Force interaction of flexible cable elements under operation conditions

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Abstract. The force interaction of cable elements is observed by the deformation of a flexible cable. To assess these forces, the cable is considered as a compound of the rod with a completely rigid cross-binding. The article represents some expressions used to define the forces, arising by the flexible cables service.

Introduction

Elastic cables are intended to connect mobile mechanisms and machines with a power source. While in operation, there are cyclic displacements of these mechanisms which lead to the deformation of flexible cables. Depending on their application (all-service, shaft cables, cables for drilling tools and etc.) the flexible cables are exposed to different types of deformation. These deformations include bending, torsional, tensile strains (deformations) and some other types. These deformations cause the destruction of the wires of a current-carrying conductor, deterioration of insulating and cable hose wrapping, its sheath and insulation breakdown. The above mentioned problems lead to the failures of actuating mechanisms and machines. The main direction in the manufacture of reliable cable products is to ensure the mechanical strength of cables aimed at improving their reliability under the impact of various deformations.

This study [1] defines the forces of the interaction between cable elements similar to the Kelley's experiment [2, 3]. The method determining the friction force and friction coefficient [4] is also known. This method allows defining the efforts of friction between the cable construction elements. These methods require some additional experiments to be carried out with end products (items) and should be conducted without any deformation, for instance, without any cable bending. During the experiment, it is difficult to find the relation between geometrical parameters and shearing forces. The task to define this shearing force has still not been solved theoretically.

Materials and methods

The producing plant is provided with operating (performance) characteristics which are necessary to comply with to ensure the cable functionality within its lifetime. The minimal bending radius by installation and service is one of the parameters to be ensured for flexible cables. Therefore, let us consider the practical example of cable reeling on the drum (cable pulley wrap) with certain radius.

The solution of the problems associated with the definition of shearing forces for cable elements is possible if the cable is represented as a compound rod with completely rigid cross-bindings using the equations of shearing forces (shear thrusts) from the general theory of compound rods [5]. To simplify the problem the following assumptions are introduced: copper wires of a cable conductor (core) are considered as solid copper rods; current-carrying conductors (cores) are arranged in parallel to each other (without their twisting). We consider this system as a laminated plate under the conditions of plane deformation. The transition to the axially symmetric problem is carried out through transformation $G(1/x)$ [6].

By reeling radius R on the drum, the bending moment (torque) affects the cable. This bending moment (torque) is determined from the following ratio [7]:

$$M^0 = \frac{EJ_x}{R},$$

where E is a reduced module of cable elasticity; J_x is the inertia moment of the cable section.

Let us determine the shearing forces (shear thrusts) between three and four-core cables (figure 1).

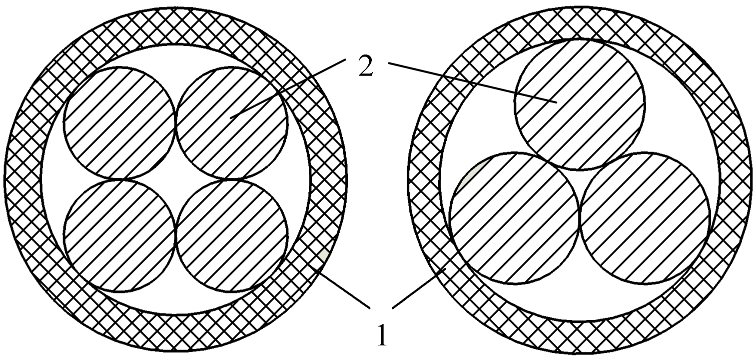

Figure 1. The section of three and four-core cables: 1) outer insulation; 2) cable cores (conductors) with inner insulation.

Each of these cables can be represented as a four-layer compound rod (figure 2). It is assumed that the current-carrying conductor (core) and its surrounding inner insulation operate as a comprehensive whole (one unit) and the contribution of the inner insulation in the core rigidity is insignificant.

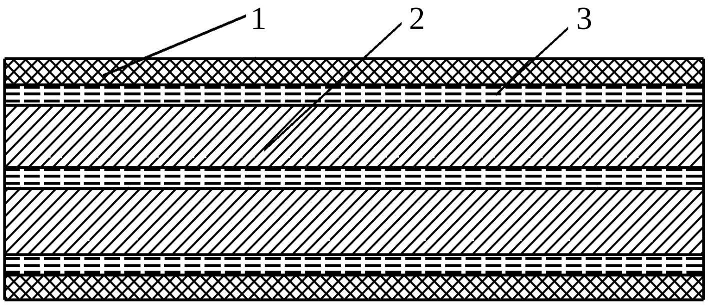

Figure 2. A four-layer compound rod 1) the rod, which bending stiffness (flexural rigidity) is equal to the rigidity of the semicylinder of the outer insulation; 2) the rod which bending stiffness (flexural rigidity) is equal to the rigidity of the stack of current-carrying cores; 3) longitudinal bracing of layer interconnection.

