

# Calculation of load-bearing capacity of prestressed reinforced concrete trusses by the finite element method

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**Abstract.** The technique of calculation of prestressed reinforced concrete trusses with taking into account geometrical and physical nonlinearity is considered. As a tool for solving the problem, the finite element method has been chosen. Basic design equations and methods for their solution are given. It is assumed that there are both a prestressed and nonprestressed reinforcement in the bars of the trusses. The prestress is modeled by setting the temperature effect on the reinforcement. The ways of taking into account the physical and geometrical nonlinearity for bars of reinforced concrete trusses are considered. An example of the analysis of a flat truss is given and the behavior of the truss on various stages of its loading up to destruction is analyzed. A program for the analysis of flat and spatial concrete trusses taking into account the nonlinear deformation is developed. The program is adapted to the computational complex PRINS. As a part of this complex it is available to a wide range of engineering, scientific and technical workers

## 1. Introduction

The design of prestressed reinforced concrete structures, including reinforced concrete trusses, is currently conducted using empirical and semi-empirical formulas [1]. These formulas do not take into account all the features of the work of prestressed systems associated with the nonlinearity of deformation, with loading, unloading and possible reloading (change in the direction of deformation) due to a sharp redistribution of forces in the event of failure of one or another element. Recommendations for accounting for the nonlinearity of the deformation of concrete and reinforcement [2], which are given in the building norms and acts and annexes to them, carry conventional character. In addition, the normative documents practically do not contain recommendations for taking into account the geometric nonlinearity. Therefore, the development of methods for analysis of prestressed reinforced concrete structures, taking into account physical and geometrical nonlinearity, which makes it possible to determine the load-bearing capacity of structures, is a vital task.

This work is devoted to the analysis of prestressed reinforced concrete trusses taking into account the nonlinearity of deformation by the finite element method. A prerequisite for the successful solution of this problem is the general theory of truss analysis, the foundations of which were laid in the nineteenth century [3] and developed later in the works of domestic and foreign scientists [4-8].



The proposed method is based on algorithms of nonlinear analysis of structures implemented and tested in the computer program PRINS for other types of structures [9]. The results obtained by the authors of this article earlier [10-11] and the domestic [12-17] and foreign [18-21] experience in the development of nonlinear methods of hinge-rod systems analysis were used.

## 2. Materials and Methods

Nonlinear analysis of structures is carried out in the program PRINS by the step-by-step method. At that on each step of loading the next equation is formulated and solved [9]:

$$\left[ K + K_{\sigma} + K_{NL_1} + K_{NL_2} \right] \{ \Delta u \} = \{ \Delta P \}, \quad (1)$$

where  $[K]$ ,  $[K_{NL_1}]$ ,  $[K_{NL_2}]$  – the stiffness matrices of the zero, first and second orders, respectively;  $[K_{\sigma}]$  – matrix of initial stresses;  $[\Delta u]$  and  $[\Delta P]$  – vectors of increments of nodal displacements and loads, respectively. The matrices  $[K_{NL_1}]$  and  $[K_{NL_2}]$  depend on the current step displacements in the first and second degree, respectively. This dependence was obtained in [9] in an explicit form.

The matrix  $K$ , which elements are determined by the properties of the material, also depends on the step values of the displacements, but it is not possible to obtain this relationship explicitly. This matrix can be calculated at the beginning of the step, taking into account the physical properties of the material at the instant of time, and under the same assumptions at the end of the step. We denote these matrices  $K_0$  and  $K_1$ , respectively. Since the properties of the material change at the loading step, the matrix  $K$  can be found approximately as the half-sum of the matrices  $K_0$  and  $K_1$ . In this way,

$K = \frac{1}{2}(K_0 + K_1)$ . We'll represent the matrix  $K$  in the form

$$K = K_0 + \Delta K. \quad (2)$$

It follows from the above that  $\Delta K = K - K_0 = \frac{1}{2}(K_0 + K_1) - K_0 = \frac{1}{2}(K_1 - K_0)$ . Taking into account formula (2), equation (1) takes the form:

$$\left[ K_0 + \Delta K + K_{\sigma} + K_{NL_1} + K_{NL_2} \right] \{ \Delta u \} = \{ \Delta P \}, \quad (3)$$

