

Volatility spillover between crude oil and exchange rate: A copula-CARR approach

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Abstract. Oil provides a powerful impetus for modern society's production and life. The influences of oil price fluctuations on socio-economic development are obvious, and it draws more attention from scholars. However, the distribution of oil is highly centralized, which leads to the vast majority of oil trading through foreign trade. As a result, exchange rate plays an important role in the oil business. Study on the relationship between exchange rate and crude oil gradually becomes a hot research topic in recent years. In this paper, we use copula and CARR model to study correlation structure and relationship between crude oil price and exchange rate. We establish CARR models as marginal models and use five copulas which are Gaussian Copula, Student-t Copula, Gumbel Copula, Clayton Copula and Frank Copula to study the correlation structure between NYMEX crude oil price range and U. S. Dollar Index range. Furthermore, we use Copula-CARR model with structural breaks to detect the change points in the correlation structure between NYMEX crude oil price range and U. S. Dollar Index range. Empirical results show that the change points are closely related to the actual economic events.

1. Introduction

20th century's several oil crises have crippled some countries' national economy, so government agencies and scholars focus on oil price's fluctuation. The world trade is one of the major channels of oil deals, while foreign exchange is also related to a country's national income. Thus, some scholars' researches focus on the oil prices and exchange rate's relationship and correlation structure.

In general, these empirical studies can be categorized into two types. One type discusses the relationship between the oil price and exchange rate. This type of analysis is typically based on VAR model, GARCH model or ECM model. For example, Ying Huang *et al* [1] analyze the impact of oil price fluctuations on the real exchange rate of RMB using four-dimensional VAR model. Adeniyi O, *et al* [2] construct GARCH model and EGARCH model to estimate the impact of oil price on the nominal exchange rate of daily data from January 2, 2009 to September 28, 2010. Oriavwote V E, *et al* [3] use the co-integration test and Granger causality test to study the relationship between the real oil price and the real exchange rate by GARCH model. Beckmann J, *et al* [4] construct Markov-switching vector error correction model (MS-VECM) model to analyze the relationship between oil price and the exchange rate. Chen H T, *et al* [5] investigate the impacts of oil price shocks on the bilateral exchange rate of the U.S. dollar against currencies in 16 OECD countries. Chou K W, *et al* [6] use the asymmetric autoregressive distributed lag model to evaluate the influence of oil price and exchange rate fluctuations on retail gasoline price in Taiwan. Yang L, *et al* [7] study the co-movement between the crude oil price and the exchange rate markets by studying their dynamics in the time and frequency



domain. These studies demonstrate the relationship between the oil price and the exchange rate in the empirical.

The other type studies the correlation structure between oil prices and exchange rate by copula which is proposed by Sklar A [8]. Copula can describe variables' relationship and correlation structures. Rémillard B, *et al* [9] analyze the relationship between Canadian / US dollar exchange rate and oil futures' price with Elliptic copulas and Archimedean copulas. Reboredo J C [10] uses copula-ARMA model to study correlation structure and relationship between oil price and exchange rate. Alouia R *et al* [11] apply a vine copula approach to investigate the dynamic relationship between energy, stock and currency markets.

It is important to note that the Copula model has change points in some cases. Some scholars have studied on copula models with structural breaks. For example, Dias A and Embrechts P [12,13] use the method put forward by Gombay E and Horváth L [14] to study the changes in the German mark and Japanese yen's relation structure using Copula models with structural breaks. This method can detect the change in parameters without assuming possible date of the change point, and it can be used in risk management, asset pricing and asset allocation. Cai X, *et al* [15] use change point detection method to describe the dependence of English Pound and Eurodollar by Mixed Gumbel Copula and generalized Pareto distribution. Boubaker H, *et al* [16] use three Archimedean copulas (Gumbel copula, Clayton copula and Frank copula) to investigate the dependence structure between daily oil price changes and stock market returns in six GCC countries (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia and United Arab Emirates). Zhu X Q, *et al* [17] propose a multiple change point detection approach from the perspective of risk dependence by using copula function, and detect the start point, end point and peak period at the subprime crisis in American banking. Therefore, we should pay attention to the structural change point when we use copula function to study the relationship between oil price and exchange rate.

To choose a right marginal distribution model is very important in Copula method. In general, most literature used ARCH models, GARCH models and SV models as the marginal distribution model. Conditional Autoregressive Price range Model (CARR model henceforth) is proposed by Ray Yeutien Chou [18] which use daily price range to model and forecast variables' volatility. In terms of volatility forecasting, CARR model can be used as the preferred option because it covers more information than GARCH model. CARR models have been widely used in the research of financial volatility. For example, Cai Y J, *et al* [19] apply DSTCC- CARR model to study the fluctuation in the stock market and CPI. Chou R Y, *et al* [20] combine CARR with a DCC structure and conclude that the price range-based volatility models have more significant economic value than the return-based volatility models. Sin C Y [21] uses CARRX model to study the factors affecting the volatilities of Asian equity markets. These empirical results show that it is feasible to apply the CARR model to the study oil price and exchange rate's volatility.

Previous studies have shown that copula function has some advantages in depicting related relation and corresponding structure, and CARR model is a better choice in volatility modeling. Therefore, this paper uses the Copula-CARR model to analyze the correlation and dependency structure of NYMEX crude oil price range and U. S. Dollar Index range, and the optimal copula model is used to detect the optimal Copula model. In addition, this paper discusses the changes in the dependency structure of NYMEX crude oil price range and U. S. Dollar Index range.

This paper proceeds as follows: Section 2 and Section 3 present a briefly introduction of CARR model and Copula. Section 4 presents a change point detection method. We use Copula-CARR model to analyze New York Mercantile Exchange (NYMEX henceforth) crude oil price range and U. S. Dollar Index range's correlation structure and detect change points for the optimal Copula-CARR model in Section 5. Finally, we conclude in Section 6.

2. CARR model

CARR model is proposed by Chou (2005), which is used to characterize the dynamic structure of asset extremum within a fixed time period. This model is similar to GARCH model.

The basic form of CARR (p, q) is:

$$\begin{aligned}
 R_t &= \lambda_t \varepsilon_t \\
 \lambda_t &= \omega + \sum_{i=1}^p \alpha_i R_{t-i} + \sum_{j=1}^q \beta_j \lambda_{t-j} \\
 \varepsilon_t &\sim iidf(\bullet) \\
 R_t &= \ln P_t^{\max} - \ln P_t^{\min}
 \end{aligned} \tag{1}$$

Where, P_t represents the price of stock observed at time t , R_t is the difference of maximum and minimum logarithm P_t ; λ_t is the conditional mean of price range based on all information up to time t , and $\lambda_t = E(R_t / I_{t-1})$, $\lambda_t \geq 0$; The distribution of ε_t , or the normalized price range $\varepsilon_t = R_t / \lambda_t$ is assumed to be distributed with a density function $f(\bullet)$ with a unit mean; ω represents the initial level of the price range, and also represents the uncertainty of price range; α_i is the coefficient of price range, represents the short-term effects of conditional mean; β_j is the coefficient of conditional mean, represents the long-term effects of conditional mean. For the process to be stationary, a condition is that the characteristic roots of the polynomial are outside the unit circle, or $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$.

