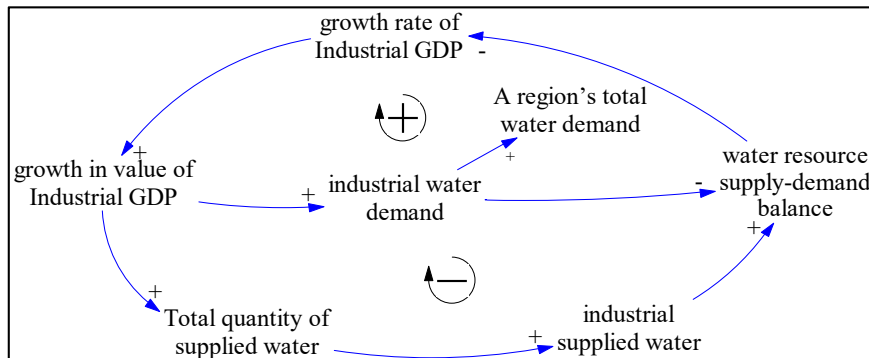


**Figure 3. Relationship of inner factors in Industrial Subsystem**

To explain the industrial subsystem properly, we have the total industrial water demand, GDP and water use per GDP involved, which can be calculated as:

$$u_3(t) = I(t) \cdot G(t),$$

(6)

where $u_3(t)$ presents the industrial water demand in sum; $G(t)$ denotes the water use per GDP at time $t$; $I(t)$ stands for output value of GDP at time $t$, which is supposed to be:

$$I(t) = I(0) \cdot (1 + r_2)^t,$$  (7)

where $I(0)$ is an initial value of $I(t)$; $r_2$ is growth rate of industrial GDP.

*2.4. Environment Subsystem*

The primary supply of the water is from the environment. Rainfall, surface water, ground water and sewage treatment plant are all involved. As for rainfall, it is divided into two main parts, flowing into surface or ground water and transforming into available water, which is used to supply other three subsystems and creates a bond among several factors of each subsystem. As for surface water, it contains lakes' and rivers' water volume, along with the difference between water intake and output unit time. The sewage, which is produced by industry and citizens, will be recycled and reused.
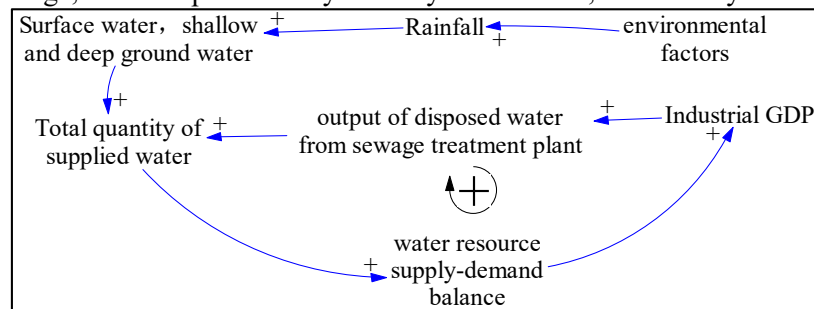


**Figure 4. Relationship of inner factors in Environment Subsystem**

Based on the above discussion, we carry out the followling equation:

$$k_1(t) = l_1(t) \cdot \gamma,$$  (8)

where $k_1(t)$ stands for the transformed water from rainfall; $l_1(t)$ is the average precipitation in a region; $\gamma$ presents the efficiency of rain, which relies on whether the relevant technique is mature. It suggests the water yield produced by rainfall.

To calculate the water withdrawal, which is produced by infrastructure, we figure out the following:

$$k_2(t) = \left[ l_1(t) \cdot \phi + l_2(t) + u_2(t) \cdot (1 - \theta) \right] \cdot \varphi,$$  (9)

where $k_2(t)$ is the total quantity of water mining underground or on surface; $\phi$ is the percentage transformed water of rainfall occupies; $l_2(t)$ denotes the total quantity of surface water and ground water; $\theta$ stands for the rate of what has been absorbed by crops in agricultural subsystem; $\varphi$ is the rate of conversion, which is provided by infrastructure.

When it comes to sewage disposal, we figure out the following equation:

$$k_3(t) = u_4(t) \cdot v,$$  (10)

where $k_3(t)$ is the output of disposed water from sewage treatment plant; $u_4(t)$ presents the sewage discharges; $v$ stands for the conversion rate of the disposal.

