

Algorithm of composing the schedule of construction and installation works

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Abstract. An algorithm for scheduling works is developed, in which the priority of the work corresponds to the total weight of the subordinate works, the vertices of the graph, and it is proved that for graphs of the tree type the algorithm is optimal. An algorithm is synthesized to reduce the search for solutions when drawing up schedules of construction and installation works, allocating a subset with the optimal solution of the problem of the minimum power, which is determined by the structure of its initial data and numerical values. An algorithm for scheduling construction and installation work is developed, taking into account the schedule for the movement of brigades, which is characterized by the possibility to efficiently calculate the values of minimizing the time of work performance by the parameters of organizational and technological reliability through the use of the branch and boundary method. The program of the computational algorithm was compiled in the MatLAB-2008 program. For the initial data of the matrix, random numbers were taken, uniformly distributed in the range from 1 to 100. It takes 0.5; 2.5; 7.5; 27 minutes to solve the problem. Thus, the proposed method for estimating the lower boundary of the solution is sufficiently accurate and allows efficient solution of the minimax task of scheduling construction and installation works.

1. Introduction

The purpose of this study was to consider the task of constructing an optimal maintenance schedule for devices in a given, specific sequence. The processes of servicing requirements by each of the devices are considered to be indivisible, i.e., having started the process of servicing the i -th requirement by the k -th device, it is necessary to bring it to the end without interrupting. Let the service times for each requirement be set for each device and the final time T_i be set for completion of the process for servicing the i -th requirement by the last device. It is necessary to build a schedule that ensures the end of the process of servicing all requirements in the shortest possible time. We will focus only on the consideration of cases when the times of transportation of requirements and readjustments of devices can be neglected. The proposed problem formulation is quite general and is one of the core problems in the production organization. The models of the theory of schedules also deal with the tasks of organizing the computing process and many others.

The problems of scheduling theory are given considerable attention to in monographs and periodicals. In the literature, the network coverage of a set of admissible schedules and the construction of approximate methods for solving a problem using generators of acceptable schedules and certain preference rules received the widest coverage. These approaches allow us in some cases to find good approximations to the solution of problems using static modeling methods. However, the restrictions on



the deadlines for completing the process of servicing the i -th requirement make it difficult for algorithms to formulate acceptable schedules and, using known methods increases the time of solving the problem. Some other approaches are based on constructing linear integer models of the problem and applying integer linear programming methods. The proposed models were reduced to problems of linear programming with Boolean variables of very large dimension, which did not allow the practical problems of scheduling theory to be solved when using these methods. The attempts to build algorithms for obtaining exact solutions to the problem on the basis of branch and boundary methods were unsuccessful, either.

2. Problem definition

Resource allocation in the problems of scheduling is a fairly well-developed branch of science, with an overall focus on meeting the requirements of organizational and technological reliability in design, construction and installation by the criterion of minimizing the time of work execution and waste of resources in each sector. Unfortunately, in the performance of works there may be deviations from the standards which would require alteration. Also, the problems of stochastic uncertainty arise because of the very nature of the construction industry. Therefore, reducing construction time due to new approaches to drawing up calendar schedules and spending resources is the most pressing task.

3. Algorithm for solving the problem

To compile a schedule of construction and installation work, it is necessary to consider the minimax algorithm. Let there be one brigade and n number of works that are performed by it. The duration of the j -th work is determined by the expression:

$$t_{ij} = \tau_{ij} + \tau_j,$$

where τ_{ij} is the time for the brigade to move from performing the i -th work to the j -th work, τ_j is the time of performing the j -th work.

It is required to determine the sequence of work execution, in which the maximum time for execution of each individual work is minimal.

The problem considered here reduces to the well-known problem of the travelling salesman in its minimax formulation.