When we assume ε_t follows exponential distribution with unit mean, such a model is called the ECARR model. The density functions are respectively:

$$f(R_t | \lambda_t) = \frac{1}{\lambda_t} \exp\left\{-\frac{R_t}{\lambda_t}\right\} \tag{2}$$

When we assume $\varepsilon_t \sim \text{Gamma}(\theta, 1/\theta)$, such a model is called the WCARR model. The density functions are respectively:

$$f(R_t | \lambda_t, \theta) = \theta \left[\Gamma\left(1 + \frac{1}{\theta}\right) \right]^\theta \frac{R_t^{\theta-1}}{\lambda_t^\theta} \exp\left\{-\left[\Gamma\left(1 + \frac{1}{\theta}\right) \frac{R_t}{\lambda_t} \right]^\theta\right\} \tag{3}$$

When we assume $\varepsilon_t \sim \text{Gamma}(\kappa)$, such a model is called the GCARR model. The density functions are respectively:

$$f(R_t | \lambda_t, \theta, \kappa) = \frac{1}{\theta^\kappa \Gamma(\kappa)} R_t^{\kappa-1} \exp\left\{-\frac{R_t}{\lambda_t}\right\} \tag{4}$$

3. Copula

Copula can be seen as a connection function of marginal distributions and joint distributions. If $F(x_1, \dots, x_n, \dots, x_N)$ is a joint distribution function with marginal distribution functions $F_1(\cdot), \dots, F_N(\cdot)$, then there exists a copula C such that:

$$F(x_1, \dots, x_n, \dots, x_N) = C(F_1(x_1), \dots, F_n(x_n), \dots, F_N(x_N)) \tag{5}$$

The function $F(x_1, \dots, x_n, \dots, x_N)$ which is defined by equation (5) is joint distribution function of marginal distribution functions $F_1(\cdot), \dots, F_N(\cdot)$.

3.1. Elliptic copulas

In general, Elliptic Copulas include Gaussian copula and Student-t copula.

3.1.1. Gaussian copula. Nelsen R B [22] defines Gaussian copula, which distribution function and density function are as follows:

$$C(u_1, \dots, u_n, \dots, u_N; \rho) = \Phi_\rho(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n), \dots, \Phi^{-1}(u_N)) \quad (6)$$

$$c(u_1, \dots, u_n, \dots, u_N; \rho) = |\rho|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \zeta^T (\rho^{-1} - I) \zeta\right) \quad (7)$$

Where, ρ means symmetric positive definite matrix; Φ_ρ means a multivariate normal distribution with correlation matrix ρ ; $\Phi^{-1}(\cdot)$ means the inverse function of normal distribution; $\zeta_n = \Phi^{-1}(u_n)$, I means a unit matrix.

3.1.2. Student-t Copula. Nelsen defines Student-t copula, of which distribution function and density function are as follows:

$$C(u_1, \dots, u_n, \dots, u_N; \rho, \nu) = T_{\rho, \nu}(t_\nu^{-1}(u_1), \dots, t_\nu^{-1}(u_n), \dots, t_\nu^{-1}(u_N)) \quad (8)$$

$$c(u_1, \dots, u_n, \dots, u_N; \rho, \nu) = |\rho|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\nu+N}{2}\right) \left[\Gamma\left(\frac{\nu}{2}\right)\right]^N \left(1 + \frac{1}{\nu} \zeta^T \rho^{-1} \zeta\right)^{-\frac{\nu+N}{2}}}{\left[\Gamma\left(\frac{\nu+1}{2}\right)\right]^N \Gamma\left(\frac{\nu}{2}\right) \prod_{n=1}^N \left(1 + \frac{\zeta_n^2}{\nu}\right)^{-\frac{\nu+1}{2}}} \quad (9)$$

Where, ρ means symmetric positive definite matrix; $T_{\rho, \nu}$ means a multivariate standard Student's t distribution with ν degrees of freedom and correlation matrix ρ ; $t_\nu^{-1}(\cdot)$ means the inverse function of univariate Student's t distribution; $\zeta_n = t_\nu^{-1}(u_n)$.

3.2. Archimedean copulas

In general, Archimedean Copulas include Gumbel copula, Clayton copula and Frank copula.

3.2.1. Gumbel copula. Gumbel Copula's distribution function and density function are as follows:

$$C_G(u, v; \alpha) = \exp\left\{-\left[(-\log u)^{\frac{1}{\alpha}} + (-\log v)^{\frac{1}{\alpha}}\right]^\alpha\right\} \quad (10)$$

$$c_G = \frac{C_G(u, v; \alpha) (\log u \cdot \log v)^{\frac{1}{\alpha}-1}}{uv \left[(-\log u)^{\frac{1}{\alpha}} + (-\log v)^{\frac{1}{\alpha}}\right]^{2-\alpha}} \left\{\left[(-\log u)^{\frac{1}{\alpha}} + (-\log v)^{\frac{1}{\alpha}}\right]^\alpha + \frac{1}{\alpha} - 1\right\} \quad (11)$$

Where, $\alpha \in (0, 1]$, relevant parameter.

3.2.2. *Clayton copula*. Clayton Copula's distribution function and density function are as follows:

$$C_C(u, v; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{1/\theta} \quad (12)$$

$$c_C(u, v; \theta) = (1 + \theta)(uv)^{-\theta-1} (u^{-\theta} + v^{-\theta} - 1)^{-2-1/\theta} \quad (13)$$

Where, $\theta \in (0, \infty)$, relevant parameter.

3.2.3. *Frank copula*. Frank Copula's distribution function and density function are as follows:

$$C_F(u, v; \lambda) = -\frac{1}{\lambda} \log \left[1 - \frac{(1 - e^{-\lambda u})(1 - e^{-\lambda v})}{1 - e^{-\lambda}} \right] \quad (14)$$

$$c_F(u, v; \lambda) = \frac{\lambda(1 - e^{-\lambda})e^{-\lambda(u+v)}}{[(1 - e^{-\lambda}) - (1 - e^{-\lambda u})(1 - e^{-\lambda v})]^2} \quad (15)$$

Where, $\lambda \neq 0$, relevant parameter; $\lambda > 0$, random variables u and v has a positive correlation; $\lambda \rightarrow 0$, u and v tend to be independent; $\lambda < 0$, u and v has a negative correlation.

