

Selecting optimal structure of burners for tubular cylindrical furnaces by the mathematical experiment planning method

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Abstract. This paper substantiates the method of mathematical planning for experimental research in the process of selecting the most efficient types of burning devices for tubular refinery furnaces of vertical-cylindrical design. This paper provides detailed consideration of an experimental plan of a 4x4 Latin square type when studying the impact of three factors with four levels of variance. On the basis of the experimental research we have developed practical recommendations on the employment of optimal burners for two-step fuel combustion.

1. Introduction

Mathematical statistics is the basis for an overwhelming majority of experimental research. Mathematical statistics is applicable when the results of experiments can be considered as random values or random processes, i.e. those connected with some uncertainty. So, research in working modes of burners of pipe furnace face a number of unregulated factors and experimental conditions, such as: the changeable composition of fuel refinery gases, dirt accumulation on water-cooled surfaces of a furnace and many other random factors (irregularities) whose precise recording is very complicated. Under such conditions, of special importance is planning the experimental procedure that is used for the mathematical description of processes and phenomena. They are described by mathematical modeling that connects values of controlled factors influencing the process with the outcomes of the experiment, called responses. The major requirement imposed on factorial plans of experiments is minimizing the number of experiments to obtain reliable evaluations of parameters calculated with simultaneous compliance of acceptable accuracy of mathematical models within the specified area of factor space. The theory of mathematical design of experiments, based on its statistical representation has developed fairly well. However, it is very rarely employed in the practice of measuring and testing in the field of heat engineering. Even in the manuals on thermotechnical testing of burners of boilers and furnaces mathematical statistics is insufficiently used for experimental works [1]. At the same time, this theory allows the study of poorly organized systems as they provide a logical plan (experiment matrix) and ways of solving tasks at different stages of the experiment.

2. Approaches to the mathematical planning of the experiment

Among the multiple experimental designs used in industry, the so-called Latin-square experiments deserve special attention. Latin squares are described in the majority of books on experimental planning [2] and constitute plans that employ not all of the combinations of levels of factors. Latin square plans are simple in use and are used only in three-factor experiments in cases when all interest factors have



the same number of levels n and it is preliminarily known that these factors do not interact or that these interactions can be neglected. Table 1 represents a sample of a Latin-matrix plan for a 3x4 factorial experiment.

Table 1. Latin square experiment plan

Levels of factor A	Levels of factor B			
	b ₁	b ₂	b ₃	b ₄
a ₁	c ₁ y ₁₁₁	c ₂ y ₁₂₂	c ₃ y ₁₃₃	c ₄ y ₁₄₄
a ₂	c ₂ y ₂₁₂	c ₃	c ₄ y ₂₃₄	c ₁ y ₂₄₁
a ₃	c ₃ y ₃₁₃	c ₄ y ₃₂₄	c ₁ y ₃₃₁	c ₂ y ₃₄₂
a ₄	c ₄ y ₄₁₄	c ₁ y ₄₂₁	c ₂ y ₄₃₂	c ₃ y ₄₄₃

Here, two factors A and B are presented correspondingly as rows and columns and form a kind of chess-board. Each row and each column of this table corresponds to a certain level of the corresponding factor. The third factor C is reflected in the squares of the table by characters – levels c_k . These characters are distributed among the squares of the table in such a way that a relevant character is encountered in each row and in each column only once. Therefore, regardless of disturbing impacts from the irregularity sources (random unaccounted factors) they will equally affect the calculation of average values along both rows and columns. That is why a Latin square allows us to execute a double check over the dispersion of experimental data, i.e. column and row effect control. Values y (indexed i, j, k) recorded in the squares of the table are experimental results (response parameters) obtained at i -level of factor A, j -level of factor B and k -level of factor C, where indexes i, j, k can possess values from 1 to 4 ($i, j, k = 1, 2, 3, 4$).

From the point of view of factorial planning, a Latin square can be considered as an example of an incomplete factorial experiment. The observations are conducted in n^2 out of n^3 of possible complexes of conditions. We need n times fewer experiments than under complete factorial experiments. We reduce the number of experiments and do not lose an opportunity to evaluate the impacts of changing levels of each factor separately at the expense of neglecting the interaction among factors.

