

Direct Numerical Simulation of Pore-scale Unidirectional Flow in Porous Media

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Abstract. The paper presents the results of mathematical and numerical modeling of single-phase fluid flow in porous media with periodic microstructure. Object of study is the area in which the cylinders are arranged in a periodic manner. At the boundaries of the area for the flow parameters is set periodic boundary condition. Also in the paper presents comparison with Darcy's law and the calculation of the permeability coefficient for different values of the radius of the cylinders.

1. Introduction

This paper presents the pore-scale direct numerical simulation of single phase flow in porous media with periodic microstructure. This simulation based on the Navier-Stokes equations for incompressible single phase fluid:

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho (\vec{u} \cdot \vec{\nabla}) \vec{u} = \rho \vec{g} - \vec{\nabla} p + \mu \nabla^2 \vec{u} \quad (1)$$

$$\vec{\nabla} \cdot \vec{u} = 0 \quad (2)$$

where ρ - density of the fluid, \vec{u} - velocity of the fluid flow, p - pressure, μ - fluid viscosity and \vec{g} - acceleration due to gravity. The main difficulty of the pore-scale modeling of the fluid flow in the porous medium is that it is very difficult to set the correct boundary and initial conditions for the system of equations (1) and (2). The paper [6] has proof of that the system of equations (1) and (2) has a solution and it is unique and depends continuously on initial and boundary conditions if zero initial condition for the velocity and periodic boundary conditions for the velocity and pressure are applied. In cases, when need to define more complex initial and boundary conditions for the velocity and pressure, we can measure these experimentally^[1-2]. In this paper the zero initial condition for the velocity and periodic boundary conditions for the velocity and pressure are considered. Analytical solution of the system of the equations (1) and (2) was found only in special cases and for simple domains. Therefore, to solve the system of equations (1) and (2) need to use numerical methods. In this paper a finite volume method with adaptive mesh refinement is used to discretize the Navier-Stokes equations (1) and (2). To solve these discretized equations, MAC (Marker-and-Cell) method is



used^[3]

2. Objective

This paper considers two-dimensional rectangular domain with size $2L \times 2L$ (where x changes from $-L$ to L and y changes from $-L$ to L), in which the cylinders are arranged in a periodic manner (figure 1). The initial condition for the velocity is defined as follows^[4-5]:

$$\vec{u}(0, x_k) = 0 \quad (3)$$

Boundary conditions for the velocity and pressure are as follows:

At the boundaries of the area:

$$\begin{aligned} \vec{u}(t, x_k)|_{x_k=-L} &= \vec{u}(t, x_k)|_{x_k=L}, \\ p(t, x_k)|_{x_k=-L} &= p(t, x_k)|_{x_k=L}, \end{aligned} \quad (4)$$

$$\left. \frac{\partial \vec{u}(t, x_k)}{\partial x_k} \right|_{x_k=-L} = \left. \frac{\partial \vec{u}(t, x_k)}{\partial x_k} \right|_{x_k=L},$$

On the surface of the cylinders (no-slip condition):

$$\vec{u}(t, x_k) = 0, \quad (5)$$

where k – number of dimensions, in our case - $k = 2$.

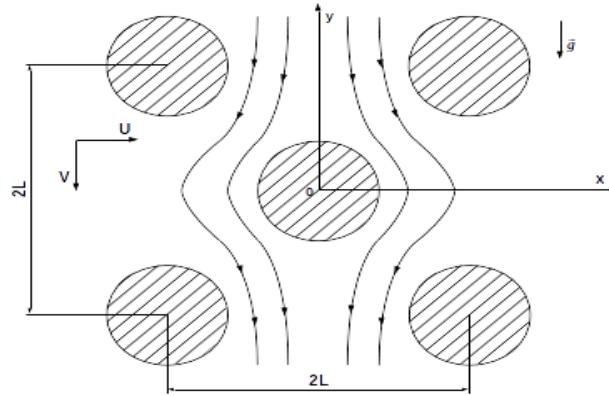


Figure 1: Two-dimensional rectangular domain with size $2L \times 2L$ (where x changes from $-L$ to L and y changes from $-L$ to L), in which the cylinders are arranged in a periodic manner

In order to find the rate of filtration - \vec{u}_d need to average the velocity over the volume:

$$\vec{u}_d = \frac{1}{\Omega_p} \int \vec{u}(t, x) dx, \quad (6)$$

where Ω_p – volume of the pore space [1]. Further, from Darcy's law [1] we can find the permeability of the porous medium:

$$K = \left| \frac{\mu \vec{u}_d}{\vec{\nabla} (p + \rho g z)} \right| \quad (7)$$

3. Results

1) Case when radius of cylinders - $r = 0,2$

Table 1: Relation between fluid viscosity - μ and filtration rate - u_d for case when radius of cylinders - $r = 0,2$

μ	u_d
0,1	-0,91
0,5	-0,163
1	-0,084
2	-0,04
5	-0,019
10	-0,0092

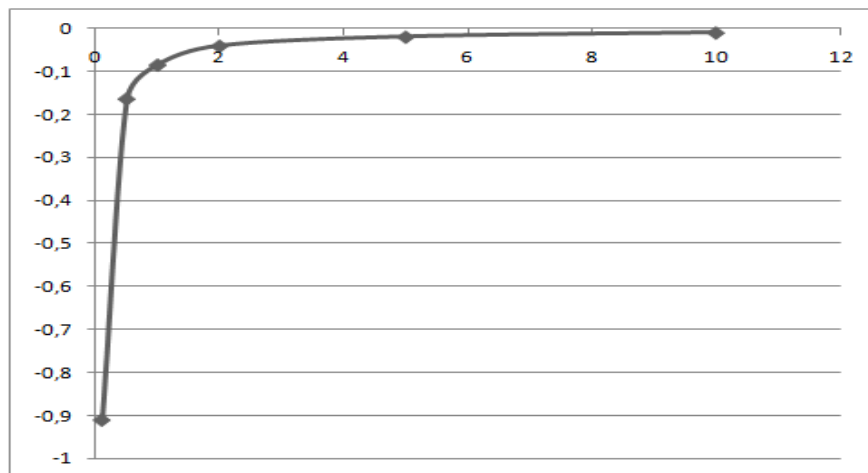


Figure 2: Relation between fluid viscosity - μ and filtration rate - u_d for case when radius of cylinders - $r = 0,2$

4. Conclusion

On this basis, we have studied the effects of porous media gap character of diffusion and mass transfer by numerical simulating the diffusion process of the porous media, and obtained the following conclusions:

- (1) Under different pore structure of the porous medium, the diffusion rate and equilibrium concentrations are different.
- (2) Under the same void structure, the diffusion rate increases, increasing the diameter of the channel did not have regular impact on the final result of the proliferation of balance.
- (3) The diffusion behavior of porous media are similar, but the changes produced by the diffusion rate and the average balance is less than the change in concentration of the resulting change in the pore structure.

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