Mathematically, the problem can be represented in the following form. It is required to determine:

$$t = \min \max_{i,j=0;n,i \neq j} t_{ij} x_{ij} \quad (1)$$

under the following restrictions:

$$\sum_{i=0}^n x_{ij} = 1, \sum_{j=0}^n x_{ij} = 1, i, j = \overline{0;n}, i \neq j, \quad (2)$$

$$U_i - U_j + (n+1)x_{ij} \leq n, i, j = \overline{0;n}, i \neq j, \quad (3)$$

$$x_{ij} \in \{0,1\}; i, j = \overline{0;n}, i \neq j, \quad (4)$$

where: $U_i \geq 0; U_j \geq 0$ are additional variables,

t_{oj} is the execution time of the j -th work, if it is performed first ($j = \overline{1;n}$),

t_{io} is the time for the brigade to move to the initial state, if the i -operation is performed last ($i = \overline{1;n}$).

Limitations (2) - (4) are typical for the traveling salesman problem. Condition (1) determines the minimax character of the problem under consideration.

Practical problems that reduce to (1) - (4) can have a very different physical meaning. For example, to (1) - (4) there reduces the problem of minimizing the system resource consumption when monitoring parameters [6].

For the solution of problem (1) - (4), the branch and boundary method can be used. In this case, it is necessary to determine the way to estimate the lower bound and the conditions for constructing a tree of possible variants.

We denote: U is a set of variables x_{ij} , $S_l = \{x_{ij}; x_{ij} = 1\}$ is a set of fixed variables that appear in separate branches of the tree of possible variants; $l = \overline{0, n}$ is the number of variables x_{ij} , fixed at 1 and included in set S_l (when $l=0$, $S_0 \neq \emptyset$), $E_{si} = \{x_{ij}; x_{ij} = 0\}$ is the set of variables x_{ij} fixed at zero, the introduction of which into set S_l leads to violation of the constraints (2) - (4) or to the non-optimal result, $G_{sl} = U \setminus (S_l \cup E_{sl})$ is the set of free variables, from which at the next step the variables are selected and included in set S_l ; T_{sl} is the lower bound of the objective function of the branch of the tree of possible variants, composed of variables $x_{ij} \in S_l$, $T_{sl}(\bar{x}_{ij})$ is the lower bound for the objective function, if $\bar{x}_{ij} = 0$. Suppose that $l=0$ and $S_0 \neq \emptyset$. In each row and each column of the original matrix $\|t_{ij}\|_{s_o}$ we find the minimal elements $t_{ih}(i = \overline{0, n})$ and $t_{Rj}(j = \overline{0, n})$. Then the lower bound of the solution can be determined with the help of the expression:

$$T_{s_o} = \max_{ij} (t_{ih}; t_{Rj}) h, k = \overline{0, n}. \quad (5)$$

To increase the probability of cutting off unpromising branches into set S_0 , it is advisable to introduce variable x_{ih} or x_{Rj} ($i; j = \overline{0, n}; i \neq j$), which leads to the maximum increase in the lower bound, provided that the variable $x_{ih}=0$ or $\bar{x}_{Rj}=0$. Substituting into the matrix $\|t_{ij}\|_{s_o}$ $t_{ih} = \infty$, we determine the value $T_{s_o}(\bar{x}_{ih}) = t_{iq} = \min_{j=o, n} t_{ij}$, which characterizes the lower bound for the objective function, if the variable $x_{ih}(\bar{x}_{ih} = 0)$ does not enter set S_0 . Using the condition:

$$T_{s_o}(\bar{x}_{ij}') = \max_{i; j} [T_{s_o}(\bar{x}_{ih}); T_{s_o}(\bar{x}_{Rj})] \quad (6)$$

we define variable x_{ij}' , S_0 that is introduced into set S_0 . To exclude from G_{s_o} the variables $x_{ij} \in E_{s_1}$ in the matrix $\|t_{ij}\|_{s_o}$, we delete the row i' and the column j' corresponding to the variable x_{ij}' , and substitute $t_{j't'} = \infty$. In all subsequent steps of the computational process, the lower bound for the objective function $T_{sl}(l = \overline{1, n})$ and the choice of the variable to be included in set S_l are made using expressions similar to (5), (6). Given that the conditions $T_{s_l} \geq T_{s_{l-1}}$, $T_{s_l}(x_{ij}') \geq T_{s_l}$ must be met, the following expressions can be used to define the lower bound and select variable x_{ij}' :

