

# Sensor Transmission Power Schedule for Smart Grids

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**Abstract.** Smart grid has attracted much attention by the requirement of new generation renewable energy. Nowadays, the real-time state estimation, with the help of phasor measurement unit, plays an important role to keep smart grid stable and efficient. However, the limitation of the communication channel is not considered by related work. Considering the familiar limited on-board batteries wireless sensor in smart grid, transmission power schedule is designed in this paper, which minimizes energy consumption with proper EKF filtering performance requirement constrain. Based on the event-triggered estimation theory, the filtering algorithm is also provided to utilize the information contained in the power schedule. Finally, its feasibility and performance is demonstrated using the standard IEEE 39-bus system with phasor measurement units (PMUs).

## 1. Introduction

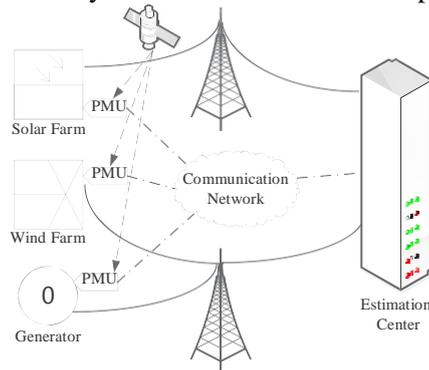
Aiming at sustaining long-term energy supply, renewable energy has attracted widespread attention. Renewable energy such as wind and solar needs a new-generation power grid infrastructure called smart grid to accommodate to the power network. Smart grid utilizes automate control, intelligent and information technology to realize a reliable and sustainable energy management system (EMS) [1–3].

Among the EMS, accurate estimation for the dynamic status of smart grid plays the key role, which is hard to be realized by conventional supervisory control and data acquisition (SCADA) systems. To satisfy the accurate and dynamic estimation needs, the real-time phasor measurement unit (PMU) is invented utilizing the GPS time-stamped technology to provide high frequency accurate synchronous phasor data. Based on PMU, Wide-area measurement systems (WAMS) is established to accomplish the synchrophasor real-time state estimation (RTSE) [4, 5]. The structure of WAMS is show in Fig. 1[6].

Amount of research on RTSE has been developed based on the observation data of PMU [7– 12]. Due to the nonlinearity of power system, nonlinear filter is wildly applied in RTSE. Extended Kalman filter (EKF), which linearizes the system functions to realize the iteration of covariance in nonlinear filtering process, was firstly introduced to estimate the dynamic states of smart grid [7]. However, the higher order terms of Taylor series translates into linearization error during the EKF process. Thus, when the system's nonlinearity is strong, EKF will lead low filtering accuracy and even filtering



divergency. The unscented Kalman filter (UKF) was introduced by Singh and Pal to solve the strong nonlinear power system's RTSE problem, which utilized the sigma points to approximate the distribution rather the system function [8]. Compared with EKF, UKF performs better when power system is strong nonlinear and the noise the Gaussian. However, UKF ineluctably suffers from the numerical stability, which makes UKF an unreliable solution for power system's RTSE. Particle filter (PF), which utilizes Monte Carlo Methodology to approximate the condition distribution, is accurate and reliable enough for nonlinear system's estimation [10]. Nevertheless, the computing power needed by PF is unpractical, thus PF can only be treated as the decade-to-be solution. Thus, EKF is wildly studied in dynamic states estimation of power systems.



**Figure 1.** System structure of RTSE for WAMS based on PMU.

A usual assumption in RTSE research is that the communication channel in Fig. 1 is ideal. However, with widespread development of wireless sensors, the communication channel between smart grid and estimation center is susceptible to environmental influence. Amid all the influence, transmission failure, which means that the observation transmitted by the smart grid is not received by the estimation center, is the one that has the most serious impact on the estimation. The probability of transmission failure typically has a positive correlation with transmission power. Because the transmission power is usually provided by limited on-board batteries in smart grid, a sensor transmission power schedule should be designed to balance the transmission failure probability and the transmission power consumption [13–15].