Equation (3) is solved in the program PRINS by an iterative method of additional loading, which is equivalent of the modified Newton-Raphson method. In this case, equation (3) is written in the form

$$\left[ K_0 + K_{\sigma} \right]_j \{ \Delta u \}_j^{(i)} = \{ \Delta P \}_j - \left[ \Delta K + K_{NL_1} + K_{NL_2} \right]_j^{(i-1)} \{ \Delta u \}_j^{(i-1)}, \quad (4)$$

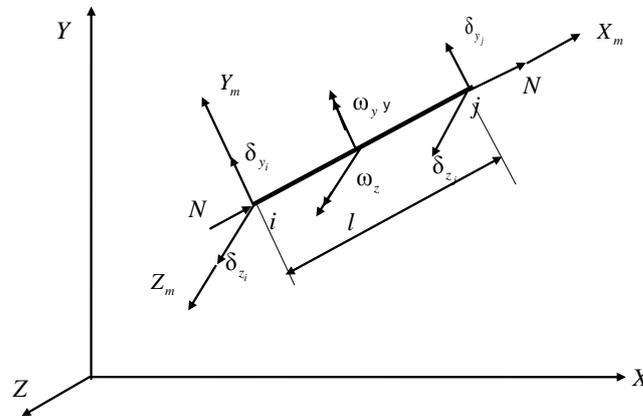
where  $j$  is the number of step loading,  $i$  is the iteration number at this step. The stresses in the elements at each loading step are calculated from the formula

$$\Delta \sigma_x = E_k (\varepsilon'_x + \varepsilon''_x), \quad (5)$$

where  $E_k$  is the tangent modulus of the bar material, and  $\varepsilon'_x$  and  $\varepsilon''_x$  – the linear and nonlinear components of the unit elongation of the bar, respectively, determined by formulas

$$\varepsilon'_x = \frac{\partial u}{\partial x}; \quad \varepsilon''_x = \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right]. \quad (6)$$

In formulas (6)  $u$ ,  $v$ ,  $w$  are the displacements of the points of the bar in the direction of the local axes  $X_m Y_m Z_m$  of the element (see Figure-1). The final values of the forces and displacements are found by summing the results obtained at each step of the loading.

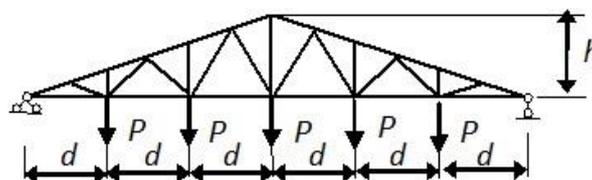


**Figure 1.** Local ( $X_m Y_m Z_m$ ) and global ( $XYZ$ ) coordinate system

Diagrams  $\sigma - \varepsilon$  in the PRINS program can be specified either in analytical or in tabular form. Currently, in the program of PRINS in the analysis of reinforced concrete trusses, two types of diagrams for concrete are realized, and one for reinforcement. For concrete, either a three-line diagram recommended by domestic building codes [2] and given in a tabular form, or curvilinear, recommended by the European Concrete Committee (ECC) [22] and given in analytical form is used. Curvilinear diagrams for compressed concrete are also recommended by domestic standards [2]. A preliminary study conducted by the authors showed that European and domestic standards give well coincident results. However, the norms recommended by the ECC are more convenient when using the finite element method, since they express stresses depending on deformations, i.e. they are given in the form of a function  $\sigma(\varepsilon)$ , and not vice versa, as recommended by domestic standards [2]. For the reinforcements the Prandtl diagram is used.

### 3. Results

The proposed method is implemented in the computer program PRINS. To check the developed methodology, the truss shown in Figure-2 was calculated.

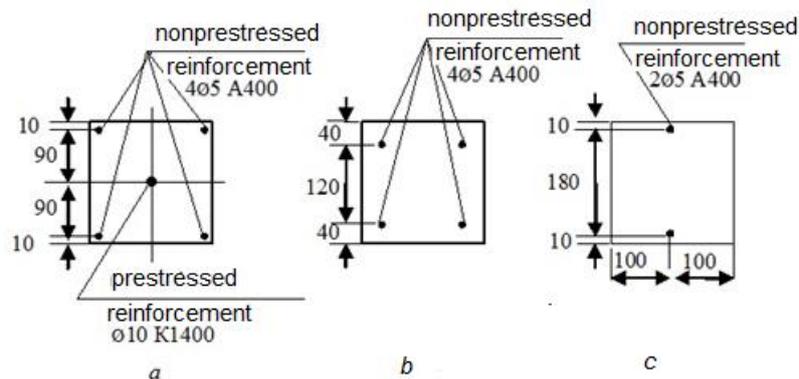


**Figure 2.** Design model of truss

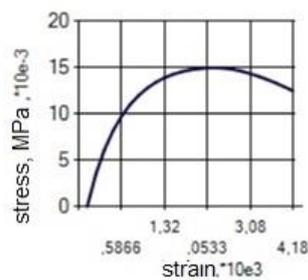
The analysis was carried out with the following input data.