Let us introduce the following symbols: r_k is the radius of cable cross-section, r_t is the radius of a current-carrying conductor (core), F_t, J_t are relatively the cross sectional area and the axial moment of inertia of a current-carrying conductor (core), E_k, E_t are modulus of material elasticity of its outer insulation and the current-carrying conductor (core).

Shearing forces (shear thrusts) in the equivalent four-layer rod can be defined from the differential equation system [5]:

$$\begin{cases} \left(\dfrac{T_1'}{\xi_1}\right)' - \Delta_{11}T_1 - \Delta_{12}T_2 - \Delta_{13}T_3 = \Delta_{10}; \\ \left(\dfrac{T_2'}{\xi_2}\right)' - \Delta_{21}T_1 - \Delta_{22}T_2 - \Delta_{23}T_3 = \Delta_{20}; \\ \left(\dfrac{T_3'}{\xi_3}\right)' - \Delta_{31}T_1 - \Delta_{32}T_2 - \Delta_{33}T_3 = \Delta_{30}, \end{cases} \tag{1}$$

where ξ_i is the shear rigidity coefficient of the *i*-seam (joint); T_i is total shearing force (thrust) in the *i*-seam (joint), which is accumulated over the rod length from its heading to the considered section; $T_i = \int_0^x \tau_i \, dx$, where τ_i is shearing force (thrust) per length unit of the *i*-seam (joint); x is the coordinate of the considered section.

The coefficient Δ_{ij} is determined as follows:

$$\begin{gathered} \Delta_{10} = \frac{M^0 c_1}{\sum EJ} = \frac{1}{R}c_1, \;\; \Delta_{20} = \frac{1}{R}c_2, \;\; \Delta_{30} = \frac{1}{R}c_3, \\ \Delta_{11} = \frac{1}{E_2F_2} + \frac{1}{E_1F_1} + \frac{c_1^2}{\sum EJ}, \\ \Delta_{22} = \frac{1}{E_3F_3} + \frac{1}{E_2F_2} + \frac{c_2^2}{\sum EJ}, \\ \Delta_{33} = \frac{1}{E_4F_4} + \frac{1}{E_3F_3} + \frac{c_3^2}{\sum EJ}, \\ \Delta_{12} = \Delta_{21} = -\frac{1}{E_2F_2} + \frac{c_1c_2}{\sum EJ}, \;\; \Delta_{23} = \Delta_{32} = -\frac{1}{E_3F_3} + \frac{c_2c_3}{\sum EJ}, \\ \Delta_{13} = \Delta_{31} = \frac{c_1c_3}{\sum EJ}, \end{gathered} \tag{2}$$

where c_i is the distance between the centers of gravity of the compound rod elements connected with each other by the *i*-seam (joint).

Taking into account that $E_1F_1 = E_kF_k$, and the tensile rigidity of the current-carrying conductors are identical and equal to E_tF_t for three and four and four-core cables (taking into account that $c_1 = c_3$), let us get the formula for defining the coefficient Δ_{ij} and the distance c_i (table 1, 2).

Table 1. Formulas for determining Δ_{ij} coefficients.

Coefficients	For three-core cable	For four-core cable
Δ_{10}	$\frac{M^0 c_1}{\sum EJ} = \frac{1}{R}c_1$	$\frac{M^0 c_1}{\sum EJ} = \frac{1}{R}c_1$
Δ_{20}	$\frac{1}{R}c_2$	$\frac{1}{R}c_2$

Δ_{30}	$\frac{1}{R}c_3$	$\frac{M^0 c_1}{\sum EJ} = \frac{1}{R}c_1$
Δ_{11}	$\frac{1}{E_t F_t} + \frac{1}{E_k F_k} + \frac{c_1^2}{\sum EJ}$	$\frac{2}{E_t F_t} + \frac{1}{E_k F_k} + \frac{c_1^2}{\sum EJ}$
Δ_{22}	$\frac{3}{E_t F_t} + \frac{c_2^2}{\sum EJ}$	$\frac{4}{E_t F_t} + \frac{c_2^2}{\sum EJ}$
Δ_{33}	$\frac{1}{E_k F_k} + \frac{2}{E_t F_t} + \frac{c_3^2}{\sum EJ}$	$\frac{2}{E_t F_t} + \frac{1}{E_k F_k} + \frac{c_1^2}{\sum EJ}$
$\Delta_{12} = \Delta_{21}$	$-\frac{1}{E_t F_t} + \frac{c_1 c_2}{\sum EJ}$	$-\frac{1}{E_t F_t} + \frac{c_1 c_2}{\sum EJ}$
$\Delta_{23} = \Delta_{32}$	$-\frac{2}{E_t F_t} + \frac{c_2 c_3}{\sum EJ}$	$-\frac{1}{E_t F_t} + \frac{c_1 c_2}{\sum EJ}$
$\Delta_{13} = \Delta_{31}$	$\frac{c_1 c_3}{\sum EJ}$	$\frac{c_1^2}{\sum EJ}$