This paper focuses on the sensor transmission power schedule problem in RTSE of power system. Considering the nonlinearity of power system, the transmission power schedule is analyzed based on EKF. The rest of this paper is organized as follows. Section 2 introduces the dynamic states estimation and the transmission power schedule problem of power system. Section 3 designs the transmission power schedule that minimizes energy consumption and guarantee proper estimation performance by determining transmission power according to the innovation of observation. Also, the filtering algorithm is provide, which utilizes the information in transmission power schedule to promote estimation performance. Section 4 utilizes IEEE 39-bus system to demonstrate the feasibility of the proposed method. The following standard notations are adopted throughout this paper. The norm of vector  $\|x\|_{\infty}$  stands for the maximum norm  $\|x\|_{\infty} = \max(|x_1|, \dots, |x_n|)$ . The expectation of random number  $x$  is described as  $E(x)$ , and the condition expectation of random number  $x$  given  $y$  is described as  $E(x/y)$ . The probability of an event is described as  $\Pr(\cdot)$ .

## 2. System description and problem statement

As most smart grid is nonlinear system, a general nonlinear model is set up as,

$$\begin{aligned} x_{k+1} &= f(x_k) + \omega_k \\ y_k &= h(x_k) + v_k \end{aligned} \quad (1)$$

where  $x_k \in \mathbb{R}^n$  and  $y_k \in \mathbb{R}^p$  are the system state and measurement output, respectively. The functions of

$f(x)$  and  $g(x)$  are continuously differentiable at  $x$ . The process noise  $\omega_k \in \mathbb{R}^n$  and measurement noise  $v_k \in \mathbb{R}^n$  are white sequences with zero means. Their covariance matrices  $E[\omega_k \omega_k^T] = Q_k \delta_{kj} > 0$  and  $E[v_k v_k^T] = R_k \delta_{kj} > 0$ . The initial distribution of  $x_0$  is Gaussian with zero mean and covariance matrix  $P_0$ . Moreover,  $x_0$ ,  $\omega_k$  and  $v_k$  are assumed to be independent with each other.

The measurement  $y_k$  is transmitted over a wireless fading channel by the wireless emission infrastructure in the smart grid, which is shown in Figure 1. A binary variable  $\gamma_k$  is denoted to reflect whether  $y_k$  is successfully received by the estimation center as,

$$\gamma_k = \begin{cases} 1, & y_k \text{ is received} \\ 0, & y_k \text{ is not received} \end{cases} \quad (2)$$

At the time instance  $k$ , both the receiving states  $\gamma_k$  and the measurement  $y_k$  make up the observation information for estimation as

$$\mathbf{Y}_k = \{\gamma_k, y_k\}. \quad (3)$$

It should be noticed that  $y_k$  is meaningless when  $\gamma_k = 0$ , because estimation only get a pure noise at time instance  $k$ . Moreover, the trigger contained information set is defined as

$$\mathbf{F}_k \square \{\mathbf{Y}_1, \mathbf{Y}_2, \dots, \mathbf{Y}_k\} \quad (4)$$

The status of  $\gamma_k$  depends on both the communication channel and the communication power. According to the law of large number, the noise in communication channel can be described by an Additive White Gaussian Noise (AWGN). At the same time, the observation signal is modulated by Quadrature Amplitude Modulation (QAM) [16]. Therefore, by the communication theory, the symbol error rate (SER) can be approximated as,

$$SER = \exp\left(-\alpha \frac{M_k}{N_0 W}\right) \quad (5)$$

here  $N_0$  is the AWGN noise power spectral density,  $W$  is the channel bandwidth,  $\alpha$  is the constant determined by the number of byte in the observation signal, and  $M_k$  is the transmission power of QAM. Amount these parameter,  $N_0$ ,  $W$  and  $\alpha$  are all positive constant and determined off-line in practice, while  $M_k$  is the only parameter need to be scheduled. According to (5) and [16], the probability of transmission failure is that,