All kinds of copulas have their own advantages to describe correlation between variables. Density function of bivariate Gaussian copula has symmetry which can capture the correlation change between variables in central, not in tail. If variables are asymptotically independent in the tail, Gaussian copula can't catch variable's "fat tail" feature. Compared to bivariate Gaussian Copula, t-Copula has thick tail, which means it can better describe tail dependence between variables. Density function of Gumbel copula has asymmetry, and its density distribution is "J" type. Gumbel copula is more sensitive to the change in upper tails, which can be used to describe upper tail dependence between variables. Clayton copula also has asymmetry, but in contrast to Gumbel copula, its density distribution is "J" type that more sensitive to the change in lower tail and can be used to describe lower tail dependence between variables. Frank copula's density distribution is "U" type, which means it is symmetrical. In contrast to Clayton copula and Gumbel copula, Frank copula can describe negative correlation between variables. Compared to Gaussian copula, Frank copula is more emphasis on tail dependence, and its upper tail dependence and lower tail dependence's growth is symmetry.

4. Diagnosis of structural breaks of Copula model: maximum likelihood function method [12]

Maximum likelihood function method can be used to detect structural breaks of copula. The maximum likelihood function method regards change point as a parameter, by constructing statistics to find the maximum value of likelihood function and estimate value of the change point.

Let (X_t, Y_t) 's joint distribution function is $C(x, y; \theta, \eta)$.

Where, θ and η is Copula's parameters and their marginal distribution functions are $F(x)$ and $G(y)$. We detect change points of θ , whereas η will be regarded as nuisance parameter. To detect whether the correlation structure between the variables in the time of $k = k^*$ has changed significantly, we can use the following test:

The null hypothesis:

$$H_0 : \theta_1 = \theta_2 = \dots = \theta_n \text{ and } \eta_1 = \eta_2 = \dots = \eta_n \quad (16)$$

The alternative hypothesis:

$$H_1 : \theta_1 = \theta_2 = \dots = \theta_{k^*} \neq \theta_{k^*+1} = \dots = \theta_n \text{ and } \eta_1 = \eta_2 = \dots = \eta_n \quad (17)$$

k^* is the location or the time of the single change point. All the parameters are supposed to be

unknown under both hypotheses. Assuming that $k = k^*$ is known, we need to test whether the two samples come from the same population. If the value of Λ_k is significant, the null hypothesis will be rejected.

$$\Lambda_k = \frac{\sup \prod_{1 \leq i \leq n} c(u_i, v_i; \theta, \eta)}{\sup \prod_{1 \leq i \leq k} c(u_i, v_i; \theta, \eta) \prod_{k \leq i \leq n} c(u_i, v_i; \theta', \eta)} \quad (18)$$

Where, c is the density function of C , and

$$c(u_i, v_i; \theta, \eta) = \frac{\partial^2 C(u_i, v_i; \theta, \eta)}{\partial u_i \partial v_i} \quad (19)$$

Λ_k can be gotten by maximum likelihood estimates.

If we denote

$$\begin{aligned} L_k(\theta, \eta) &= \sum_{1 \leq j \leq k} \log c(u_i, v_i; \theta, \eta) \\ L_k^*(\theta, \eta) &= \sum_{k \leq j \leq n} \log c(u_i, v_i; \theta, \eta) \end{aligned} \quad (20)$$

For each $k (1 \leq k \leq n)$, there is an unique corresponding $\hat{\theta}_k$. The likelihood ratio can be written as:

$$-2 \log(\Lambda_k) = 2[L_k(\hat{\theta}_k, \hat{\eta}_k) + L_k^*(\hat{\theta}_k^*, \hat{\eta}_k^*) - L_T(\hat{\theta}_T, \hat{\eta}_T)] \quad (21)$$

If Z_n 's value is greater than the critical value of 9, the null hypothesis will be rejected. It means there exists a structural break.

$$Z_n = \max_{1 \leq k \leq n} (-2 \log(\Lambda_k)) \quad (22)$$

5. Empirical research

5.1. Data and summary statistics

Light, sweet crude futures on the NYMEX is often used as a major international pricing benchmark because of its good trading liquidity and price transparency. U. S. Dollar Index (USDIX henceforth) is a measure of the value of the U. S. dollar relative to majority of its most significant trading partners, which can comprehensively reflect the exchange rate of the dollar in the international foreign exchange market. Therefore, we choose NYMEX Light Crude (CL) and USDIX to represent crude oil

Table 1. Descriptive statistics for price ranges of NYMEX crude oil and USDIX.

	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera	ADF
						Test P Value Value
NYMEX	2.36821	4.560	1.1329	5.4991	965.3706	-6.02760.0000
crude oil price						
USDIX	0.76070	4.006	1.7821	9.3249	4471.2950	-4.93180.0000

prices and exchange rates. We use daily data from the period 2005/5/31 to 2014/5/30.

According to equation (1), we calculate price ranges of NYMEX crude oil and USDX. Summary statistics of the price range series and ADF test are presented in table 1.

All ADF test statistics reject the hypothesis of a unit root at the 1% confidence level, which means that they are stationary time series and we can model them directly.

5.2. Analysis based on copula-CARR models

5.2.1. Analysis based on CARR models. Previous empirical study indicates that it is not easy to determine the orders of the model. As a result, researchers mostly used low orders' model, such as GARCH (1,1) model, GARCH (2,1) model and GARCH (1,2) model, etc. Based on the order selection experience of GARCH model, in this paper, we construct CARR models with 2 orders for the price range of NYMEX crude oil and the price range of USDX. We assume the random error ratio term is respectively assumed to follow standard exponential distribution, standard Weibull distribution and standardized generalized Gamma distribution, respectively denoted by ECARR, WCARR, GCARR.

Assuming that the residual items respectively follow standard exponential distribution, standard Weibull distribution and standardized generalized Gamma distribution, we construct CARR model for the price range of NYMEX crude oil and the price range of USDX.

CARR (p, q) model for the price range of NYMEX crude oil is as follows:

$$\begin{aligned}
 R_{Nt} &= \lambda_{Nt} \varepsilon_{Nt} \\
 \lambda_{Nt} &= \omega + \sum_{i=1}^p \alpha_i R_{Nt-i} + \sum_{j=1}^q \beta_j \lambda_{Nt-j} \\
 \varepsilon_{Nt} &\sim iidf(\bullet) \\
 R_{Nt} &= \ln P_{Nt}^{\max} - \ln P_{Nt}^{\min}
 \end{aligned} \tag{23}$$

Where, R_{Nt} represents price range of NYMEX crude oil; P_{Nt}^{\max} and P_{Nt}^{\min} represents NYMEX crude oil's daily maximum price and daily minimum price; λ_{Nt} is the conditional mean of NYMEX crude oil's price range based on all information up to time t ; ε_{Nt} is residual item.