Panel length  $d = 3$  m, height  $h = 3$  m; cross section dimensions for all bars  $20 \times 20$  cm, the bars reinforcement schemes are shown in Figure-3; heavy concrete of B20 class, non prestressed reinforcement of class A400, prestressed reinforcement of class K1400 were used; the truss was loaded with concentrated forces  $P = 25$  kN at the nodes of the bottom chord of truss. The stress-strain

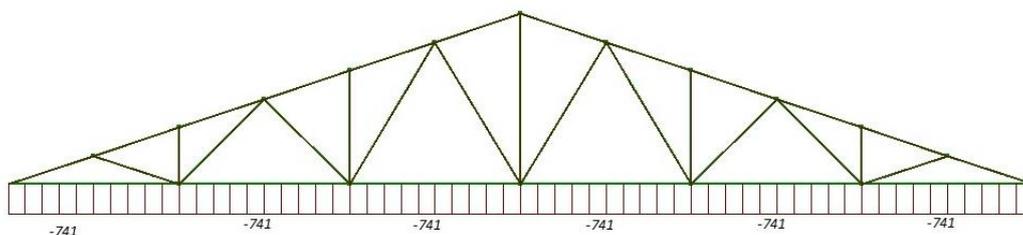
diagram for a compressed zone of concrete is shown in Figure 4. The load was applied in steps. At the first step, the prestress was carried out by setting of the temperature effect on the prestressed reinforcement of the bottom chord. The stress diagram in concrete at the first step of loading obtained with the aid of developed program is shown in Figure 5. At subsequent steps, an external nodal load was applied with a multiplier, the value of which was assumed to be 0.05 for steps from 2 to 15, and 0.025 for the remaining steps. The purpose of the analysis was to determine the ultimate load for the truss and to study its behavior during the loading process.



**Figure 3.** The bars reinforcement schemes: *a* – bottom chord, *b* – top chord, *c* – lattice



**Figure 4.** The stress-strain diagram for a compressed zone of concrete



**Figure 5.** Preliminary stresses in concrete, kPa

The destruction of the truss occurred at the 18th step with nodal load equal to  $0.775P$ . We give some of the analysis results that allow us to understand the causes of destruction.

Figure 6 and Figure 7 show the diagrams of concrete stresses and the stresses in non prestressed reinforcement, respectively, at the 17th step of loading. It is seen from these figures that at the 17th step of loading the stresses in the concrete of the bars of the bottom chord and in the central vertical post become zero due to the cracking of the concrete, and the stresses in the armature of the central vertical reach the yield point. Consequently, at the 17th step the truss turns into a mechanism, and its further loading becomes impossible.

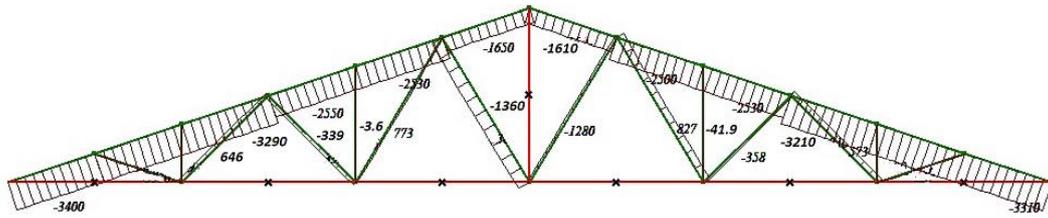


Figure 6. Stresses in concrete at 17th step of loading

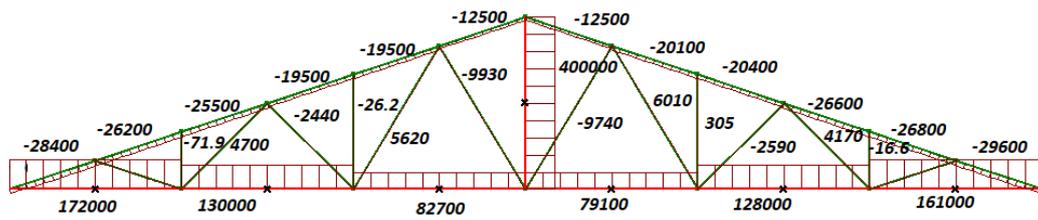


Figure 7. Stresses in non prestressed reinforcement at 17th step of loading

Figure 8 shows the diagram of the total values of the forces in the bars at the 17th step of loading.