$$T_{s_l}(\bar{x}_{ij}') = \max_{i, j = \overline{0, n}} [T_{s_l}(x_{ih}'); T_{s_l}(\bar{x}_{Rj}); T_{s_l}], \quad (7)$$

$$i \neq \rho, j \neq q, x_{pq} \in S_l, l = \overline{1, n},$$

$$T_{s_l} = \max_{i, j = \overline{0, n}} (t_{ih}; t_{Rj}; T_{s_{l-1}}), i \neq p, j \neq q, x_{pq} \in S_l, l = \overline{1, n}. \quad (8)$$

Consistently using conditions (7), (8), we construct a branch of the tree of possible variants, which includes the variables $x_{ij} \in S_l$. With $L=n$, we get the first record solution $t^0 = T_{S_n}$, which is used to cut off unpromising branches by checking the inequality:

$$T_{S_l}(x'_{ij}) < t^0, l = n-1, n-2, \dots, L, \dots, 0. \quad (9)$$

We assume that inequality (9) does not hold with $l = n-1, n-2, \dots, L+1$ and is satisfied for $l=L$. Since all branches of the tree of variants for which inequality (9) does not hold are unpromising, in order to cut them off in the matrix $\|t_{ij}\|_{S_l}$, we substitute the element corresponding to the variable x'_{ij} introduced into set S_l , by ∞ . Using conditions (7), (8), we define the lower bound for the objective function and choose a new variable x'_{ij} to include in set S_l . When constructing a new branch of the tree of possible variants, we check the inequality:

$$T_{S_l} < t^0, l = L, L+1, \dots, n. \quad (10)$$

If inequality (10) is satisfied for $l = n$, we get a new record solution T_{S_n} , which is used later to verify inequality (9), (10). The computational process terminates if condition (9) is not satisfied when $l=n-1, n-2, \dots, 0$. In this case, the last record solution t^0 and the corresponding set of variables S_n is optimal.

Let's consider the presented way of solving the problem (1) - (4) on a concrete example, with its initial data being shown in table 1.1. Using expression (5), we find the lower boundary of the objective function $T_{S_0} = 5$. Looking through column t_{iq} and row T_{pj} in Table 1.2, we determine that the condition (6) corresponds to a variable $x'_{0,3}$, which is introduced into set S_0 . In this case, the value $T_{S_0}(\vec{x}'_{0,3}) = 7$. Striking out the first row and the fourth column in Table 1.1 and substituting by $t_{3,0} = \infty$, we get the matrix $\|t_{ij}\|_{S_1}$, which is shown in Table 1.2. Looking through row t_{Rj} and column t_{ih} in this table, we find $T_{S_1} = 5$.

Table 1.1.

i	j					
	0	1	2	3	4	
						t_{ih} t_{iq}
0	∞	7	8	5	9	5 7
1	4	∞	6	2	7	2 4
2	2	7	∞	4	5	2 4
3	1	5	4	∞	6	1 4
4	3	6	8	7	∞	3 6
t_{Rj}	1	5	4	2	5	- -
t_{ij}	2	6	6	4	6	- -

Table 1.2.

i	j					
	0	1	2	4	t_{ih}	t_{iq}
1	4	∞	6	7	4	6
2	2	7	∞	5	2	5
3	∞	5	4	6	4	5
4	3	6	8	∞	3	6
t_{Rj}	2	5	4	5	5	-
t_{ij}	3	6	6	6	-	-

Table 1.3.

i	j			t_{ih}	t_{iq}
	1	2	4		
2	7	∞	5	5	7
3	∞	4	6	4	6
4	6	8	∞	6	8
t_{Rj}	6	4	5	-	-
t_{ij}	7	8	6	-	-

