

Numerical study of the stress-strain state of reinforced plate on an elastic foundation by the Bubnov-Galerkin method

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Abstract. The stress-strain state of a rectangular slab resting on an elastic foundation is considered. The slab material is isotropic. The slab has stiffening ribs that directed parallel to both sides of the plate. Solving equations are obtained for determining the deflection for various mechanical and geometric characteristics of the stiffening ribs which are parallel to different sides of the plate, having different rigidity for bending and torsion. The calculation scheme assumes an orthotropic slab having different cylindrical stiffness in two mutually perpendicular directions parallel to the reinforcing ribs. An elastic foundation is adopted by Winkler model. To determine the deflection the Bubnov-Galerkin method is used. The deflection is taken in the form of an expansion in a series with unknown coefficients by special polynomials, which are a combination of Legendre polynomials.

1 Introduction

Stress-strain analysis of reinforced elements of various structures is widely used in the different stage of the reinforced concrete structures design [1-3]. Winkler's models are widely used in road and civil construction in the design of road clothes and foundation slabs under static [4] and dynamic loads [5].

Among the many technical problems associated with the assessment of the state of structures and buildings, an important place is taken by methods of quality control of material based on impact indentation. In [6] the impact conical indentation on a circular slab resting on Winkler foundation is considered. The solution of this problem takes into account the plastic flow of material from the punch. Non-linear integral equation for impact force $P(t)$ was solved digitally.

Static analysis of functionally graded circular plates resting on Winkler elastic foundation is considered in [7]. The differential transforms method is utilized to solve the governing differential equations of bending of the thin circular plate under various boundary conditions.

The buckling analysis of composite orthotropic truncated conical shells under a combined axial compression and external pressure and resting on a Winkler foundation is discussed in [8]. The governing equations were obtained for the orthotropic truncated conical shell and solved by applying the Superposition and Bubnov-Galerkin method.

A nonlinear investigation is presented in [9] for impact response of functionally graded material doubly curved panels, which are resting on Winkler

The relaxation constants of the material included in the nonlinear equation of Maxwell-Gurevich are considered in [10] for creep models.

In order to develop methods of digital analysis of stress-strain state of slab resting on Winkler foundation the method of Bubnov-Galerkin is considered.



2 Problem statement

We consider a rectangular plate with ribs of rigidity directed parallel to the edges of the plate, loaded with a distributed load q perpendicular to the median plane rigidly fixed at the edges (figure 1).

The plate is considered as constructively orthotropic. The differential equation of the bending of an orthotropic plate on an elastic base has the form

$$D_1 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^4} + 2D_3 \lambda^2 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^2 \partial \eta^2} + D_2 \lambda^4 = \left(\frac{a}{2}\right)^4 [q(\xi, \eta) - kw(\xi, \eta)] \quad (1)$$

$$\text{where } D_1 = D + \frac{E_1 J_1}{b_1}; \quad D_2 = D + \frac{E_2 J_2}{a_1}; \quad D_3 = D + \frac{1}{2} \left(\frac{J_{2t}}{a_1} + \frac{J_{1t}}{b_1} \right);$$

D - cylindrical stiffness of a plate; $E_1 J_1$, $E_2 J_2$ - rib bending stiffness; J_{2t} , J_{1t} - moments of inertia in torsion of ribs; $w(x, y)$ - plate deflection; k - modulus of subgrade reactions; $a_1 = a/n_1$ is the distance between the stiffeners of the parallel axis y ; $b_1 = b/n_2$ is the distance between the stiffeners of the parallel axis x ; n_1 , n_2 - the number of stiffener.

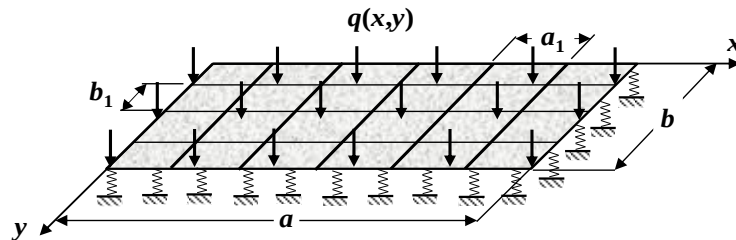


Figure 1. Slab resting on Winkler foundation scheme

We assume that the plate is rigidly constrained along the contour, then the deflection $w(x, y)$ satisfies the following boundary conditions

$$\begin{aligned} w(0, y) = \frac{\partial w(0, y)}{\partial x} = 0; \quad w(a, y) = \frac{\partial w(a, y)}{\partial x} = 0; \quad w(x, 0) = \frac{\partial w(x, 0)}{\partial y} = 0; \\ w(x, b) = \frac{\partial w(x, b)}{\partial y} = 0; \end{aligned} \quad (2)$$

The dimensionless variables ξ, η are related to the coordinates x, y by the relations

$$x = \frac{a}{2}(\xi + 1); \quad y = \frac{b}{2}(\eta + 1); \quad -1 \leq \xi \leq 1; \quad -1 \leq \eta \leq 1; \quad \lambda = \frac{a}{b}. \quad (3)$$