According to p-values (significance at the 5% level) in table 2, all the parameters in WCARR (1, 1), GCARR (1, 1) and GCARR (2, 2) are significant. As a result, we need to use Akaike information criterion (AIC henceforth) to select the optimal model. Akaike H [23] proposed the Akaike information criterion to measure the statistical model's goodness of fit, which is built on the basis of the concept of entropy. Statistical models often appear over-fitting, which means use the extra parameter to make the model has higher goodness of fit. AIC is an evaluation standard to choose the optimal model in alternative models which can best explain the data and with the minimum free parameter. In the multiple alternative models, the minimum AIC value model is the best choice.

AIC is defined as follows:

$$\begin{aligned}
 AIC &= -2 \ln(\text{Maximum likelihood function value of the model}) \\
 &+ 2(\text{The number of parameters of the model})
 \end{aligned} \tag{24}$$

Table 2. CARR models using daily price range of NYMEX crude oil, T=2036.

Model	Parameters							Log-likelihood
	ω	α_1	α_2	β_1	β_2	θ	K	
ECARR(1, 1)	0.0179 [0.7074]	0.0832 [3.3378]		0.9091 [30.6324]				-3666.2423

	-0.4793	-0.0008	0.0000			
ECARR(1, 2)	0.0207	0.0983	0.6483	0.2445		
	[0.6618]	[1.9335]	[0.9405]	[0.3800]		-3661.4211
	-0.5081	-0.0532	-0.3470	-0.7041		
ECARR(2, 1)	0.0157	0.1036	-0.0283	0.9179		
	[0.6849]	[1.7897]	[-0.4705]	[31.414]		-3661.3747
	-0.4934	-0.0735	-0.6380	0.0000		
ECARR(2, 2)	0.0368	0.0961	0.0684	-0.0653	0.8851	
	[0.7395]	[3.0251]	[2.2266]	[-1.2219]	[17.6205]	-3661.6390
	-0.4596	-0.0025	-0.0260	-0.2217	0.0000	
WCARR(1, 1)	1.6057	0.3050	0.0449		1.9444	
	[36.8627]	[15.0821]	[1.9379]		[76.2278]	-3114.0022
	0.0000	0.0000	-0.0526		0.0000	
WCARR(1, 2)	1.2064	0.2944	-0.0627	0.2844	1.9665	
	[30.3869]	[15.8392]	[-2.5195]	[10.2929]	[73.3458]	-3085.2257
	0.0000	0.0000	-0.0118	0.0000	0.0000	
WCARR(2, 1)	1.4221	0.2215	0.2526	-0.0509	2.0264	
	[31.7628]	[11.9713]	[14.8120]	[-1.9676]	[73.9352]	-3035.8639
	0.0000	0.0000	0.0000	-0.0491	0.0000	
WCARR(2, 2)	1.2599	0.2365	0.2088	-0.1207	0.1648	2.0265
	[27.8028]	[12.7464]	[10.7632]	[-4.9549]	[5.2210]	[74.0252]
	0.0000	0.0000	0.0000	0.0000	0.0000	-3029.3285
GCARR(1, 1)	1.4884	0.3252	0.0765		1.4700	1.6806
	[30.4592]	[13.2419]	[2.6685]		[23.5340]	[14.4002]
	0.0000	0.0000	-0.0076		0.0000	0.0000
GCARR(1, 2)	1.0345	0.3075	-0.0445	0.3271	1.4375	1.7919
	[24.8558]	[13.8125]	[-1.4211]	[9.5506]	[21.9289]	[13.6415]
	0.0000	0.0000	-0.1553	0.0000	0.0000	0.0000
GCARR(2, 1)	1.3325	0.2350	0.2576	-0.0281	1.5638	1.6127
	[26.8919]	[10.7259]	[12.5846]	[-0.8679]	[22.3464]	[14.1013]
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
GCARR(2, 2)	1.0806	0.2553	0.2000	-0.1363	0.2486	1.5030
	[22.0897]	[11.8241]	[9.3941]	[-4.3541]	[6.4125]	[22.2672]
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: [] is t statistic; () is the p-values.

Table 3. The alternative models' AIC value of NYMEX crude oil' CARR model.

Model	AIC Value
WCARRX(1, 1)	6066.6569
GCARRX(1, 1)	6199.9475
GCARRX(2, 2)	6030.1538

From table 3, GCARR (2, 2) model with the smallest AIC value is an optimal CARR model for price range of NYMEX crude oil.

$$\begin{aligned}
 R_{N_t} &= \lambda_{N_t} \varepsilon_{N_t} \\
 \lambda_{N_t} &= 1.0806 + 0.2553 R_{N_{t-1}} + 0.2000 R_{N_{t-2}} - 0.1363 \lambda_{N_{t-1}} + 0.2486 \lambda_{N_{t-2}} \\
 \varepsilon_{N_t} &\sim \text{Gamma}(1.7403)
 \end{aligned} \tag{25}$$

Where, R_{Nt} represents price range of NYMEX crude oil; λ_{Nt} is the conditional mean of NYMEX crude oil's price range based on all information up to time t ; ε_{Nt} is residual item.

The price range of USDX's CARR (p, q) model is as follows:

$$\begin{aligned}
 R_{Dt} &= \lambda_{Dt} \varepsilon_{Dt} \\
 \lambda_{Dt} &= \omega + \sum_{i=1}^p \alpha_i R_{Dt-i} + \sum_{j=1}^q \beta_j \lambda_{Dt-j} \\
 \varepsilon_{Dt} &\sim iidf(\bullet) \\
 R_{Dt} &= \ln P_{Dt}^{\max} - \ln P_{Dt}^{\min}
 \end{aligned} \tag{26}$$

Where, R_{Dt} represents price range of USDX; P_{Dt}^{\max} and P_{Dt}^{\min} represents USDX's maximum and minimum values of the day; λ_{Dt} is the conditional mean of USDX's price range based on all information up to time t ; ε_{Dt} is residual item.

Table 4. CARR model using daily price range of USDX, T=2036.