$$P\{\gamma_k = 0\} \square \lambda_k = (1 - \theta)^{M_k}, \quad (6)$$

where  $\theta$  is the constant parameter satisfying

$$\theta = 1 - \exp\left(\frac{-\alpha}{N_0 W}\right). \quad (7)$$

It should be noticed that  $\theta \in (0, 1)$ , thus, the transmission failure probability satisfies  $\lambda \in (0, 1)$ . The task of sensor transmission power schedule is calculating the minimum transmission power  $M_k$  that guarantee proper estimation performance.

### 3. EKF based transmission power schedule and filtering algorithm

This section establish the transmission power schedule based on EKF, also the filtering algorithm under the designed schedule is also proposed to promote the estimation accuracy inspired by the event-trigger filter theory.

#### 3.1. EKF theory

In this part, the two-step EKF is introduced, and execute the filtering algorithm when the observation  $y_k$  is received. For convenience, the one step state prediction and the prior respective error covariance matrices are denoted as  $\hat{x}_{k|k-1} = E(x_k/\mathbf{F}_{k-1})$  and  $P_{k|k-1} = Cov(x_k/\mathbf{F}_{k-1})$ , respectively. At the same

time, the state estimation and the posterior respective error covariance matrices are denoted as  $\hat{x}_{k|k-1} = E(x_k/F_k)$  and  $P_k = Cov(x_k/F_k)$  respectively.

Then, if the previous time estimation result  $\hat{x}_{k|k-1}$  and  $P_{k-1}$  are already know, the iteration of EKF includes the prediction and measurement update steps. The prediction step is described as

$$\begin{aligned}\hat{x}_{k|k-1} &= f(\hat{x}_{k-1}) \\ \hat{P}_{k|k-1} &= A_k \hat{P}_{k-1} A_k^T + Q\end{aligned}\quad (8)$$

where  $A_k$  is got by Taylor expansion of the continuous function  $f(x)$  as,

$$A_k = \left. \frac{\partial f}{\partial x} \right|_{x=\hat{x}_{k-1}}. \quad (9)$$

Moreover, if the observation  $y_k$  is received by the estimation center, the measurement update step satisfies that,

$$\begin{aligned}\hat{x}_k &= \hat{x}_{k|k-1} + K_k (y_k - h(\hat{x}_{k|k-1})) \\ \hat{P}_k &= \hat{P}_{k|k-1} - K_k C_k \hat{P}_{k|k-1}\end{aligned}\quad (10)$$

where  $C_k$  is the first order Taylor polynomial of the continuous function  $h(x)$  as,

$$C_k = \left. \frac{\partial h}{\partial x} \right|_{x=\hat{x}_{k|k-1}}, \quad (11)$$

and  $K_k$  satisfies that,

$$K_k = \hat{P}_{k|k-1} C_k^T (C_k \hat{P}_{k|k-1} C_k + R)^{-1} \quad (12)$$

### 3.2. Transmission power schedule

It can be noticed from (10) that different value of  $y_k$  has different influence on the measurement update step of EKF process. In filtering theory,  $(y_k - \hat{y}_{k|k-1})$  is known as the ‘‘innovation’’ of  $y_k$ , which evaluate the importance of the observation  $y_k$ .