The solution of equation (1) is made in the form of a double row

$$w(\xi, \eta) = \sum_{m=1}^M \sum_{n=1}^N A_{mn} Q_{m+3}(\xi) Q_{n+3}(\eta), \quad (4)$$

where A_{mn} are unknown coefficients, $Q_i(\xi)$ are special orthonormal polynomials of degree i , which can be represented as a combination of three classical Legendre polynomials [8], satisfy the boundary conditions

$$Q_i(\pm 1) = \frac{\partial Q_i(\pm 1)}{\partial \beta} = 0, \quad \beta = \xi, \eta \text{ and the normalization conditions}$$

$$\int_{-1}^1 \frac{\partial^4 Q_i(\xi)}{\partial \xi^4} Q_j(\xi) d\xi = \delta_{ij}, \quad \delta_{ij} - \text{Kronecker symbol.}$$

Using the Bubnov-Galerkin method [11,12,13], we obtain a system of linear algebraic equations with respect to the unknown A_{mn} :

$$\sum_{m=1}^M \sum_{n=1}^N B_{kl}^{mn} A_{mn} = q_{kl},$$

$$\begin{aligned}
B_{kl}^{mn} = & D_1 \int_{-1}^1 \frac{\partial^4 Q_{m+3}(\xi)}{\partial \xi^4} Q_{k+3}(\xi) d\xi \int_{-1}^1 Q_{n+3}(\eta) Q_{l+3}(\eta) d\eta \\
& + 2D_3 \lambda^2 \int_{-1}^1 \frac{\partial^2 Q_{m+3}(\xi)}{\partial \xi^2} Q_{k+3}(\xi) d\xi \int_{-1}^1 \frac{\partial^2 Q_{n+3}(\eta)}{\partial \eta^2} Q_{l+3}(\eta) d\eta \\
& + D_2 \lambda^4 \int_{-1}^1 Q_{m+3}(\xi) Q_{k+3}(\xi) d\xi \int_{-1}^1 \frac{\partial^4 Q_{n+3}(\eta)}{\partial \eta^4} Q_{l+3}(\eta) d\eta \\
& + k \left(\frac{a}{2}\right)^4 \int_{-1}^1 Q_{m+3}(\xi) Q_{k+3}(\xi) d\xi \int_{-1}^1 Q_{n+3}(\eta) Q_{l+3}(\eta) d\eta \\
q_{kl} = & \left(\frac{a}{2}\right)^4 \int_{-1}^1 Q_{k+3}(\xi) d\xi \int_{-1}^1 q(\xi, \eta) Q_{l+3}(\eta) d\eta; k, l = 1, 2, 3 \dots \dots
\end{aligned}$$

The bending moments arising in the plate are determined by the equations

$$\begin{aligned}
M_x(\xi, \eta) &= -4 \frac{D_1}{a^2} \left(\frac{\partial^2 w(\xi, \eta)}{\partial \xi^2} + \nu \lambda^2 \frac{\partial^2 w(\xi, \eta)}{\partial \eta^2} \right); \\
M_y(\xi, \eta) &= -4 \frac{D_2}{a^2} \left(\lambda^2 \frac{\partial^2 w(\xi, \eta)}{\partial \eta^2} + \nu \frac{\partial^2 w(\xi, \eta)}{\partial \xi^2} \right).
\end{aligned}$$

3 Results and discussion

Calculation of the concrete plate was carried out with the following initial data: $a=3$ m; $b=2$ m; $h=5$ cm; $E_c=0.2 \cdot 10^5$ MPa; $\nu_c=0.2$. Stiffeners: width $b = 1$ cm; height $h = 2.5$ cm; Material-steel: $E_{st}=2 \cdot 10^5$ MPa; $G_{st}=8 \cdot 10^4$ MPa. The plate is loaded with a uniformly distributed load $q=100$ kN/m²