Model	Parameters							
	ω	α_1	α_2	β_1	β_2	θ	K	Log-likelihood
ECARR(1, 1)	0.0024	0.0782		0.9183				
	[0.2418]	[1.3456]		[14.5865]				-1375.4225
	-0.8090	-0.1784		0.0000				
ECARR(1, 2)	0.0028	0.0743		0.9558	-0.0344			
	[0.2569]	[0.5184]		[0.4757]	[-0.0184]			-1374.3413
	-0.7972	-0.6042		-0.6343	-0.9853			
ECARR(2, 1)	0.0024	0.0665	0.0143	0.9157				
	[0.2407]	[0.4652]	[0.0919]	[13.2726]				-1374.3093
	-0.8098	-0.6418	-0.9268	0.0000				
ECARR(2, 2)	0.0046	0.0767	0.0722	0.0596	0.7848			
	[0.1979]	[0.7269]	[0.1925]	[0.0110]	[0.1576]			-1373.5884
	-0.8432	-0.4673	-0.8473	-0.9912	-0.8748			
WCARR(1, 1)	0.0060	0.0859		0.9055		2.6178		
	[4.1530]	[11.7003]		[109.2228]		[67.2153]		-291.3359
	0.0000	0.0000		0.0000		0.0000		
WCARR(1, 2)	0.0056	0.0800		0.9962	-0.0842	2.6179		
	[3.0611]	[4.4504]		[4.2336]	[-0.3884]	[66.2398]		-291.0326
	-0.0022	0.0000		0.0000	-0.6977	0.0000		
WCARR(2, 1)	0.0062	0.0730	0.0170	0.9011		2.6182		
	[4.1081]	[3.9738]	[0.8082]	[93.7680]		[65.8385]		-290.9305
	0.0000	-0.0001	-0.4190	0.0000		0.0000		
WCARR(2, 2)	0.0119	0.0817	0.0881	-0.0898	0.9029	2.6245		
	[4.1319]	[9.7869]	[11.9773]	[-9.7552]	[106.7649]	[64.4726]		-288.5163
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
GCARR(1, 1)	0.0021	0.0779		0.9191		0.8020	10.4213	
	[1.4006]	[8.7979]		[96.2944]		[16.2917]	[9.7143]	-171.4014

	-0.1613	0.0000		-0.0526		0.0000	0.0000
GCARR(1, 2)	0.0019	0.0719		1.0142	-0.0890	0.8051	10.3347
	[1.3212]	[3.3875]		[3.4880]	[-0.3292]	[16.2684	[9.7030] -171.1503
	-0.1864	-0.0007		-0.0005	-0.7420	0.0000	0.0000
GCARR(2, 1)	0.0021	0.0653	0.0153	0.9162		0.8005	10.4564
	[1.3940]	[3.0407]	[0.6580]	[87.5041]		[16.2053	[9.6721] -171.0746
	-0.1633	-0.0024	-0.5106	0.0000		0.0000	0.0000
GCARR(2, 2)	0.0035	0.0690	0.0606	0.2945	0.5708	0.8239	9.8379
	[1.0976]	[3.4294]	[0.9296]	[0.3123]	[0.6577]	[16.2767	[9.7053] -171.1206
	-0.2724	-0.0006	-0.3526	-0.7548	-0.5107	0.0000	0.0000

Note: [] is t statistic; () is the p-values.

Table 5. The alternative models' AIC value of USDX' CARR model.

Model	AIC Value
WCARR(1, 1)	586.6718
WCARR(2, 2)	585.0327

According to p-values (significance at the 5% level) in table 4, all the parameters in WCARR (1, 1) and WCARR (2, 2) are significant.

From table 5, WCARR (2, 2) with the smallest AIC value is an optimal CARR model for price range of USDX.

$$\begin{aligned}
 R_{Dt} &= \lambda_{Dt} \varepsilon_{Dt} \\
 \lambda_{Dt} &= 0.0119 + 0.0817 R_{Dt-1} + 0.0881 R_{Dt-2} - 0.0898 \lambda_{Dt-1} + 0.9029 \lambda_{Dt-2} \\
 \varepsilon_{Dt} &\sim \text{Weibull}(2.6245)
 \end{aligned} \tag{27}$$

Where, R_{Dt} represents price range of USDX; λ_{Dt} is the conditional mean of USDX's price range based on all information up to time t ; ε_{Dt} is residual item.

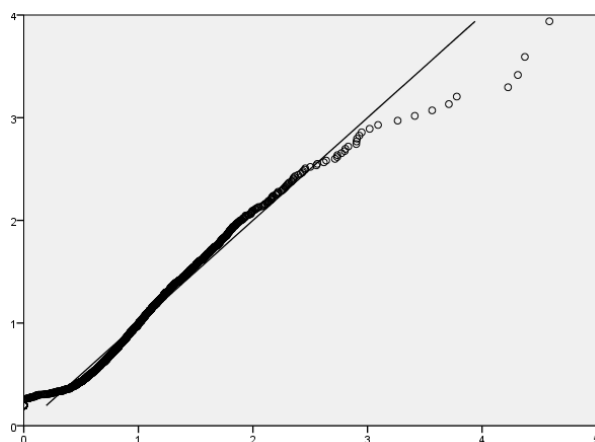


Figure 1. QQ-plots of NYMEX crude oil returns against Gamma distribution.

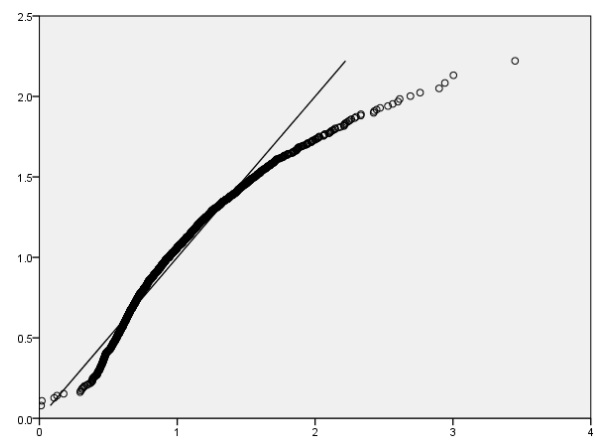


Figure 2. QQ-plots of USDX returns against Weibull distribution.