Finally, we set $u_5(t)$ as the total supply of water in one region, which is the sum of $k_1(t)$, $k_2(t)$ and $k_3(t)$, and it goes like:

$$u_5(t) = k_1(t) + k_2(t) + k_3(t),$$  (11)

*2.5. Water Scarcity Measurement Model*

To sum up, the coupling relationship between the four subsystems can be refined as the following differential equations:

$$\begin{cases} \dfrac{du_1(t)}{dt} = r\left[1 - \dfrac{u_1(t)}{\lambda P_m}\right] \cdot u_1(t) \\ u_2(t) = a(0) \cdot (1 + r_1)^t \cdot S \\ u_3(t) = I(0) \cdot (1 + r_2)^t (\lambda_2 - a) \\ u_5(t) = l_1(t) \cdot \gamma + \left[l_1(t) \cdot \phi + l_2(t) + u_2(t) \cdot (1 - \theta)\right] \cdot \varphi + u_4(t) \cdot \nu \end{cases} \tag{12}$$

Here the sewage discharges is the composing of $u_1(t)$ and $u_2(t)$, that is

$$u_4(t) = u_1(t) \cdot (1 - m_1) + u_2(t) \cdot (1 - m_2), \tag{13}$$

where $m_1$ is the rate of gross water consumption that cannot be recycled; $m_2$ is the rate of industrial water consumption that cannot be recycled. The water deficit is obtained as

$$W = u_1 + u_2 + u_3 - u_5. \tag{14}$$

## 3. The application on the South Africa

South Africa is tropical savanna climate. It is generally arid to semiarid, which makes it have less rainfall and strong evaporation. Generally speaking, there is little precipitation, sparse vegetation (only 1.01% of cover rate) and expending desert area. Besides, uneven surface water distribution exacerbate water scarcity. Meanwhile, infrastructures remain in disrepair, which exacerbates the problem of water supply. To overcome many obstacles, we consider designing an intervention plan to help solving its problem, and mitigate water scarcity by constructing reservoirs and other storage facilities, which will be benefit the burden area, and it really makes sense.

Water resource system is a relatively complex system, which can be solved by applying Sysytem Dynamics. When emulating the dynamic evaluating and forecasting model, we determine several parameters in advance. As we can see, population size is the key to work out a total region's domestic water demand. So the first step is to evaluate its quantity. According to *Logistic Population Growth Model*, we simulate the population and display it in figure 5:
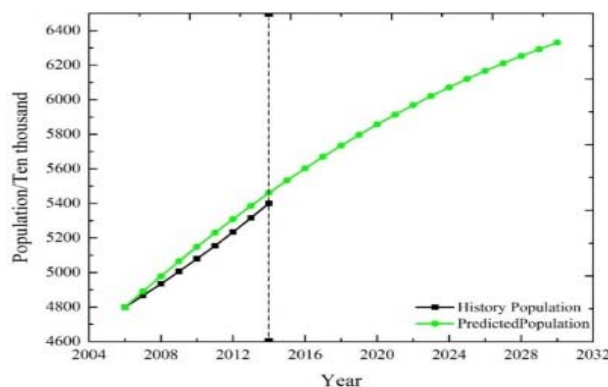


**Figure 5. Population Gross Prediction in South Africa**

where *r* is 0.065; Pm is 70 million. There are other necessary parameters needed to be settled, just as the following table.

**Table 1. Other Necessary Values Set**

| Values | size |
|---|---|
| Rainfall(Cubic kilometers) | 603 |
| Rate of rainfall's conversion (%) | 8.6 |

| | |
|---|---|
| Surface water, shallow and deep ground water(Cubic kilometers) | 48 |
| Industrial GDP(initial value)(Ten million) | 3400 |
| Growth rate of Industrial GDP (%) | 0.015 |
| GIQ(Cubic kilometer/square kilometer) | 0.007836 |
| cultivated area(Cubic kilometer) | 10000 |
| Change rate of cultivated area (%) | 0 |

**\*Minor factors used are not displayed above**

We use Venism to simulate the dynamic evaluating and forecasting model. From figure 6, we can see that the water supply from nature stays at a high level, though it will fall off in the future. It suggests that water existing in nature is abundant. However, as shown in figure 7, the water is always in the state that lags behind demand since 2015. It is unnatural that high level of supply corresponds to low level of water deficit. Since abundant water in nature can no longer be one of physical factors that worsen water deficit, the major reason is related to economic aspects, such as irrational arrangement of resources and poor technologies required for deep mine.
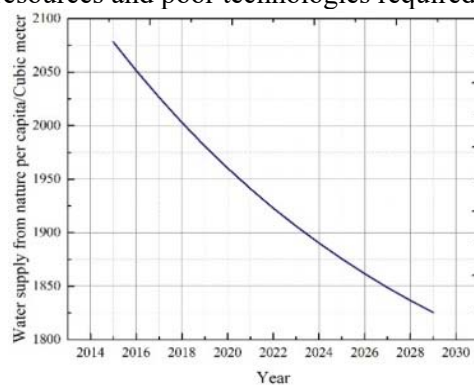