To obtain an appropriate estimation accuracy, the transmission power should be determined by the innovation of  $y_k$ . When the innovation is big, the information contained in  $y_k$  is important for filter, thus paying plenty of transmission power is worth. On the contrary,  $y_k$  with small innovation is less important for estimation, which does not need too much transmission power. Before utilizing the innovation of  $y_k$ , the weighted average can be carried out by

$$\varepsilon_k = (y_k - \hat{y}_{k|k-1})^T Z_k (y_k - \hat{y}_{k|k-1}), \quad (13)$$

where  $Z_k$  is the weighted matrix to be designed and  $\hat{y}_{k|k-1}$  is the prior condition mean of  $y_k$  as,

$$\hat{y}_{k|k-1} = E(y_k | F_{k-1}) = h(f(\hat{x}_{k-1})) \quad (14)$$

The transmission power schedule in this paper is designed as,

$$M_k = \frac{\varepsilon_k N_0 W}{2\alpha}. \quad (15)$$

Thus the transmission failure probability under the condition that observation equals  $y_k$  in can be deduced as,

$$\begin{aligned}
P(\gamma_k = 0 | y_k) &= \left( \exp\left(\frac{-\alpha}{N_0 W}\right) \right)^{\frac{\varepsilon_k N_0 W}{2\alpha}} \\
&= \exp\left(\frac{-\alpha}{N_0 W} \cdot \frac{(y_k - \hat{y}_{k|k-1})^T Z_k (y_k - \hat{y}_{k|k-1}) / 2}{2\alpha} N_0 W\right) \\
&= \exp\left(-\frac{(y_k - \hat{y}_{k|k-1})^T Z_k (y_k - \hat{y}_{k|k-1})}{2}\right)
\end{aligned} \tag{16}$$

Because  $y_k$  is the random variable, and its prior condition distribution can be approximated by the Gaussian distribution that

$$p(y_k | F_{k-1}) \sim N(\hat{y}_{k|k-1}, P_{y_k}), \tag{17}$$

where,  $P_{y_k}$  is the prior condition covariance of  $y_k$  calculated by

$$P_{y_k} = C_k (A_k \hat{P}_{k-1} A_k^T + Q) C_k^T + R \tag{18}$$

Thus the expectation of the transmission failure probability is calculated by

$$\begin{aligned}
P(\gamma_k = 0 | F_{k-1}) &= \int_{\mathbb{R}^p} \frac{1}{(2\pi)^{p/2} \det(P_{y_k})^{1/2}} \exp\left(\frac{-(y_k - \hat{y}_{k|k-1})^T P_{y_k}^{-1} (y_k - \hat{y}_{k|k-1})}{2}\right) P(\gamma_k = 0 | y_k) dy_k \\
&= \int_{\mathbb{R}^p} \frac{1}{(2\pi)^{p/2} \det(P_{y_k})^{1/2}} \exp\left(\frac{-(y_k - \hat{y}_{k|k-1})^T (P_{y_k}^{-1} + Z_k) (y_k - \hat{y}_{k|k-1})}{2}\right) dy_k \\
&= \frac{1}{\det(I_{p \times p} + P_{y_k} Z_k)^{1/2}}
\end{aligned} \tag{19}$$

According to related work with EKF, the performance of EKF depends on the expectation of transmission failure probability. To get proper performance of EKF, it must be satisfied that  $P(\gamma_k = 0 | F_{k-1}) \leq \lambda_c$ , where  $\lambda_c$  is the gate depended on different system [17]. Due to (19), if  $Z_k$  is selected as

$$Z_k = \rho P_{y_k}^{-1}, \tag{20}$$

the expectation of transmission failure probability can be calculated as,

$$P(\gamma_k = 0 | F_{k-1}) = 1 / (1 + \rho)^{p/2}. \tag{21}$$

If  $\lambda_c$  is determined and the aim of transmission power schedule is to minimize energy consumption as much as possible,  $\rho$  can be calculated as,

$$\rho = \lambda_c^{-2/p} - 1. \tag{22}$$

### 3.3. Filtering algorithm in estimation center

Section 3.2 established the transmission power schedule in smart grid, this section will provide the corresponding filtering algorithm in estimation center. From Section 3.2 it can be noticed that the observation  $y_k$  can either successful or failed to be received by the estimation center. When  $\gamma_k = 1$ , filtering algorithm can be realized by EKF in Section 3.1. Otherwise, when  $\gamma_k = 0$ , filtering algorithm should be delicately design by taking advantage of event-trigger theory to make full use of information containing in the transmission power schedule.