Figure 2 shows the distribution of deflections $w(\xi, \eta)$,

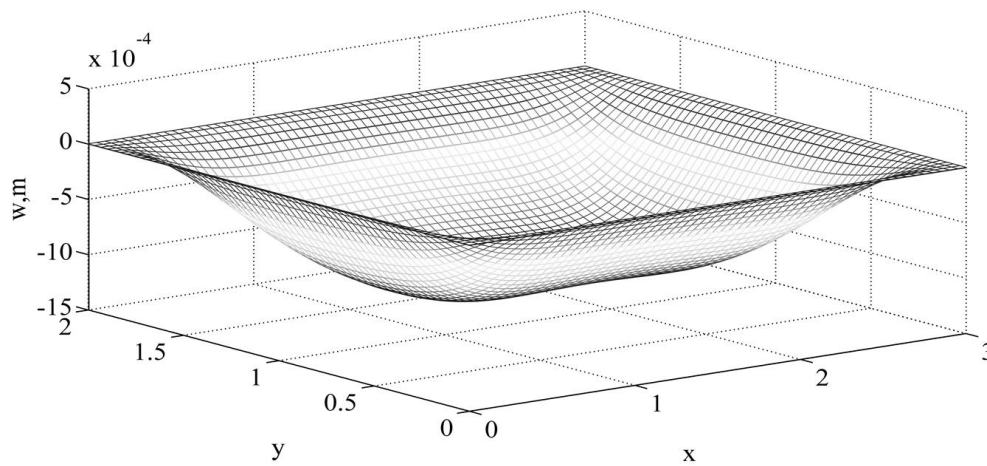


Figure 2. The distribution of deflections $w(\xi, \eta)$ over the plate

Figure 3 shows the bending moments $M_x(\xi, \eta)$,

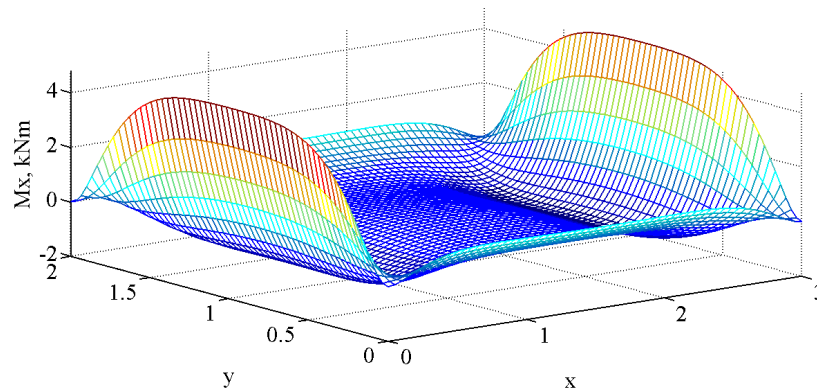


Figure 3. The distribution of the bending moments $M_x(\xi, \eta)$

Figure 4 shows $M_y(\xi, \eta)$ over the plate.

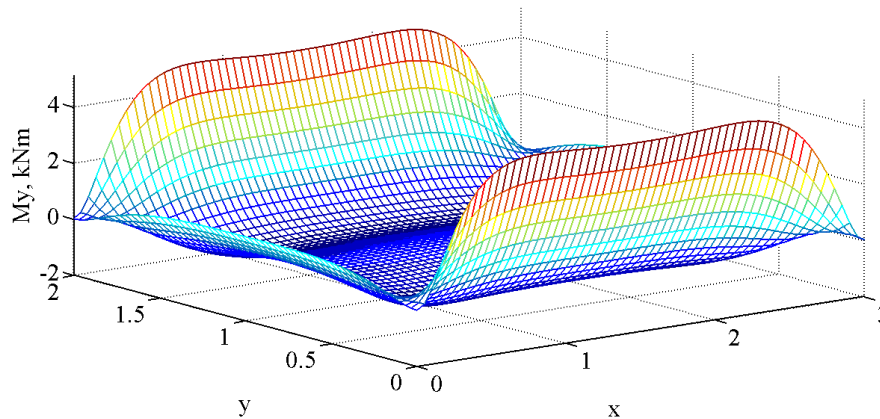


Figure 4. The distribution of the bending moments $M_y(\xi, \eta)$

Tables 1 and 2 show the results of calculating the plate for different modulus of subgrade reactions and a different number of stiffeners.

Table 1. The results of calculating the plate for $k=100$ MPa/m

$k=100$ MPa/m (Sandy-clay soil); $N=15$; $M=15$							
y				x			
n	n	w	M	M	w	M	M
0	0	0.11156	4.8948	0.97896	0.10987	0.93268	4.6634
6	6	0.11163	4.9854	0.98552	0.11001	0.95551	4.7222
12	12	0.11168	5.0744	0.99216	0.11014	0.97801	4.7806
12	0	0.11163	5.0673	0.9454	0.11009	1.0008	4.6681
0	12	0.11161	4.9016	1.0274	0.10992	0.91141	4.7758

Table 2. The results of calculating the plate for $k=500$ MPa/m

$k=500 \text{ MPa/m (limestone); } N=15; M=15$							
Number of edges		$y = b/2, 0 \leq x \leq a$			$x = a/2, 0 \leq y \leq b$		
n_1	n_1	w	M	M	w	M	M
0	0	0.021073	2.0878	0.41756	0.020767	0.41644	2.0822
6	6	0.021075	2.1257	0.42023	0.020756	0.42632	2.1069
12	12	0.021076	2.1631	0.42293	0.020746	0.43602	2.1313
12	0	0.021046	2.1617	0.4033	0.020762	0.44637	2.082
0	12	0.021103	2.0892	0.43789	0.020751	0.40678	2.1315

As can be seen from the above calculations, the number of reinforcing bars does not affect the magnitude of the maximum deflection and give insignificant changes in the values of the maximum moments, and accordingly the stresses. The influence of the bed coefficient on deflection and bending moments is significant. An increase in the bed coefficient results in a decrease in the deflection by a factor of five and a reduction in bending moments by 50%.

The proposed method makes it possible to calculate the strength and rigidity of arbitrarily fixed, randomly loaded rectangular reinforced plates on an elastic foundation.

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