

Optimal design of earth-moving machine elements with cusp catastrophe theory application

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Abstract. This paper deals with the optimal design problem solution for the operator of an earth-moving machine with a roll-over protective structure (ROPS) in terms of the catastrophe theory. A brief description of the catastrophe theory is presented, the cusp catastrophe is considered, control parameters are viewed as Gaussian stochastic quantities in the first part of the paper. The statement of optimal design problem is given in the second part of the paper. It includes the choice of the objective function and independent design variables, establishment of system limits. The objective function is determined as mean total cost that includes initial cost and cost of failure according to the cusp catastrophe probability. Algorithm of random search method with an interval reduction subject to side and functional constraints is given in the last part of the paper. The way of optimal design problem solution can be applied to choose rational ROPS parameters, which will increase safety and reduce production and exploitation expenses.

1. Introduction

Current standards in the field of self-propelled earth-moving machinery (ISO 3471, ISO 3449) require equipping tractor cabins with protection structures (ROPS) to reduce the risk of injuries to the operator caused by tractor rollovers or falling stones, trees etc. In some cases experimental assessment of a ROPS adequacy can be time-consuming, rather expensive and often technically difficult to perform because it requires testing full-scale samples of ROPS in a special laboratory.

Prevention of tractor rollover fatalities is an important problem attracting interest of many experts both in Russia and abroad. Machine, environmental and human factors causing tractor roll-over are analyzed by Melvin L. Myers [1, 2]. Dumitrache P. investigated the behavior of earth-moving machine protective structures under load by the method of parametric modeling application [3]. John R. Powers et al [4] performed the way of improvement protective properties using the AutoROPS (automatically deploying, telescoping ROPS). Harris J. R. et al [5] used finite element modeling to compare maximum stress values in ROPS for simulated static tests and dynamic rearward rollovers.

The engineering approach to rollover protection includes application of classic suspension systems (springs, dampers), structural protective devices incorporated into operator's seat, additional energy-absorbing components etc. The alternative way of ROPS efficiency upgrading is optimization of structural parameters (geometrical dimensions, tolerances, defect sizes). The statement of an optimal design problem with the catastrophe theory application was presented in [6].

The aim of this study is to present the way of choosing rational ROPS parameters via cusp catastrophe theory application at the design stage.



2. Materials and methods

The catastrophe theory was first recognized in the middle of the 1970s [7]. The catastrophe theory enables researching sudden quantitative and qualitative changes in any state of a system. Thom's list [7] includes seven elementary catastrophes. The cusp catastrophe is the most often used among them. Potential energy of the system can be expressed in this case in the canonical form:

$$V_{ab}(x) = \frac{1}{4}x^4 + \frac{1}{2}ax^2 + bx, \quad (1)$$

where x – state variable; a, b – manage variables.

Cusp catastrophe manifold M (Figure 1) can be expressed through the relationship:

$$\frac{d}{dx}V_{ab}(x) = x^3 + ax + b. \quad (2)$$

Cubic equation (Eq. 2) may have from one to three real roots. The nature of these roots depends on discriminant

$$D = 4a^3 + 27b^2. \quad (3)$$

B_1 and B_2 curves (Figure 1) correspond to $D = 0$; P – cusp point, where $a = b = 0$. B_1 , B_2 and P constitute a bifurcation set. When $D < 0$, points a and b belong to I domain. E domain corresponds to $D > 0$.

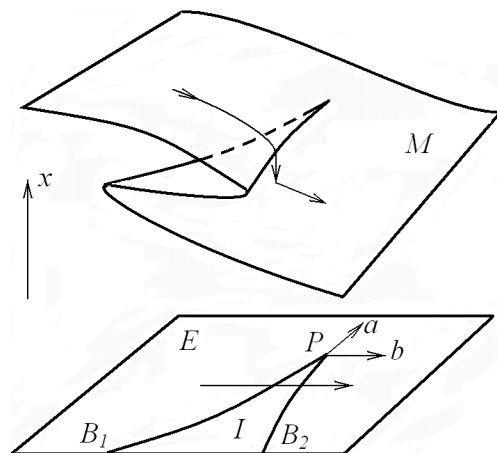


Figure 1. Manifold M and cusp catastrophe representation

Manifold M points disposed in the internal cusp surface correspond to an unstable equilibrium system state. The trajectory that defines the system state can leave domain I due to changing variance a, b . It causes a sudden system change or catastrophe. According to Thom's perfect delay principle, the catastrophe happens only when the trajectory leaves I domain.