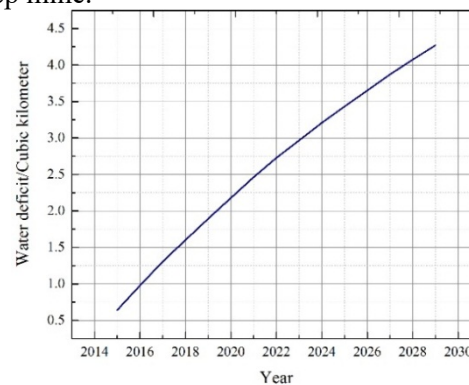**Figure 6. Water Supply from Nature**         **Figure 7. Water Deficit**

In figure 8, we change different values of rainfall to study the influence on water deficit. When the precipitation increases, it will put off the arrival of dry years; and when the precipitation decreases, it will exacerbate water deficit. Nevertheless, unpredictable and uncontrollable property is the main characteristic of rainfall, which makes it hard to mitigate water deficit by changing precipitation. Therefore, we should reinforce the harvest of rainfall to postpone the arrival of dry years. Judging from figure 9, we can see that the less water demand per capita, the less water deficit will be, which suggests that if we save water personally, the arrival of water shortages year will be lagged behind. The other reason is the large population in South Africa, which makes it tough for people to get enough water, as well as meeting their demand.
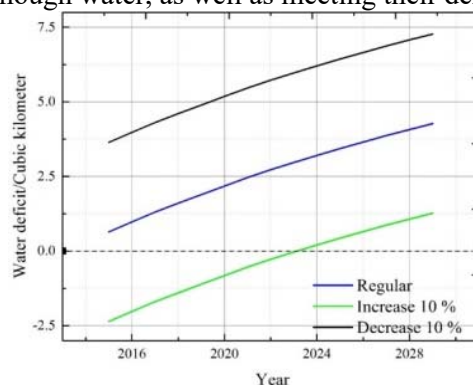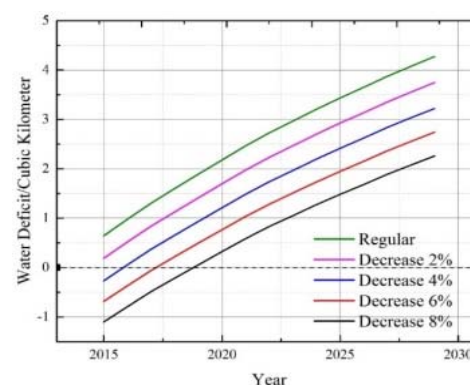


**Figure 8. Impact of Different Precipitation**     **Figure 9. Impact of Different Water**
**on Water Deficit**                              **Demand per capita on Water Deficit**

As seen in figure 10, if we decrease the rate of sewage recovery step by step, water

deficit will be more severe. So if we increase the water-handling force, it will have the same effect as changing precipitation does. When considering how to intervene water scarcity, building as much sewage treatment plants as possible can be one of the solutions.
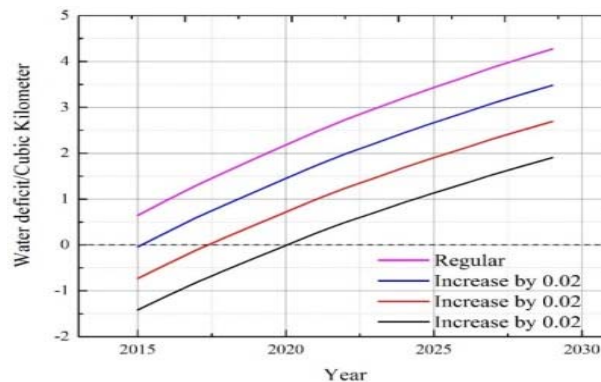


**Figure 10. Impact of Different Rate of Sewage Recovery on Water Deficit**

## 4. Conclusions

Human being is experiencing severe water scarcity problem, which is an important constrain factor for the development of society, economy and environment. The paper proposes a water ecosystem model to measure the ability of a region to provide clean water. The water system is divided into 4 subsystems, which are population subsystem, agricultural subsystem, industrial subsystem and environment subsystem respectively. These subsystems are subtle and can simulate water system comprehensively.

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