

Research on Agricultural Product Options Pricing Based on Lévy Copula

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Abstract: China is a large agricultural country, and the healthy development of agriculture is related to the stability of the whole society. With the advancement of modern agriculture and the expansion of agricultural scale, the demand for farmers to avoid market risks is increasingly urgent. Option trading has the effect of attracting farmers' intervention, promoting order agriculture development, perfecting agricultural support policy and promoting the development of agricultural futures market. Relative to the futures, the option transaction because the margin is low, reducing the trader's entry threshold, you can make more small and medium investors to participate. This is not only active in the futures market, but also for many small and medium investors to provide effective financial management tools.

1. Introduction

For agricultural producers, the risks of using options to avoid price fluctuations in agricultural products are more convenient and flexible than futures. Under the condition of market economy, farmers face the market directly, and the fluctuation of the price of agricultural products has great influence on the income of farmers. Therefore, a tool is needed to avoid this risk.

Since the introduction of options trading in agricultural production activities in the United States, the price risk of food products has been socialized by option trading, and some of these spreads have been borne by traders. Part of the national financial commitments are reduced, the options market to replace the agricultural subsidy policy, not only successfully using the price mechanism of the market to protect the price of agricultural products, food products in favor of "commercialization", and effectively control the amount of food supplement countries^[1].

Because of the particularity of weather, man-made and policy influence, the price fluctuation process is different from that of other assets, so its pricing model should also have its particularity. A good option pricing model can guide the option trade, reduce the risk in the option transaction, and make the option transaction order. Because the option can lock the amount of loss, making it a good tool to avoid risks.^[2] China is a big agricultural country, the fluctuation of agricultural prices has a great impact on the income of the majority of farmers, so an option pricing model adapted to our market can effectively reduce the price risk in China's agricultural trade.

The research on option pricing started late in China, and most of the work is close to the work of translation and review. Few papers have been designed to fit China's national conditions and market conditions. This paper puts forward the option pricing model in our country's market condition, which is meaningful in innovation and research.

Although the option of agricultural products is a tool of risk management, it may become a source



of risk when applied improperly. For the majority of farmers and agriculture related entities to provide sophisticated risk management services to prevent excessive speculation. Reasonable pricing of agricultural products options is a crucial issue.

We believe that financial assets have a very important feature, their changes there is a sudden jump. In this paper, Lévy copula function with jumps is used to price the agricultural product options.

2. Definition of Lévy Copula

2.1 Lévy Copula

$F(x, y) : [-\infty, \infty]^2 \rightarrow [-\infty, \infty]$ is Lévy Copula, the following characteristics must be met.

- (1) F is 2-increasing;
- (2) $\forall x$, satisfy $F(0, x) = F(x, 0) = 0$;
- (3) $F(x, \infty) - F(x, -\infty) = F(\infty, x) - F(-\infty, x) = x$.

If $x < 0$ or $y < 0$, then $F(x, y) = 0$.

When the Lévy Copula and the tail marginal integral are very smooth, it is very easy to construct the Lévy Copula model with the Lévy density. ^[3,4]

Theorem 1 (build Lévy Copulas from general Copulas)

Let C is the Copula function on $[0, 1]^d$, which is strictly incrementing the continuous function. $\phi(1) = \infty, \phi(0) = 0$. There is d -order non-negative derivative on $(0, 1)$, then

$$F(u_1, u_2, \dots, u_d) := \phi(C(\phi^{-1}(u_1), \phi^{-1}(u_2), \dots, \phi^{-1}(u_d)))$$

is Lévy Copula on $[0, \infty]^d$.

2.2. Archimedes Lévy Copula

Let $\phi : [-1, 1] \rightarrow [-\infty, \infty]$ is strictly increasing continuous function, $\phi(1) = \infty, \phi(0) = 0$ $\phi(-1) = -\infty$. There are d -order derivatives on $(-1, 0)$ and $(0, 1)$ that are satisfied

$$F(0, x) = F(x, 0) = 0$$

Let

$$\phi(u) := 2^{d-2} \{\phi(u) - \phi(-u)\}$$

where $u \in [-1, 1]$, then

$$F(u_1, u_2, \dots, u_d) := \phi\left(\prod_{i=1}^d \tilde{\phi}^{-1}(u_i)\right), \quad (u_1, u_2, \dots, u_d) \in (-\infty, \infty]^d$$

called the general Archimedes Lévy Copula.