Obviously (a, b) points trajectory can be accidental. In a common case a and b manage variables change during the time and system state will be defined by stochastic function (process) $D(t)$. Naturally, it is necessary to solve the stochastic process of I domain overshoot problem (Figure 1).

Catastrophe theory methods are used in fracture mechanics, stability problems of shells, beams, plates etc [8,9]. Operational loads, defect sizes and machine part dimensions in consequence of dispersion in manufacturing tolerances, mechanical characteristics of materials, etc. may be described

by the probability theory. Therefore, it is necessary to consider constitution elements design problems when using the catastrophe theory and probability approach.

2.1. The cusp catastrophe probability evaluation

In the general case a and b are control variables changing during the time t and system state, which will be defined by the random process. The problem of the random process (function) $D(a, b, t)$ overshoot was solved in [10].

Catastrophe happens when (a, b) points trajectory is leaving I domain, and $D(a, b)$ sign is changing from negative to positive. It is evident that catastrophe origin probability is:

$$P = P\{D(a, b) > 0\}. \quad (4)$$

The reliability function in this case is:

$$R(a, b) = 1 - Q = P\{D(a, b) < 0\}. \quad (5)$$

The solution of this problem depends on specific applied situation. If a and b are stochastic quantities with means \bar{a}, \bar{b} , variances σ_a^2, σ_b^2 and beivariate distribution law $f_1(a, b)$, catastrophe probability can be determined [6]:

$$Q = \int_0^{\infty} f_2(D) dD, \quad (6)$$

where $f_2(D)$ is the discriminant $D(a, b)$ distribution law.

Monte Carlo and statistical linearization methods can also be applied for $f_2(D)$ evaluation. For example, using the statistical linearization method, it is possible to estimate discriminant $D(a, b)$ mean value \bar{D} and variance σ_D^2 :

$$\bar{D} = 4\bar{a}^3 + 27\bar{b}^2; \quad (7)$$

$$\sigma_D^2 = \left(\frac{\partial \bar{D}}{\partial \bar{a}}\right)^2 \sigma_a^2 + \left(\frac{\partial \bar{D}}{\partial \bar{b}}\right)^2 \sigma_b^2 = 144\bar{a}^4 \sigma_a^2 + 2916\bar{b}^2 \sigma_b^2. \quad (8)$$

Normal distribution law of the D can be accepted for catastrophe probability rough evaluation:

$$f_2(D) = \frac{1}{\sigma_D \sqrt{2\pi}} \exp\left[-\frac{(D - \bar{D})^2}{2\sigma_D^2}\right]. \quad (9)$$

The cusp catastrophe probability is:

$$Q = \frac{1}{2} + \Phi(\gamma), \quad (10)$$

where $\Phi(\gamma)$ is the Laplace function,

$$\Phi(\gamma) = \frac{1}{\sqrt{2\pi}} \int_0^{\gamma} e^{-\frac{t^2}{2}} dt, \quad \gamma = \frac{\bar{D}}{\sigma_D}. \quad (11)$$

2.2. Optimal design problem statement

The statement of the optimization problem includes the establishment of system limits, choice of the objective function and independent design variables (optimized parameters) [11].

Let us begin with the determination of the objective function. In a number of situations, it is impossible to achieve maximum life of a machine part (reliability of the system). Therefore, it is better to choose an efficiency index considering both reliability during exploitation and manufacturing cost. The single criterion design of the minimum total cost expenses is formulated as follows: to minimize

$$C_T(X^*) = \min_{X \in \Omega} C_T(X), \quad (12)$$

under conditions

$$X_{\min} \leq X \leq X_{\max} - \text{side constraints},$$

$$F(X) \leq 0 - \text{functional constraints},$$

where X^* is the value of the optimized parameters vector X , minimizing the objective function;

Ω is the vector X acceptable region, including side and functional constraints.