When planning according to Latin square impact of four sources of dispersion is studied: the first source is row (factor A), the second source is column (factor B), the third source is the Latin character “c”, recorded in squares of the table (factor C) and the fourth source is an experiment error.

Analysis-of-variance breakdown for $n \times n$ Latin square plan used in this work is presented in Table 2.

Table 2. Analysis-of-variance breakdown of Latin square plans

Factors	Sum of squares	Number of degrees of freedom	Average square (dispersion MS)	Mean-square ration (F_{obsrv})
A	$S_a = n^{-1} \sum_{i=1}^n \left(\sum_{j,k=1}^n y_{ijk} \right)^2 - n^{-2} \left(\sum_{i,j=1}^n y_{ijk} \right)^2$	$n-1$	$MS_a = \frac{S_a}{(n-1)}$	$\frac{MS_a}{MS_{rmnd}}$
B	$S_b = n^{-1} \sum_{j=1}^n \left(\sum_{i,k=1}^n y_{ijk} \right)^2 - n^{-2} \left(\sum_{i,j=1}^n y_{ijk} \right)^2$	$n-1$	$MS_b = \frac{S_b}{(n-1)}$	$\frac{MS_b}{MS_{rmnd}}$
C	$S_c = n^{-1} \sum_{k=1}^n \left(\sum_{i,j=1}^n y_{ijk} \right)^2 - n^{-2} \left(\sum_{i,j=1}^n y_{ijk} \right)^2$	$n-1$	$MS_c = \frac{S_c}{(n-1)}$	$\frac{MS_c}{MS_{rmnd}}$
Sum	$S_{gen} = \sum_{i,j=1}^n (y_{ijk})^2 - n^{-2} \left(\sum_{i,j=1}^n y_{ijk} \right)^2$	$n^2 - 1$	$MS_{gen} = \frac{S_{gen}}{n^2 - 1}$	—
Remainder	$S_{rmnd} = S_{gen} - (S_a + S_b + S_c)$	$(n-1)(n-2)$	$MS_{rmnd} = \frac{S_{rmnd}}{(n-1)(n-2)}$	—

The Table does not practically require explanations. Here, n is the number of levels of each of three factors: A, B and C. Sums of squares for these factors are calculated in an absolutely identical way. The error sum of squares is found by subtraction. In fact, all three factors fit into the plan of an experiment in a symmetrical way: in any plan of a Latin squares type factors presented by rows, columns and characters can be rearranged in any order and still producing Latin squares.

Variance analysis of the significance of the impact of factors A, B, C on the values of response parameters employs Fisher's test that allows comparing the values of selective variances MS of two independent samplings. To calculate F_{obsrv} – of the observed value of F we need to find the ratio of variances of two samplings. At that, the larger variance should be in the numerator and the lesser variance should be in the denominator. As the value of the nominator, according to the test, must be larger or equal to the value of the denominator, the value of F_{obsrv} will always be larger or equal to unity (1). If this is not so, and $F_{obsrv} < 1$, the inverse value $(F_{obsrv})^{-1}$ should be used for application of the criterion F .

The critical point $F_{crit}(\alpha, k_1, k_2)$ is found in the table of critical point of F distributions [3] for a given significance level α and numbers of degrees of freedom k_1 and k_2 , where k_1 is the number of degrees of freedom of a larger variance in the numerator of the ratio of these two variances, and k_2 is the number of degrees of freedom in the denominator of the ratio. If experiment results in $F_{obsrv} > F_{crit}$, the null hypothesis H_0 on the significance of the impact of factors is accepted; if not, an alternative hypothesis H_1 on insignificance of the impact of corresponding factors. If variance analysis shows the significance of the impact of linear effects (factors), i.e. it shows the significance of difference in the averages, the following question emerges: exactly which average values are different? Different criteria are used to check the differences of average values, in particular Duncan's rank test [4].