Table 1.4.

i	j			t_{ih}	t_{iq}
	2	4			
2	∞	5	5	∞	
3	4	∞	4	∞	
t_{Rj}	4	5	-	-	
t_{ij}	∞	∞	-	-	

Condition (8) is satisfied by variables $x_{1,0}, x_{4,0}, x_{3,1}, x_{2,4}$, for which $T_{S_1}(\bar{x}_{ij}) = 6$. We choose any of these variables, for example $x'_{1,0}$, which we include in set S_1 . All further calculations are carried out in a similar way. Tables 1.3, 1.4 show the matrices $\|t_{ij}\|_{S_l}$ ($l=2, 3$) and the elements t'_{ij} that correspond to the variables x'_{ij} introduced into set S_l .

Figure 1 represents a tree of possible variants, about each vertex of which the evaluation of lower bound of the objective function is indicated. The process of forming set S_l ends when $l=5$. The first record solution $t^0=6$ corresponds to the variables $x_{0,3} = x_{3,2} = x_{2,4} = x_{4,1} = x_{1,0} = 1$ that enter set S_5 . Since condition (9) is satisfied when $l=4, 3, 7, \dots, 0$, the solution obtained is optimal.

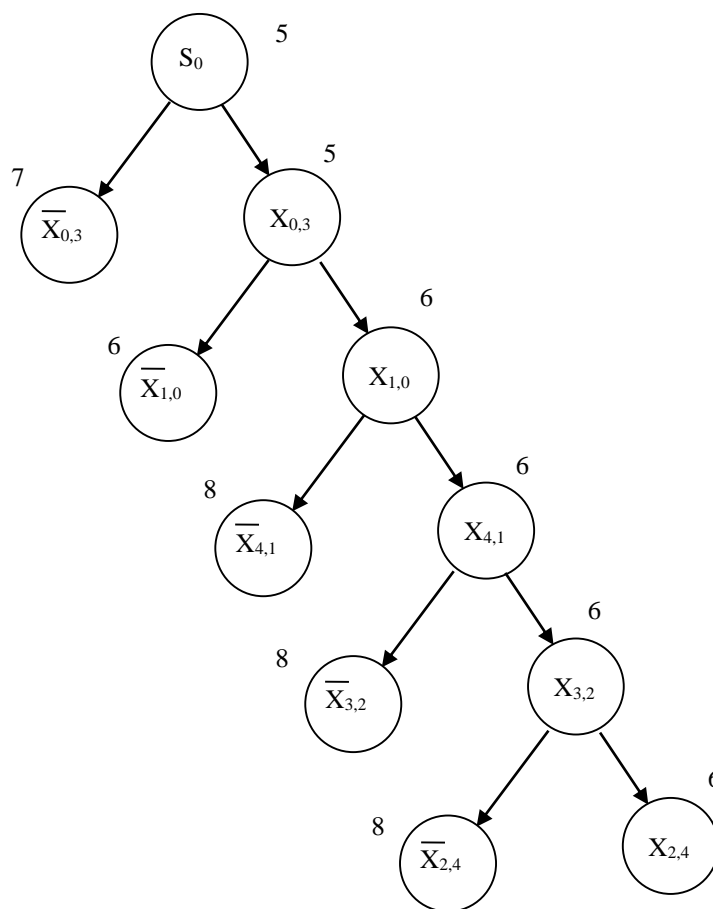


Figure 1. Tree of possible solutions

Estimation of the efficiency of the considered method for determining the lower bound for the solution was made by means of computational experiments. The program of the computational algorithm was compiled in the MatLAB-2008 program. For initial data of the matrix $\|t_{ij}\|$, random numbers uniformly distributed in the range from 1 to 100 were taken. The time of solving the problem for $n=20, 30, 40, 50$, is 0.5; 2.5; 7.5; 27 minutes respectively. Thus, the proposed method for estimating the lower boundary for the solution is sufficiently accurate and allows efficient solution of the minimax task of scheduling construction and installation works.