Table 2. The distance between centers of gravity of a compound rod.

Distance c_i	For three-core cable	For four-core cable
c_1	$\frac{4}{3\pi} \cdot \frac{r_k^3 - \left(\frac{2}{\sqrt{3}}+1\right)^3 r_t^3}{r_k^2 - \left(\frac{2}{\sqrt{3}}+1\right)^2 r_t^2} - \frac{2r_t}{\sqrt{3}}$	$\frac{4}{3\pi} \cdot \frac{r_k^3 - \left(\sqrt{2}+1\right)^3 r_t^3}{r_k^2 - \left(\sqrt{2}+1\right)^2 r_t^2} - r_t$
c_2	$r_t\sqrt{3}$	$2r_t$
c_3	$\frac{4}{3\pi} \cdot \frac{r_k^3 - \left(\frac{2}{\sqrt{3}}+1\right)^3 r_t^3}{r_k^2 - \left(\frac{2}{\sqrt{3}}+1\right)^2 r_t^2} - \frac{r_t}{\sqrt{3}}$	$\frac{4}{3\pi} \cdot \frac{r_k^3 - \left(\sqrt{2}+1\right)^3 r_t^3}{r_k^2 - \left(\sqrt{2}+1\right)^2 r_t^2} - r_t$

The cross-sectional areas of the compound rod elements are as follows:

$$F_k = \frac{\pi}{2}\left(r_k^2 - r_t^2\right);$$

$$F_t = \pi r_t^2.$$

The moment of semi cylinder inertia of the outer insulation of a three-core cable can be represented as:

$$J_k = 0{,}11\left(r_k^4 - \left(\frac{2}{\sqrt{3}}+1\right)^4 r_t^4\right) - 0{,}238 r_k^2 r_t^2 \left(\frac{2}{\sqrt{3}}+1\right)^2 \frac{r_k - r_t\left(\frac{2}{\sqrt{3}}+1\right)}{r_k + r_t\left(\frac{2}{\sqrt{3}}+1\right)}.$$

The moment of semi cylinder inertia of the outer insulation of a four-core cable is as follows:

$$J_k = 0{,}11\left(r_k^4 - \left(\sqrt{2}+1\right)^4 r_t^4\right) - 0{,}238 r_k^2\left(\sqrt{2}+1\right)^2 r_t^2 \frac{r_k - \left(\sqrt{2}+1\right) r_t}{r_k + \left(\sqrt{2}+1\right) r_t}.$$

The moment of inertia of a current-carrying conductor (core) can be calculated as:

$$J_t = \frac{\pi}{4} r_t^4.$$

The total bending rigidity of a three-core cable is:

$$\sum EJ = 2E_k J_k + 3E_t J_t.$$

The total bending stiffness (flexural rigidity) of a four-core cable is relatively:

$$\sum EJ = 2E_k J_k + 4E_t J_t.$$

The combined equation of equilibrium (1) can be approximately brought to one equation based on the principle of minimal potential energy of internal forces. The condition of minimal potential energy can be represented as follows:

$$T'' - \lambda_0^2 T = \Delta_0, \tag{3}$$

where T is some force which does not depend on the number of the seam (joint), when

$$T_i = \alpha_i T, \quad (i = 1, \ldots, n); \tag{4}$$

$$\lambda_0 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} \Delta_{ij}\alpha_i\alpha_j}{\sum_{i=1}^{n} \alpha_i^2 / \xi_i}; \quad \Delta_0 = \frac{\sum_{i=1}^{n} \Delta_{i0}\alpha_i}{\sum_{i=1}^{n} \alpha_i^2 / \xi_i},$$

where n is the number of seams (joints) of the compound rod (in this case $n = 3$).