The iteration under  $\gamma_k = 0$  is same with EKF as,

$$\begin{aligned}
\hat{x}_{k|k-1} &= f(\hat{x}_{k-1}) \\
\hat{P}_{k|k-1} &= A_k \hat{P}_{k-1} A_k^T + Q
\end{aligned} \tag{23}$$

where  $A_k$  is

$$A_k = \frac{\partial f}{\partial x} \Big|_{x=\hat{x}_{k-1}}. \quad (24)$$

However, the measurement update step is quite different from EKF, because the observation  $y_k$  is not available for estimation center. Even though, the information contained in the transmission power schedule can be utilized by the filtering algorithm. Noticing that (16) is same with the trigger strategy in [18], the stochastic event-triggered filter in [18] can be utilized to realize the filtering algorithm when  $\gamma_k = 0$ , which is shown as,

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} \\ \hat{P}_k &= \hat{P}_{k|k-1} - \tilde{K}_k C_k \hat{P}_{k|k-1} \end{aligned} \quad (25)$$

where  $C_k$  is that,

$$C_k = \frac{\partial h}{\partial x} \Big|_{x=\hat{x}_{k|k-1}}, \quad (26)$$

And  $\tilde{K}_k$  satisfies that,

$$\tilde{K}_k = \hat{P}_{k|k-1} C_k^T (C_k \hat{P}_{k|k-1} C_k + R + Z^{-1})^{-1} \quad (27)$$

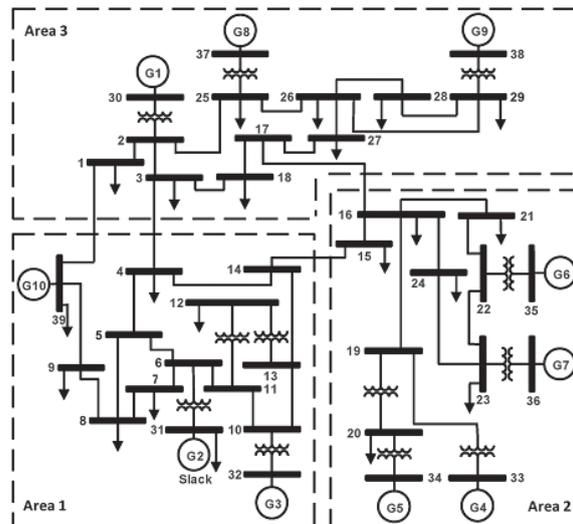
Comparing the posterior condition covariance in (25) utilizing the transmission power schedule information with the prior condition covariance. It can be found that the posterior condition covariance is smaller than the prior condition covariance due to the existence of the term  $\tilde{K}_k C_k \hat{P}_{k|k-1}$ , which actually reflects the influence of the information contained in transmission power schedule.

Take advantage of the binary variable  $\gamma_k$ , the filtering algorithm regardless of the receiving of the observation  $y_k$  can be united into

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} + \gamma_k \hat{K}_k (y_k - h(\hat{x}_{k|k-1})) \\ \hat{P}_k &= \hat{P}_{k|k-1} - \hat{K}_k C_k \hat{P}_{k|k-1} \end{aligned} \quad (28)$$

where  $\hat{K}_k$  satisfies

$$\hat{K}_k = \hat{P}_{k|k-1} C_k^T (C_k \hat{P}_{k|k-1} C_k + R + (1 - \gamma_k) Z^{-1})^{-1} \quad (29)$$



**Figure 2.** IEEE 39-bus 10-generator (New England) system.

#### 4. Case study and simulation

IEEE 39-bus system, which is also known as the 10-generator New England system, is widely applied to evaluate the estimation performance of smart grid [8, 10]. Fig. 2 describes the constitution of IEEE 39-bus system, where each grid is a generator. The parameters of generators quotes from [10]. The simulation executes 15 seconds after a fault breaks out at the line connecting the bus 14 to bus 15. By selecting the sampling interval as 0.02s, 750 discrete time interval estimation is performed.