3. Planning of experimental research aimed at selection of optimal design of burners

We set forth the task of experimental research as follows: to display the influence of four types of burners on working efficiency of vertical cylindrical tubular furnaces of different heat power without heat recycling of waste gases in the process of burning of different refinery gases. To resolve this task we planned the experiment according to the scheme of 4x4 Latin square to study the impact of three factors A, B, and C on the process under study with four variance levels for each of those factors. Factor A is the type of gas burner, factor B is the heat power of tubular furnace and factor C is the combustion heat of burning refinery gases. Four types of burners were studied: diffusive with free air delivery; injective; diffusive-kinetic; and wind-box burner with two-step fuel combustion [5, 6].

Technical specifications and construction of the above mentioned industrial burners are described in [7, 8]. The burners are installed on the floor of cylindrical tubular furnaces with the following heat powers: 3.5; 5.0; 8.0; and 10.5 Megawatt. Refinery gases combustion heat varied at four levels: 52.3; 65.8; 77.3; 88.4 MJ/m³.

Table 3 presents the design of the experiment with the results of measurements. The working efficiency of the tubular furnace "y" (response parameter) was assessed according to the sum of waste heat loss with emitted gases.

Table 3. Design of experiment with the results of measurements

Levels of factor A	Levels of factor B			
	b ₁	b ₂	b ₃	b ₄
a ₁	c ₁ y ₁₁₁ = 19.7	c ₂ y ₁₂₂ = 21.0	c ₃ y ₁₃₃ = 18.5	c ₄ y ₁₄₄ = 18.8
a ₂	c ₂ y ₂₁₂ = 17.8	c ₃ y ₂₂₃ = 18.4	c ₄ y ₂₃₄ = 17.5	c ₁ y ₂₄₁ = 16.3
a ₃	c ₃ y ₃₁₃ = 20.1	c ₄ y ₃₂₄ = 19,5	c ₁ Y ₃₃₁ = 18.2	c ₂ y ₃₄₂ = 16.6
a ₄	c ₄ y ₄₁₄ = 18.9	c ₁ y ₄₂₁ = 18.3	c ₂ y ₄₃₂ = 18.3	c ₃ y ₄₄₃ = 15.9

Table 4 presents the results of processing experimental data according to the scheme of variance analysis presented in Table 2 at the number of variance levels for each factor $n = 4$.

Table 4. Results of processing experimental data

Factors	Sum of squares	Number of degrees of freedom	Average square (variance MS)	Mean-square ratio (F_{obsrv})
A	$S_a = 9.5$	3	$MS_a = 3.17$	$\frac{MS_a}{MS_{rmd}} = 6.10$
B	$S_b = 14.6$	3	$MS_b = 4.87$	$\frac{MS_b}{MS_{rmd}} = 9.4$
C	$S_c = 0.7$	3	$MS_c = 0.23$	$\frac{MS_c}{MS_{rmd}} = 0.44$
Sum	$S_{gen} = 27.9$	15	$MS_{gen} = 1.86$	—
Remainder	$S_{rmd} = 3.1$	6	$MS_{rmd} = 0.52$	—

Significances of factors A, B and C on response parameter “y” were tested on the basis of Fisher’s variance ratio. For this purpose we used dispersion relations for the observed sizes of values of the criterion from Table 4:

$$F_{obsrv}^a = MS_a / MS_{rmd} = 6.1; \quad F_{obsrv}^b = MS_b / MS_{rmd} = 9.4; \quad F_{obsrv}^c = MS_c / MS_{rmd} = 0.44.$$

As the value of the criterion observed for factor C turned out to be less than unity, the inverse value from this value $(F_{obsrv}^c)^{-1} = MS_{rmd} / MS_c = (0.44)^{-1} = 2.27$ should be used to evaluate the significance of factor C. Here, a larger variance $MS_{rmd} = 0.52$ will be placed in the numerator, and the lesser variance $MS^c = 0.23$ will be placed in the denominator. Accordingly, the sequence of numbers of freedom k_1 and k_2 , used to search for critical points of F-distribution is substituted here for inverse. Thus, for factors A and B at the significance level $\alpha = 0.05$ and the number of degrees of freedom of compared variances $k_1 = 3$ and $k_2 = 6$, the value of the critical point is $F_{0.05}(3.6) = 4.76$; and for factor C, at the same significance level, the value of the critical point is $F_{0.05}(6.3) = 8.94$.