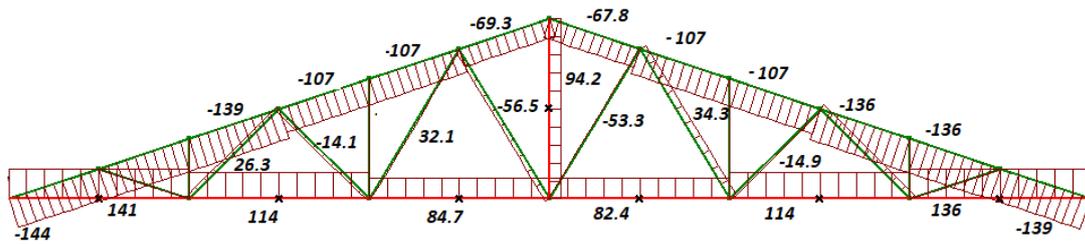


Figure 8. The diagram of the total values of the forces in the bars at the 17th step of loading

The deformed state of the truss at the 17th step of loading is shown in Figure 9.

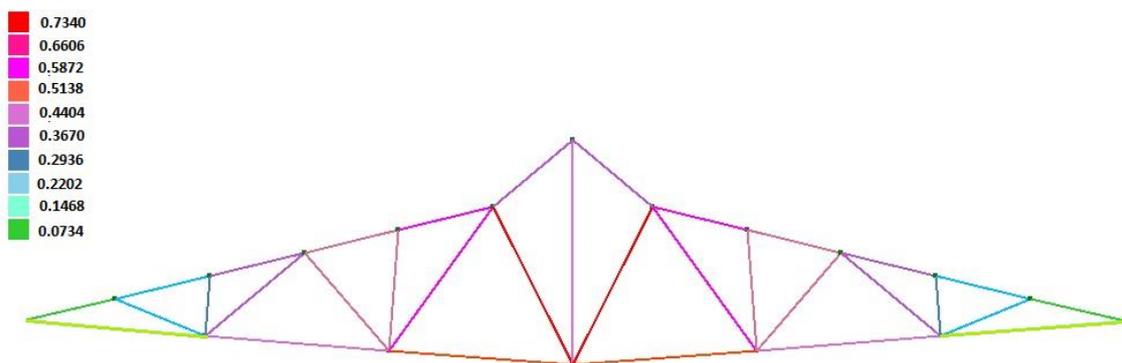
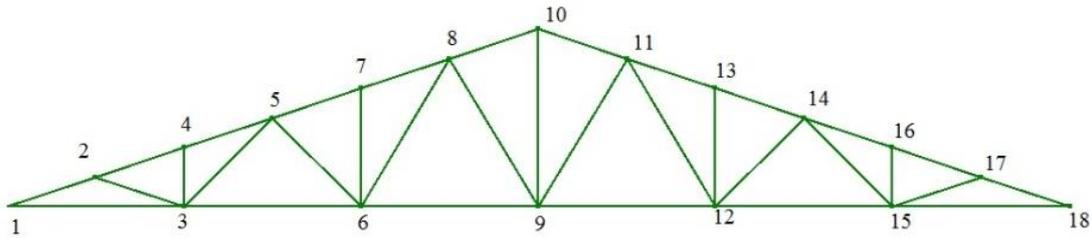


Figure 9. Deformed state of the truss at the 17th step of loading (displacement scale 1: 1)

#### 4. Discussion

The sharp fracture of the top chord in Figure-9 is explained by the fact that when the vertical 9-10 breaks down, the central fragment of the truss 8-9-11-10 (see Figure 10) changes to a hinged quadrilateral in which it becomes possible the displacements of the nodes without deformation of the elements.