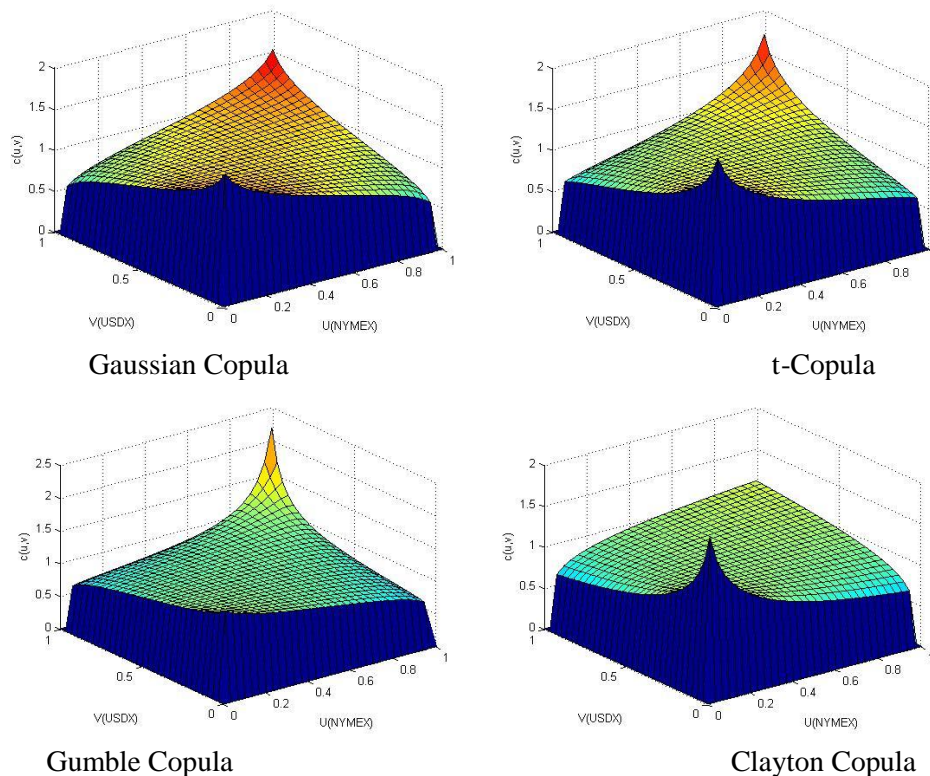
We employ Q-Q plot to test whether the models are applicable. The results in figures 1 and 2 shows Q-Q plot of NYMEX crude oil and USDX. From figures 1 and 2, we apply equation (25) and equation (27) as marginal distribution models are appropriate.

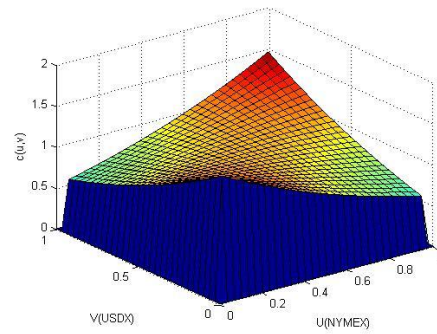
5.2.2. Analysis based on Copula-CARR models. Table 6 shows estimation results of Copula for NYMEX crude oil and USDX. Z value and its p value can be used to measure whether the parameters of various types of Copula are significant. Kendall rank correlation coefficient (τ) is used to measure the consistency degree of their change direction. Spearman rank correlation (ρ) is used to measure the degree of correlation.

Table 6. Estimates of the dependence parameters of different copula models for NYMEX crude oil and USDX.

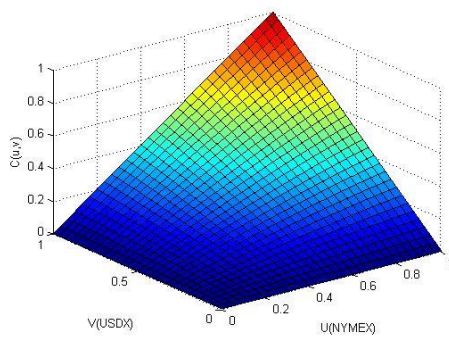
	Parameters		Z value		Goodness of fit	τ	ρ	Log-likelihood
	Name	Value	Test value	P value				
Gaussian Copula	ρ^N	0.1522	6.9930	0.0000	0.0599	0.0957	0.1431	23.0400
t-Copula	ρ^t	0.1569	6.6870	0.0000	0.0796	0.1058	0.1582	-7.6560
Gumbel Copula	α	1.0966	65.2200	0.0000	0.0792	0.0892	0.1331	22.1100
Clayton Copula	θ	0.1401	4.5870	0.0000	0.2027	0.0774	0.1159	12.1400
Frank Copula	λ	1.0095	7.4820	0.0000	0.0337	0.1143	0.1709	28.0700

From Z value and its p value in table 6, we can see that all the parameters of Copula are significant under 5% significance. In addition, goodness of fit of Clayton Copula is the largest one. So Clayton Copula is the best copula for NYMEX crude oil and USDX's correlation structure. Clayton copula's density distribution is "J" type, which means that it is more sensitive to lower tail change of variables and can be used to describe lower tail dependence. Figures 3 and 4 show the density function diagram and distribution function diagram of copulas for NYMEX crude oil and USDX.

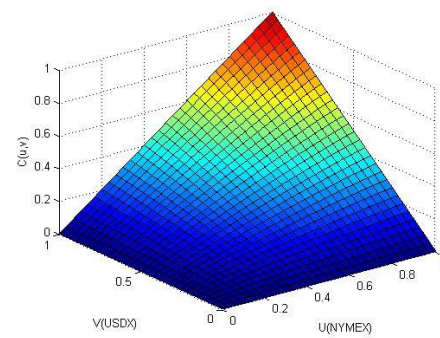




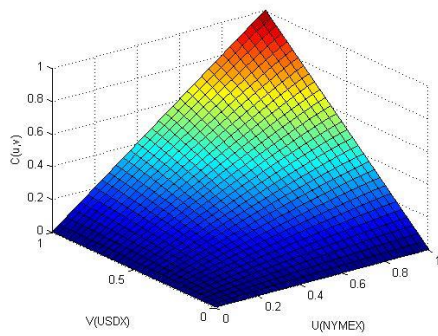
Frank Copula

Figure 3. The density function diagram of copulas for NYMEX crude oil and USDx.

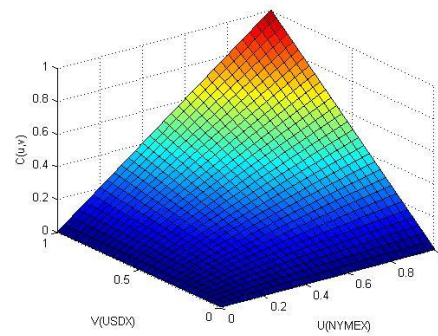
Gaussian Copula



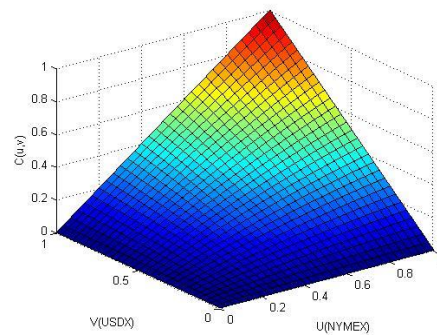
t-Copula



Gumble Copula



Clayton Copula



Frank Copula

Figure 4. The distribution function diagram of copulas for NYMEX crude oil and USDx.

5.3. Copula-CARR model with structural breaks

We analyze the correlation between NYMEX crude oil and USDX using Copula-CARR model with structural breaks.

We will detect change points of Clayton Copula-CARR model, which is the optimal Copula-CARR model for NYMEX crude oil's price range and USDX's price range.

Firstly, we get Z_n value of the overall sample shown in figure 5. From figure 5, we can see that Z_n obtains the maximum value in the 1005th data and it is over 9. It means the 1005th data is a change-point.