Example 1. Let the number of works is 4. The initial data are given in Table 1.5.

Table 1.5 The initial data

τ_i	1	2	3	4	5	6	7	8
K_1	2	1	1	4				
K_2		3	3	2	1			
K_3			4	4	3	6		
K_4				5	5	2	3	

Then for the duration of the work we have:

$$10 = 1 + 2 + 3 + 4 \leq T \leq 4 + 5 + 6 + 7 = 22$$

Suppose that the duration of consecutive works is 15, therefore, it is required to reduce the work package by 7 units. We select 7 times the smallest numbers K_i , and the following cases are possible:

- Variant 1.

$$\tau_1 = 1, \tau_2 = 3, \tau_3 = 5, \tau_4 = 6, T = 15$$

with costs:

$$S_1 + S_2 + S_3 + S_4 = 8 + 6 + 9 + 5 = 28$$

- Variant 2.

$$\tau_1 = 1, \tau_2 = 2, \tau_3 = 6, \tau_4 = 6, T = 15$$

with costs :

$$S_1 + S_2 + S_3 + S_4 = 8 + 6 + 9 + 5 = 28$$

Consequently, the minimax solution of the problem requires to determine $(D-T)$ of the minimum coefficients of cost increments K_i with the reduction in the work duration by unit. Both variants of the solution make it possible to efficiently calculate the value of minimizing the time of performance of works by the parameters of organizational and technological reliability.

Example 2. Let's set up the work schedule (Figure 2).

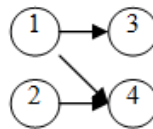


Figure 2. The work schedule

Then, Figure 3 shows the network graph obtained by the aggregated transformation, and the aggregation tree itself is shown in Figure 4:

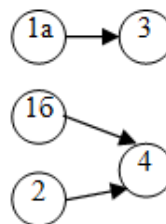


Figure 3. Network graph obtained by the aggregated transformation

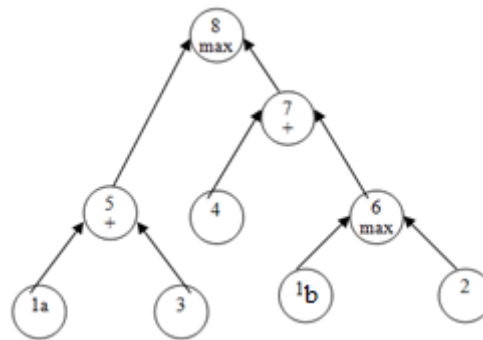


Figure 4. The aggregation tree itself

Table 1.6 describes the work data.

Table 1.6. The work data

τ_i	1	2	3	4	5	6	7	8
K_{1a}		3	2	5				
K_{1b}		3	2	5				
K_2	2	1	3					
K_3			4	3	3			
K_4			5	3	2			

Step 1. We generate the resulting table for aggregated work 5. We get:

τ_5	9	8	7	6	5
S_5	8	10	13	16	20

Step 2. The same is done for aggregated work 6. We get:

τ_6		4	3	2
S_6		8	14	14

Step 3. The same thing goes for aggregated work 7. We get:

τ_7	9	8	7	6	5
S_7	10	12	15	19	24

Step 4. We finish for aggregated work 8. We get:

τ_8	9	8	7	6	5
S_8	18	22	28	35	44

We define a minimax version of the work duration. This option is the only one:

$$T = 9, S = 18.$$

Thus, an algorithm for scheduling construction and installation work was obtained, taking into account the schedule for the movement of brigades, which is characterized by the ability to efficiently calculate the values of minimizing the work performance time in the parameters of organizational and technological reliability through the use of the branch and boundary method

4. Conclusions

On the basis of the studies carried out, models have been created that allow, subject to contractual terms, to select options for performing works that ensure the minimization of additional funds directed to shorten the period of work performance.

Applying the developed models and mechanisms allows a repeated usage of the developments, their replication and mass introduction and leads to significant reduction of the duration of labor and funds

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