In Fig. 1, each generator consists a smart grid, and its model can be described as,

$$\begin{aligned} E_{d'}(k+1) &= \frac{\Delta t}{T'_{qo}} \left( -E'_d(k) + (X_q - X'_q)i_q(k) \right) + E'_d(k), \\ E_{q'}(k+1) &= \frac{\Delta t}{T'_{do}} \left( -E'_q(k) - (X_d - X'_d)i_d(k) + E_{fd}(k) \right) + E'_q(k) \\ \delta(k+1) &= \Delta t \left( \omega(k) - \omega_s \right) \omega_b + \delta(k), \end{aligned} \quad (30)$$

$$\omega(k+1) = \frac{\Delta t}{2H} \omega_s \left( T_m - E'_d(k)i_d(k) - E'_q(k)i_q(k) - (X'_q - X'_d)i_d(k)i_q(k) - D(\omega(k) - \omega_s) \right) + \omega(k).$$

This simulation pays attention on the smart of the generator 2 in Fig. 1, which is consisted by the generator and IEEE DC1A AVR. The generator can be described by (30), and the AVR can be modeled as,

$$\begin{aligned} E_{fd}(k+1) &= \frac{\Delta t}{T_E} \left( - \left( K_E + A_x e^{B_x E_{fd}(k)} \right) E_{fd}(k) + V_R(k) \right) + E_{fd}(k), \\ R_f(k+1) &= \frac{\Delta t}{T_F} \left( -R_f(k) + \frac{K_F}{T_F} E_{fd}(k) \right) + R_f(k), \\ V_R(k+1) &= \frac{\Delta t}{T_A} \left( - \frac{K_A K_F}{T_F} E_{fd}(k) - V_R(k) + K_A R_f(k) + K_A (v_{ref} - v(k)) \right) + V_R(k). \end{aligned} \quad (31)$$

PMU measures the magnitude and phase for both voltage of the smart grid and current flowing through generator. For convenience, the measurement data of PMU is denoted as  $v(k)$ ,  $\theta(k)$ ,  $i_d(k)$ ,  $i_q(k)$ . By Kirchhoff Law, the observation function can be established as

$$\begin{aligned} i_d(k) &= \frac{1}{X'_d} \left( E'_q - v(k) \cos(\delta(k) - \theta(k)) \right), \\ i_q(k) &= \frac{1}{X'_q} \left( -E'_d + v(k) \sin(\delta(k) - \theta(k)) \right). \end{aligned} \quad (32)$$

The system noise is set as Gaussian noise with the covariance matrix  $diag[10^{-18}, 10^{-18}, 10^{-9}, 10^{-9}, 10^{-9}, 10^{-9}, 10^{-9}]$  and the observation noise is set as Gaussian noise with covariance matrix  $diag[10^{-10}, 10^{-8}, 10^{-10}, 10^{-8}]$ . The estimator is placed at the estimation center, and the observations is transmitted by the power schedule in Section 3.2.