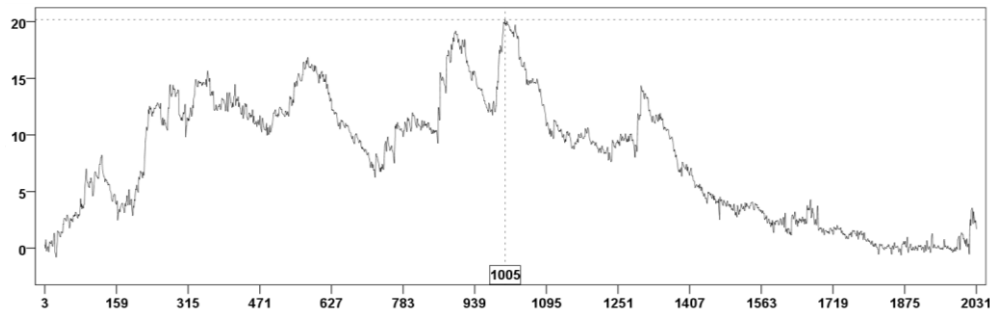


Figure 5. Z_n values of sample 1-2036.

Secondly, the overall sample is divided into two sub-samples by the 1005th data. Figure 6 depicts Z_n value of the sample from 1 to 1005. From figure 6, we can get the second change point - the 731th data (Z_n value is 10.2416). Figure 7 depicts Z_n value of the sample from 1005-2036. From figure 7, we can get the third change point - the 731th data (Z_n value is 9.1728).

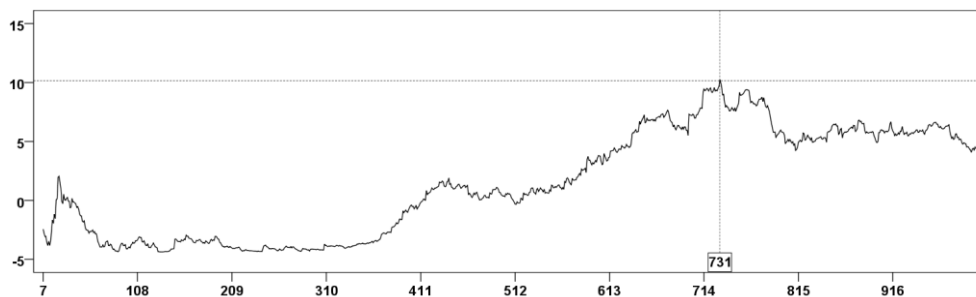


Figure 6. Z_n values of sample 1-1005.



Figure 7. Z_n values of sample 1005-2036.

Thirdly, the sample from 1 to 1005 is divided into two sub-samples by the 731th data. Figure 8 depicts the Z_n value of the sample from 1 to 731. From figure 8, we can get the fourth change point -

the 605th data (Z_n value is 15.1194). Figure 9 depicts the Z_n value of the sample from 731 to 1005. From figure 9, we can see that all Z_n values are less than 9, which means there doesn't exist change point of the sample from 731 to 1005.

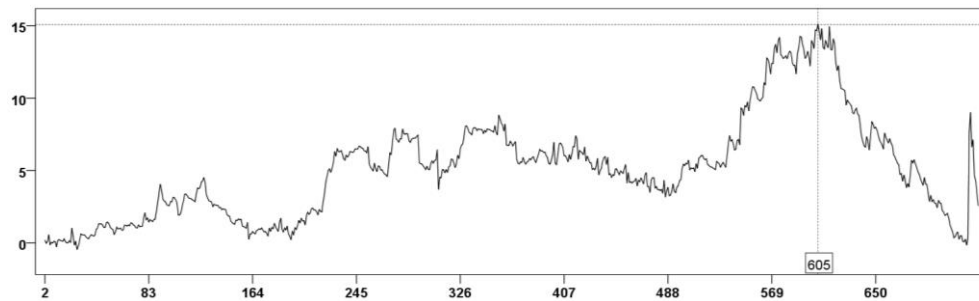


Figure 8. Z_n values of sample 1-731.

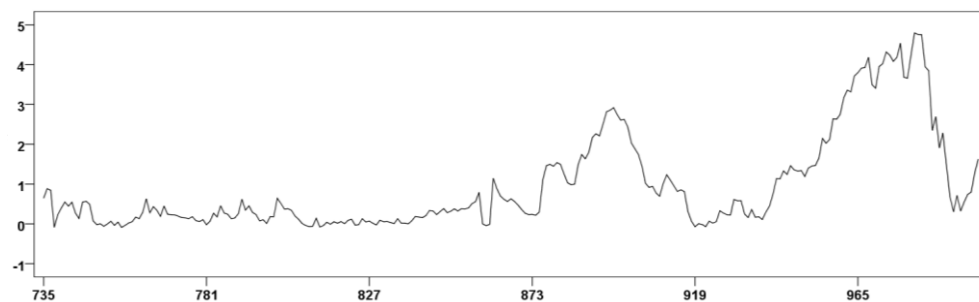


Figure 9. Z_n values of sample 731-1005.

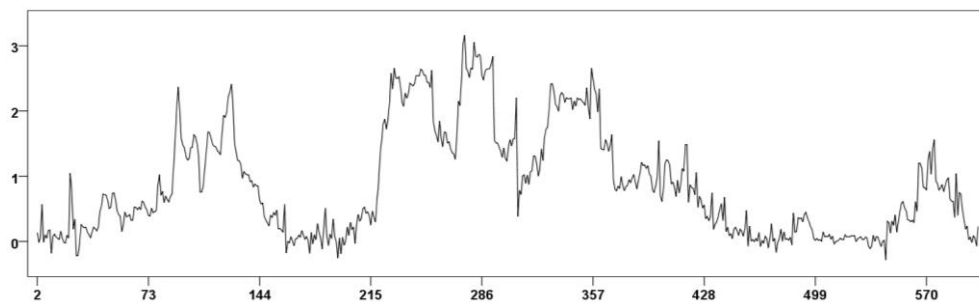


Figure 10. Z_n values of sample 1-605.

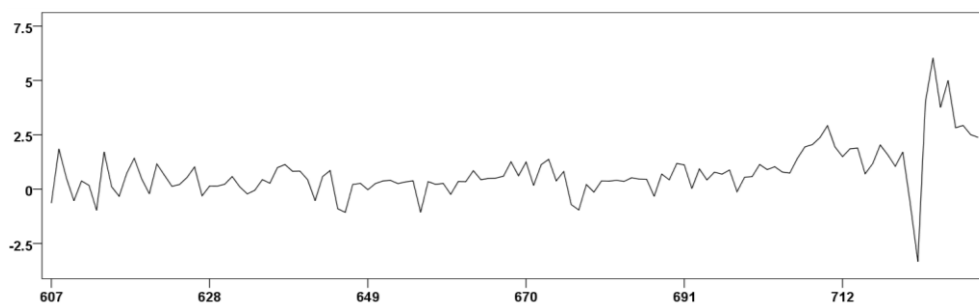


Figure 11. Z_n values of sample 605-731.