We discuss the bimodal compound Poisson distribution with positive jump, which means that the Lévy measure is $[0, \infty)^2 \setminus \{0\}$ not in $\mathbb{R}^2 \setminus \{0\}$. The hypothesis of jumping is reasonable in the operational risk environment, so we limit the Lévy Copula to the range of the positive Lévy Copula. ^[5,6]

2.3 Positive Lévy Copula and Sklar theorem

The two-dimensional positive Lévy Copula $C : [0, \infty]^2 \rightarrow [0, \infty]$ is a two-dimensional incremental zero-based grounded function with regular boundaries. Two-dimensional incremental nature of the description for $(u_1, u_2) \in [0, \infty]^2$ $(v_1, v_2) \in [0, \infty]^2$. we have

$$C(u_1, u_2) - C(u_1, v_2) - C(v_1, u_2) + C(v_1, v_2)$$

Zero base surface property states if $u_1 = 0$ $u_2 = 0$, then $C(u_1, u_2) = 0$. Finally, the boundary is

defined as $C(u_1, \infty) = 0$ $C(\infty, u_2) = u_2$. Positive Lévy Copula is $C(u_1, \infty) = u_1$ $C(\infty, u_2) = u_2$.

Like the distribution Copula joins the marginal distribution function into a joint distribution function, Lévy Copula connects the marginal tail integral into a tail integral. For a Lévy process with a positive jump, the tail integral is defined as the following formula:

$$U(x_1, x_2) = \begin{cases} 0 & \text{if } x_1 = \infty \text{ and/or } x_2 = \infty \\ \nu([x_1, \infty) \times [x_2, \infty)) & \text{if } (x_1, x_2) \in [0, \infty)^2 \setminus \{0\} \\ \infty & \text{if } (x_1, x_2) = 0 \end{cases}$$

The boundary tail integral is defined as

$$U_1(x_1) = U(x_1, 0), U_2(x_2) = U(0, x_2).$$

The following theorem is derived from Cont and Tankov (2004) about the Sklar theorem of Lévy Copula.^[7,8,9]

Theorem Let (S_1, S_2) is a Lévy process with positive jump, tail integral U , boundary tail integral U_1 and U_2 . There are two-dimensional positive Lévy Copula C dependent structures S_1 and S_2 , for all $x_1, x_2 \in [0, \infty]$

$$U(x_1, x_2) = C(U_1(x_1), U_2(x_2))$$

If U_1 and U_2 are continuous, then Lévy Copula is the only. Otherwise, it is unique on the $RanU_1 \times RanU_2$.

3. Empirical study

The basket price of agricultural products at maturity date is $(S(T) - K)^+$. The price of the basket option of the agricultural products is as follows

$$C[K, T] = e^{-rT} E[S(T) - K]^+$$

We consider white sugar, corn and wheat as 3 basket options for agricultural products with a storage ratio of 0.5%, 0.3% and 0.6%, where $r = 6\%$, $\nu = 0.9$. The model parameters and weights are shown in the table1. The expiration date of the option is 2 months. The execution price K is 200. The initial price $S_i(0)$ is the unit (per ton price).

Table 1 three underlying assets of agricultural product options

	1	2	3
$S_i(0)$	100	100	100
μ_i	-0.15	-0.06	-0.2
σ_i	0.1	0.2	0.04
ω_i	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Using the Monte Carlo method to simulate 10^5 times, the agricultural option price are 71.1441 and 71.2570 respectively by Black-Scholes model and Lévy copula model. The value of Lévy copula model is higher.

Relative error

$$\varepsilon[K] = \frac{|BS[K, T] - LC[K, T]|}{LC[K, T]}$$

The Black-Scholes model and Lévy copula model are compared by a set of execution prices, where

$K \in [200, 300]$. The results of the simulation are shown in the Table2.