It is reasonable to assume that $\alpha_i \approx c_i$, since the shearing force (thrust) of the seams (joints), which is ξ_i determined by the area of core contact and outer insulation, is relatively small. The solution of this equation (3) is as follows:

$$T = A\,\mathrm{sh}\,\lambda_0 x + B\,\mathrm{ch}\,\lambda_0 x + \frac{1}{\lambda}\int_0^x \Delta_0\,\mathrm{sh}\,\lambda_0 (x-t)\,dt = A\,\mathrm{sh}\,\lambda_0 x + B\,\mathrm{ch}\,\lambda_0 x + \frac{1}{\lambda}\Delta_0\left(\mathrm{ch}\,\lambda_0 x - 1\right),$$

then

$$T' = A\lambda_0\,\mathrm{ch}\,\lambda_0 x + B\lambda_0\,\mathrm{sh}\,\lambda_0 x + \Delta_0\,\mathrm{sh}\,\lambda_0 x.$$

The constant coefficients A, B are found from the boundary conditions:

$$x = 0; \quad T' = 0; \quad \Rightarrow A = 0;$$

$$x = l; \quad T = 0; \quad \Rightarrow B = \frac{\Delta_0\left(1 - \mathrm{ch}\,\lambda_0 l\right)}{\lambda_0\,\mathrm{ch}\,\lambda_0 l}.$$

Finally, there is:

$$T = \frac{\Delta_0\left(1 - \mathrm{ch}\,\lambda_0 l\right)}{\lambda_0\,\mathrm{ch}\,\lambda_0 l}\mathrm{ch}\,\lambda_0 x + \frac{1}{\lambda}\Delta_0\left(\mathrm{ch}\,\lambda_0 x - 1\right).$$

Eventually, the forces (thrusts) T_i are defined as follows:

$$T_1 = c_1\left(\frac{\Delta_0\left(1-\operatorname{ch}\lambda_0 l\right)}{\lambda_0 \operatorname{ch}\lambda_0 l}\operatorname{ch}\lambda_0 x+\frac{1}{\lambda}\Delta_0\left(\operatorname{ch}\lambda_0 x-1\right)\right);$$
$$T_2 = c_2\left(\frac{\Delta_0\left(1-\operatorname{ch}\lambda_0 l\right)}{\lambda_0 \operatorname{ch}\lambda_0 l}\operatorname{ch}\lambda_0 x+\frac{1}{\lambda}\Delta_0\left(\operatorname{ch}\lambda_0 x-1\right)\right); \quad (5)$$
$$T_3 = c_3\left(\frac{\Delta_0\left(1-\operatorname{ch}\lambda_0 l\right)}{\lambda_0 \operatorname{ch}\lambda_0 l}\operatorname{ch}\lambda_0 x+\frac{1}{\lambda}\Delta_0\left(\operatorname{ch}\lambda_0 x-1\right)\right).$$

Discussion

The defined shearing forces (thrusts) are the friction force between the cable elements. The mechanism of cyclical deterioration of insulation layers can be implemented if a) the above considered mechanism of cable bending takes place with low intensity; b) current-carrying conductors (cores) experience free cyclical shifts interacting with each other by means of the forces of sliding friction. Since the law of shearing (shift) distribution alongside the contact surfaces within the length of deformation zone has a zone of extreme shearing (shift), the mechanism of cyclical tear and wear allocates just in this zone [8].

Cyclical deterioration leads to the failure caused by a short circuit between conductors due to wear-out of the double layer of insulation of the adjacent current-carrying conductors. The parameters, which define the operability and efficiency of the cycle, are the amplitude of shearing (shifts) and shearing stress on the contact surface, which in turn depends on the material frictional properties, insulation layers and forces of normal interaction.

The proposed solution is simplified. Therefore, the results of calculation using these formulas should be considered as estimating (evaluating) and require more accurate definition, for example, applying numerical methods.

On the basis of the obtained formulas (5), it is possible to claim that shearing forces in cables depend on element materials, geometrical parameters, the bending radius and the length of the cable deformation area.

The obtained results of calculations allow assessing (estimating) the shearing forces (shearing stresses) on the contact surface of the cable elements. The reliability assessment of insulation and its mechanical strength is carried out by comparison of the real values of stresses for contacting elements with the assumed values for cable materials [9 - 12]. This is a necessary prerequisite for prognostication of operability, efficiency and mechanical strength, taking into account the contact interaction of cable construction elements. The advantages of the proposed solutions are in their simplicity and the possibility to use them for more complicated cable constructions.

Conclusion

The expressions (5) were obtained to assess the value of shearing forces in three-core and four-core cables originated by their reeling. The calculations were carried out based on the representation of multiple-core cable as a compound rod with completely rigid cross binding. The approximate solution was obtained through the substitution of the differential equation system for a multilayer rod by one equation of minimum potential energy**.** The obtained solution is not applied to the fields close to the cable ends, which are equal to five diameters of the rod.

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