Figure 10 depicts the Z_n value of the sample from 1 to 605. Figure 11 depicts the Z_n value of the

sample from 605 to 731. From figure 10 and figure 11, we can see that all Z_n values are less than 9, which means there doesn't exist change point.

Finally, the sample from 1005 to 2036 is divided into two sub-samples by the 1101th data. Figure 12 depicts the Z_n value of the sample from 1005 to 1101. Figure 13 depicts the Z_n value of the sample from 1101 to 2036. From figure 12 and figure 13, we can see that all Z_n values are less than 9, which means there doesn't exist a change point.

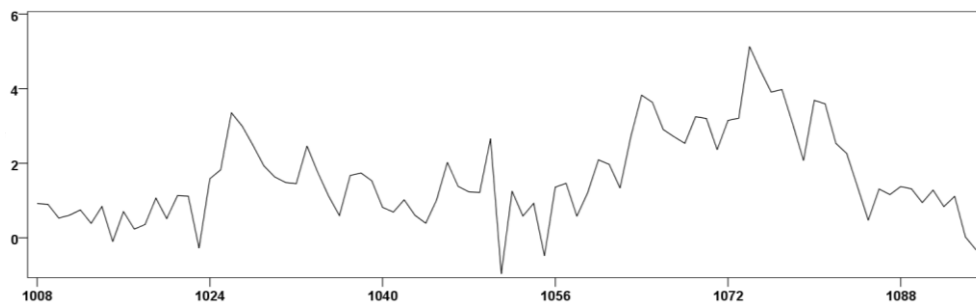


Figure 12. Z_n values of sample 1005-1101.

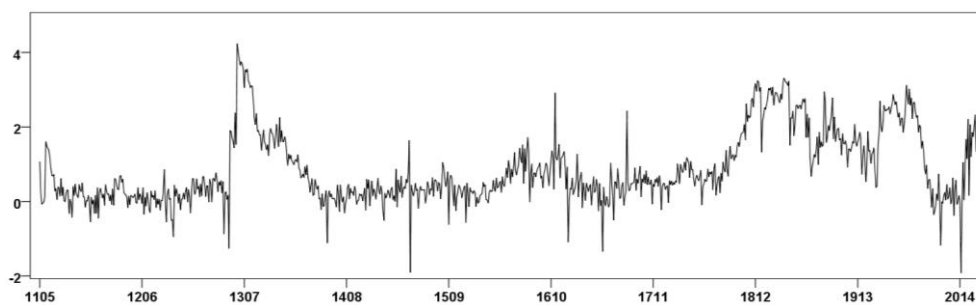


Figure 13. Z_n values of sample 1101-2036.

We get four change points in the optimal Copula-CARR model of NYMEX crude oil price's range and USDX's price range, which correspond to the dates of March 17, 2008, October 30, 2008, January 4, 2010 and June 1, 2010.

The dates of change points are closely related to the economic events.

- On March 12, 2008, the Federal Reserve announced that it will expand the securities lending program. On March 16, the Federal Reserve decided to lower the discount rate from 3.5% to 3.25%, and create a new discount window for primary dealers. International oil prices continued to rise in early 2008, impacted by the situation continued unrest in Nigeria, a major oil producer in Africa. On March 12, New York oil prices hit a record high, breaking the \$ 110 / barrel.
- On September 15, 2008, the fourth largest US investment bank- Lehman Brothers declared bankruptcy. On the same day, the US investment bank - Merrill Lynch & Co. was acquired by BOA (Bank of America). These events had a direct impact on the subsequent global stock market's crash. Due to the rise in the US dollar exchange rate and the US financial crisis, oil demand in the United States and other developed countries fell sharply. International oil prices tumbled. When the international financial crisis intensified, oil prices fell more than 30 US dollars / barrel. On October 16, WTI crude oil fell to \$ 69.85 / barrel, down 51.9%.
- In December, 2009, world's three major credit rating agencies (Standard & Poor's, Fitch and Moody's) downgraded Greece's sovereign debt rating, which meant the beginning of the European debt crisis. Euro exchange rate fell sharply. In early 2010, due to the high northern latitudes' temperature dropped to a historic low in winter, U. S. oil inventories continued to

fall in December. The international oil market continued to push on. WTI crude oil price hit its peak since the middle of October 2008.

- Crude oil futures were gradually overshadowed by the high U. S. crude oil inventories in April, 2010. With the unfavorable economic data, the international oil's price went down. Due to the serious European debt crisis and the rising oil inventories, crude oil market's situation worsened in May, 2010. Until the end of May, 2010, as gasoline consumption season and the Atlantic hurricane season came, larger rebound appeared in international oil prices.

6. Conclusion

In recent years, the impact of crude oil price fluctuations and exchange rate fluctuations on the national economy have been closely concerned by governments around the world. Many scholars have found that there is a correlation between the two fluctuations in the empirical study. The study of the two fluctuations can provide some implications for the government departments to formulate relevant policies and measures to deal with the crisis. Based on these considerations, we use Copula-CARR model with structural breaks to study the correlation structure and relationship of NYMEX crude oil price range and U. S. Dollar Index range.

Different from previous studies, we propose a new approach--CARR models to discuss the fluctuations of NYMEX crude oil price range and U. S. Dollar Index range. That is to say, we study their ranges' fluctuations. Then, based on CARR model, we use Gaussian Copula, Student-t Copula, Gumbel Copula, Clayton Copula and Frank Copula to study the correlation structure for NYMEX crude oil price range and U. S. Dollar Index range. From the empirical results, we can see that Clayton Copula is the optimal copula function for NYMEX crude oil price range and U. S. Dollar Index range's correlation structure. There is an obvious lower tail dependence between them.

In addition, we detect change points for NYMEX crude oil price range and U. S. Dollar Index range using Copula-CARR model with structural breaks. We get four change points and prove that they are closely related to the economic events, especially the 2008 financial crisis and the 2010 European debt crisis.

The empirical results of this paper offer us several policy implications:

Firstly, significant fluctuations of oil price give us a sign of exchange rate's abnormal fluctuation. Thus, enterprises and governments should take timely measures to manage exchange rate risk when the signal appears.

Secondly, the fluctuations of U.S. dollar, which is the world's currency, will have great effect on other currency's exchange rate. For other countries, precautionary measures should be taken in advance to manage their currency's risk by paying close attention to oil price fluctuations.

Thirdly, financial crisis will lead to the fluctuations of oil prices and exchange rate. As a result, enterprises and governments should manage their financial risk and prevent financial crisis.

Acknowledgments

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