Table 2. Call price of agricultural products option

T	K	$BS[K, T]$	$LC[K, T]$	$\varepsilon[K]$
Two months	200	71.1441	71.2570	0.14%
	205	66.1690	66.2819	0.15%
	210	61.1939	61.3069	0.16%
	215	56.2189	56.3318	0.17%
	220	51.2438	51.3567	0.18%
	225	46.2687	46.3817	0.20%
	230	41.2937	41.4066	0.22%
	235	36.3186	36.4316	0.24%
	240	31.3436	31.4565	0.27%
	245	26.3685	26.4814	0.31%
	250	21.3934	21.5064	0.36%
	255	16.4184	16.5313	0.43%
	260	11.4433	11.5562	0.53%
	265	6.4682	6.5812	0.69%
	270	1.4932	1.6061	0.98%
	275	0	0	NaN
	280	0	0	NaN
	285	0	0	NaN
	290	0	0	NaN
	295	0	0	NaN
	300	0	0	NaN

The values obtained by Lévy copula model are higher than the values of the Black-Scholes model. The results are consistent with the reality.

4. Conclusion

This paper studies the pricing of agricultural options on Lévy copula. Then the Monte Carlo simulation technique was used to solve the Black-Scholes model and Lévy copula respectively. The Lévy copula function, which is closer to the financial market in real life, is used to express the stochastic process of asset price, and the distribution characteristics of the underlying assets can show other characteristics. This can be more in line with the actual market, can better describe the financial asset prices and benefits, so as to better avoid risks.

Since the rapid development of the option, the main reason is that it has its own unique advantages such as low cost, simple and efficient, and the risk and return to separate, so that traders to avoid the same time, to retain the opportunity to obtain income. China's development of agricultural options market can achieve attracting people to participate in, improve the agricultural support policies to promote the order of agricultural development and agricultural futures market.

In order to further towards the modernization of agriculture, the implementation of effective risk management in agriculture, China needs to learn from each other, learn the advanced developed countries agricultural risk management experience, deepen model innovation, exploring new mechanism of agricultural risk management. The use of options and other financial instruments for agricultural risk management, is to ensure that China's agricultural risk management mechanisms continue to improve, and the healthy development of agricultural products market is an important measure.

At present, the Chinese government in order to protect the interests of the majority of food growers, in addition to the implementation of some varieties of protection of the purchase price, in recent years in the main grain varieties are based on the land area of the implementation of the subsidy policy.

Although the implementation of the food subsidy policy to a certain extent, increased farmers benefit, promote the development of agriculture, but increased the financial burden of the country. With the agricultural options market will make the government more effective protection of agricultural development.

The option market of agricultural products has just been established in China, and the products option should be tested by the market and investors. The exchange should improve the contracts and keep up with the requirements of the market development. At the same time, and actively preparing for the timely introduction of new varieties of agricultural options such as corn, wheat and other options, options, options can not only enrich the variety system, storage system reform and agricultural supply side structural reform can better serve important food varieties.

References

- [1] Tomek W G, Kaiser H M. Agricultural product prices[M]. Cornell University Press, 2014.
- [2] Understanding options for agricultural production[M]. Springer Science & Business Media, 2013.
- [3] Black F, Scholes M. The Pricing of Options and Corporate Liabilities[J]. The Journal of Political Economy, 1973, 81(3): 637-654.
- [4] Carr P, Helyette G, Dilip B, et al. The Fine Structure of Asset Returns: an Empirical Investigation[J]. The Journal of Business, 2002, 75(1): 305-332.
- [5] Tankov P. Financial modelling with jump processes[M]. CRC press, 2003.
- [6] Deelstra G, Petkovic A. How they can jump together: Multivariate Lévy processes and option pricing[J]. Belgian Actuarial Bulletin, 2010, 9(1): 29-42.
- [7] Applebaum D. Lévy processes and stochastic calculus[M]. Cambridge university press, 2009.
- [8] Luciano E, Semeraro P. Multivariate time changes for Lévy asset models: Characterization and calibration[J]. Journal of Computational and Applied Mathematics, 2010, 233(8): 1937-1953.
- [9] Kawai R. A multivariate Lévy process model with linear correlation[J]. Quantitative Finance, 2009, 9(5): 597-606.