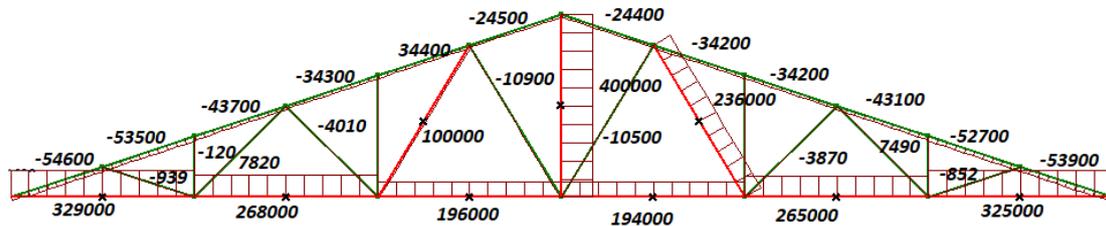


**Figure 10.** The numbering of the nodes

We note that when geometric nonlinearity is taken into account, the initially symmetric design scheme of the structure is somewhat distorted, which leads to a distortion of the symmetry in the stress-strain state.

Analysis of the causes of the destruction of the truss allows us to understand how to increase its load-bearing capacity. It is obvious that in this case it is necessary to strengthen the vertical 9-10 (Fig. 10). As calculations show, replacing the reinforcement of this bar from type "c" to type "b" (Fig. 3) leads to an increase in the maximum load to 1.275 P. At the same time the nature of the destruction does not change, which gives the foundation to suggest that the central vertical remains the weak link. Indeed, as can be seen from Figure 11, the yield stress as before is achieved primarily in the central bar.

Varying the dimensions of the cross sections and the reinforcement of the bars, and, possibly, the scheme of the truss, it is possible to achieve a further increase in its bearing capacity.



**Figure 11.** The stresses in non-prestressed reinforcement at 37th step of load

## 5. Conclusion

The method proposed in this paper and the computer program based on it and implemented in the computational complex PRINS, make it possible to analyze in detail the processes of deformation of prestressed trusses under load up to their destruction, taking into account physical and geometric nonlinearity. Similar approaches to the calculation of reinforced concrete trusses in domestic design practice have not yet been used. The PRINS program is accessible to a wide range of specialists and can be useful in the analysis and design of reinforced concrete trusses

## References

- [1] Russian standard SP 52-102-2004
- [2] Russian standard SP 63.13330 2012
- [3] Zhuravskij D I 1855 *About the bridges of the Ghau girder frame* (Saint- Petersburg)
- [4] Galerkin B G 1926 *To the calculation of un-girder trusses and rigid frames* (Moscow: Gostekhizdat)
- [5] Filin A P 1966 *Matrices in the statics of rod systems* (Moscow: Gosstrojizdat)
- [6] Gofman SH M and Agapov V P 1972 *The buckling analysis of three-dimensional hinged-rod systems Proceedings of universities. Building and Architecture* **1** pp 31-35
- [7] Bajkov V N and Sigalov EH E 1991 *Reinforced concrete structures. General course* (Moscow: Strojizdat)

- [8] Pecol'da T M and Tura V V 2003 *Reinforced concrete structures. Fundamentals of theory and design* (BSTU Publishing House)
- [9] Agapov V P 2005 *Finiteelement method in static, dynamic and buckling of structures* (ASV Moscow)
- [10] Agapov V P and Ajdemirov K R 2016 *Scientific review* **2** p 31
- [11] Agapov V P and Ajdemirov K R 2016 *Industrial and Civil Engineering* **11** p 4
- [12] Hejdari A and Galishnikova V V 2013 *Herald of RUDN* **1** p 118
- [13] Kirsanov M N 2016 *Magazine of Civil Engineering* **4** p 52
- [14] Akaev N K and Yusupov A K 2016 *Herald of Dagestan State Technical University* **42**(3) p 119
- [15] Tin'kov D V 2015 *Magazine of Civil Engineering* **5**(57) p 66
- [16] M N Kirsanov 2016 *Structural Mechanics and Analysis of Constructions* **3** p 16
- [17] Karpenko N I, Eryshev V A, Latysheva E V and Kokarev S A 2015 *Industrial and Civil Engineering* **3** 12
- [18] Iwicki P and Krajewski M 2013 *Mechanics and Mechanical Engineering* **17**(1) p 49
- [19] Coarita E and Flores L 2015 *IACSIT International Journal of Engineering and Technology* 2015 **7**(3)
- [20] Toklu Y C 2004 *Computer&Structures* **22** 1581
- [21] Shi H, Salim H, Shi Y and Wei F 2015 *Mechanics Based Design of Structures and Machines An International Journal* **43** p 38
- [22] 1993 *Comitee Euro-International De Beton. Ceb-Fip\_Model Cod, (1990)* (London: Thomas Telford House)