The capacity of optimized parameters includes cusp catastrophe manage variables. Let us impose side constraints on geometrical dimensions of machinery parts. Functional constraints may be also imposed on some machine parameters, such as mass, stress-strain properties, rigidity etc. Functional constraints for an earth-moving machine with a roll-over protective structure include deflection-limiting volume, lateral force, absorbed energy etc.

Mean total expected costs are:

$$C_T(a,b) = C_{IT} + Q(a,b) \cdot C_{2T}, \quad (13)$$

where C_{IT} is the initial cost and C_{2T} is the cost of failure according to the cusp catastrophe probability $Q(a,b)$ (Equations (4) – (11)).

Cusp catastrophe manage variables a, b depend on optimized parameters:

$$a = f_a(X); \quad b = f_b(X).$$

3. Results and discussion

A great many algorithms of optimal problem solution are based on direct search methods. An algorithm of the random search method with an interval reduction is used to solve the original problem.

Statistical modeling of the optimized parameters vector X is accomplished by using Monte Carlo technique simulation:

$$X_I^I = (x_I^I; x_2^I; \dots; x_n^I). \quad (14)$$

If obtained vector X_I^I is contained in acceptable region Ω , including side and functional constraints, then cusp catastrophe manage variables a, b can be defined as:

$$a_I^l = f_a(X_I^l); \quad b_I^l = f_b(X_I^l). \quad (15)$$

The cusp catastrophe probability and the reliability function are:

$$Q_I^l = Q(a_I^l, b_I^l); \quad R_I^l = 1 - Q_I^l; \quad (16)$$

and the objective function value is:

$$C_T^l = C_T(a_I^l, b_I^l). \quad (17)$$

The optimization method allows performing computations in accordance with the given number of random tests N

$$\begin{aligned} X_I^2 &\rightarrow a_I^2, b_I^2 \rightarrow R_I^2 \rightarrow Q_I^2 \rightarrow C_T^2; \\ &\dots\dots\dots \\ X_I^j &\rightarrow a_I^j, b_I^j \rightarrow R_I^j \rightarrow Q_I^j \rightarrow C_T^j; \\ &\dots\dots\dots \\ X_I^N &\rightarrow a_I^N, b_I^N \rightarrow R_I^N \rightarrow Q_I^N \rightarrow C_T^N. \end{aligned} \quad (18)$$

Therefore, value X_I^* of the optimized parameters vector, minimizing the objective function $C_T^{I*} = C_T(X_I^*)$, is obtained. This value is entered for storage and is used as the starting point of the next set of tests; at the same time the boundaries of a new interval were reduced.

Using the same procedure (Equations (14) – (18)), let us perform k test sets and obtain:

$$\begin{aligned} X_{II}^* &\rightarrow C_T^{II*} = C_T(X_{II}^*); \\ &\dots\dots\dots \\ X_k^* &\rightarrow C_T^{k*} = C_T(X_k^*). \end{aligned}$$

The amount of test sets depends on the required accuracy of objective function calculations Δ :

$$|C_T(X_k^*) - C_T(X_{k-1}^*)| \leq \Delta.$$

4. Conclusion

Prevention and protection concepts of ROPS efficiency improvement provide different directions of investigation, such as research of new promising materials (for example, TWIP or TRIP steel), application of additional safety devices (structural protective devices incorporated into the operator's seat, air bags, additional energy-absorbing components, etc), improvement of classic suspension systems (springs, dampers). The alternative way of ROPS efficiency upgrading is solution of an optimization problem to choose substantiated structural parameters.

This approach to designing enables us to carry out statistical analysis of stability near critical points and to give recommendations for engineering of earth-moving machine and equipment elements in terms of the statistical catastrophe theory.

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