As the values of the criterion observed for factors A and B $F_{obsrv}^a = 6.1$ and $F_{obsrv}^b = 9.4$ exceed the values of the corresponding critical point $F_{0.05}(3.6) = 4.76$, the impact of each of these factors on the response function is significant. Along with this, as the value $(F_{obsrv}^c)^{-1} = 2.27$ is less than the value of the corresponding critical point $F_{0.05}(6.3) = 8.94$, the impact of factor C on the response parameter will be insignificant.

Now, let us define which averages are significantly different for significant factors A and B. For this purpose we will use the Duncan multiple rank test, having a preliminary calculated standard error of mean $S_y = \sqrt{MS_{rmd}/n} = \sqrt{0.52/4} = 0.36$. From the corresponding Tables for this criterion we will extract significant ranks for a number of degrees of freedom $k = (4-1)(4-2) = 6$ and significance levels $\alpha = 0.05$ (Table 5):

Table 5. Significant ranks for a number of degrees of freedom

p	2	3	4
Ranks r	3.46	3.58	3.64
Ranks, multiplied by standard error			
$r \times S_y$	1.24	1.29	1.31

We determine the averages for factor A according to the formula $\bar{y}_i = (1/4) \sum_{j=1}^4 y_{ijk}$, $i = 1, 2, 3, 4$ and arrange them in increasing order of the values of levels a_i of this factor: $\bar{y}_2 = 17.5$; $\bar{y}_4 = 17.85$; $\bar{y}_3 = 18.6$; $\bar{y}_1 = 19.5$. Then, to evaluate the significance of differences among the heat powers of tubular furnaces we deduct lesser values from larger to find the difference between averages and compare them with $r \times S_y$: $\bar{y}_1 - \bar{y}_2 = 2.0 > 1.24$ – difference is significant; $\bar{y}_1 - \bar{y}_4 = 1.65 > 1.29$ – difference is significant; $\bar{y}_1 - \bar{y}_3 = 0.9 < 1.31$ – difference is insignificant; $\bar{y}_3 - \bar{y}_2 = 1.1 < 1.24$ – difference is insignificant; $\bar{y}_3 - \bar{y}_4 = 0.75 < 1.29$ – difference is insignificant; $\bar{y}_4 - \bar{y}_2 = 0.35 < 1.24$ – difference is insignificant.

Averages for factor B are determined according to the formula $\bar{y}_j = (1/4) \sum_{i=1}^4 y_{ijk}$, $j = 1, 2, 3, 4$. Then we arrange them in ascending order of the values of levels b_j of this factor: $\bar{y}_4 = 16.9$; $\bar{y}_3 = 18.13$; $\bar{y}_1 = 19.13$; $\bar{y}_2 = 19.3$. To evaluate the significances of differences between the types of burners, we deduct lesser values from larger, determine the difference between the averages and compare them with $r \times S_y$: $\bar{y}_2 - \bar{y}_4 = 2.4 > 1.31$ – significant difference; $\bar{y}_2 - \bar{y}_3 = 1.17 < 1.29$ – insignificant difference; $\bar{y}_2 - \bar{y}_1 = 0.17 < 1.24$ – insignificant difference; $\bar{y}_1 - \bar{y}_4 = 2.23 > 1.29$ – significant difference; $\bar{y}_1 - \bar{y}_3 = 1.0 < 1.24$ – insignificant difference; $\bar{y}_3 - \bar{y}_4 = 1.23 < 1.29$ – insignificant difference.

Thus, determination of the difference between averages and evaluation of significances of the levels of factors A, B and C resulted in the following conclusions.

4. Conclusions

The results obtained show that the burners installed on the floor of tubular cylindrical furnaces influence the efficiency of tubular furnaces (heat power of 3.5 Megawatt) but when changing over to furnaces with increased single capacity (5 – 10.5 Megawatt) the structure of burner does not exert significant impact on the efficiency of the work of furnace.

The difference in the working efficiency of furnaces with injective burners, diffusive burners with free air delivery and diffusive-kinetic burners is insignificant.

According to the results of experiment planning we have found the most optimal constructions of burners – wind-box burners of two-step fuel combustion. In addition, these burners as research [9–11] shows are low-toxic and are sufficiently used in Oil Refinery Plants in Russia.

References

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