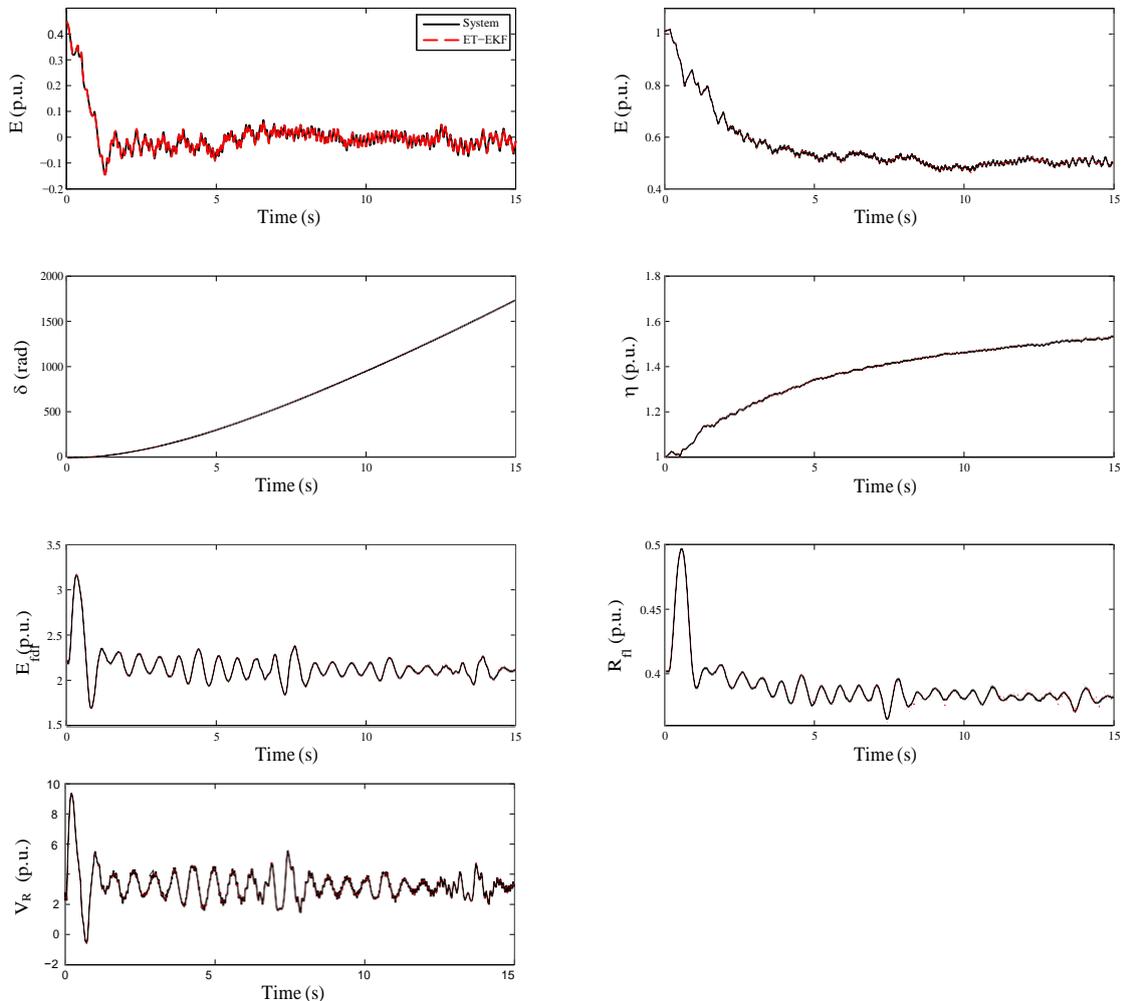
For convenient, this section denotes the filtering algorithm in Section 3.3 is denoted as ET-EKF. The gate for the expectation of transmission failure probability is selected as  $\lambda_c = 90\%$ , which means that only 10% of the measurement by PMU is received by the estimation center.

The simulation result is shown in Fig. 3. It can be seen that ET-EKF keeps stable in the statistics sense and provides accuracy filtering results, even only one-tenth observations are received by the estimation center.

#### 5. Conclusion

Considering the RTSE problem for WAMS under practical communication channel, the event-

trigger based transmission power schedule is designed. Moreover, the corresponding filter is designed based on EKF, which utilizes the information contained in the power schedule to promote the filtering accuracy. The simulation verifies that the power schedule and filter designed in this paper can reduce the transmission power and achieve high accuracy estimation result at the same time.



**Figure 3.** States and estimation results of generator when  $\lambda = 10\%$  and plot of PMU transmissions.

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