

Simulation of car movement along circular path

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Abstract. Under operating conditions, suspension system performance changes which negatively affects vehicle stability and handling. The paper aims to simulate the impact of changes in suspension system performance on vehicle stability and handling. **Methods.** The paper describes monitoring of suspension system performance, testing of vehicle stability and handling, analyzes methods of suspension system performance monitoring under operating conditions. The mathematical model of a car movement along a circular path was developed. Mathematical tools describing a circular movement of a vehicle along a horizontal road were developed. Turning car movements were simulated. Calculation and experiment results were compared. Simulation proves the applicability of a mathematical model for assessment of the impact of suspension system performance on vehicle stability and handling.

1. Introduction

Monitoring of vehicle stability and handling performance under operating conditions is a topical research issue as it impacts crash rates. Vehicle stability and handling depend on suspension system performance. The paper deals with development of an efficient method to monitor the performance of the suspension system of a motor vehicle under operating conditions.

At present, the stability of a car is assessed by performance of brake and steering systems and tyres rather than by suspension system performance [1].

Technical requirements and methods of testing for stability and handling are specified in Government Standard 31507-2012 [2]. It covers six types of tests:

1. Roll-over test (assessment of static stability and sprung mass roll angles);
2. Steering hitch (assessment of roadholding ability);
3. Turning (assessment of maximum velocity);
4. Moose test (assessment of maximum velocity);
5. Direct motion (assessment of average angular velocity of correcting steering inputs);
6. Running (assessment of acceptable velocity under operating conditions).

According to the Technical Rules, all these tests are carried out only on vehicles which are not in circulation (chassis). The tests give satisfactory assessment results for vehicle stability and handling. However under real operating conditions, it is difficult to carry them out because of increased testing risks, demanding requirements for test engineers and test drivers, and need for special-purpose equipment and testing tracks.



2. Materials and methods

The researchers have been attempting to solve the issue of suspension system performance for a long time. For example, they developed a method for determining a lateral response of wheels when monitoring the performance of the suspension system on vibro-test benches [3]. The evaluation criterion is a lateral tyre grip rate which is minimum relation of the lateral response to the normal static one. However, this method does not link test bench and road test parameters. So, it needs to be improved.

Suspension system performance can be evaluated on test benches and by safe road tests. One of them is analysis of angular variations of a vehicle body and path when moving along the circular path and crossing a single obstruction [4]. To develop the method, one need reliable mathematical tools to determine angular variations of a body and a path of the vehicle for variable parameters of suspension system performance.

Mathematical tools are based on generally accepted equations of classical mechanics and automobile theory describing spatial movements of sprung and unsprung masses of the vehicle [5] as well as interaction of elastic tyres with a support surface [6, 7].

At the first stage, let us construct diagrams of vehicle movement along a circular path on a flat level surface. The diagrams are a five-mass model of the vehicle projected in three planes: XOZ , XOY and ZOY (Fig. 1-3).

Absolute coordinate system $X_1; Y_1; Z_1$ is closely connected with a support road surface. Moving coordinate system $X; Y; Z$ is closely connected with a sprung mass of the vehicle in such a way that its origin aligns with the sprung mass center.

Thus, the position of the sprung mass of the vehicle in space is determined by coordinates X_1 and Y_1 of its mass center as well as the turning angle γ of the whole mass of the vehicle about the z-axis (a course angle). Sprung mass can produce longitudinal vibrations along the z-axis and angular vibrations along the x-axis (a roll angle β) and y-axis (a trim angle α).

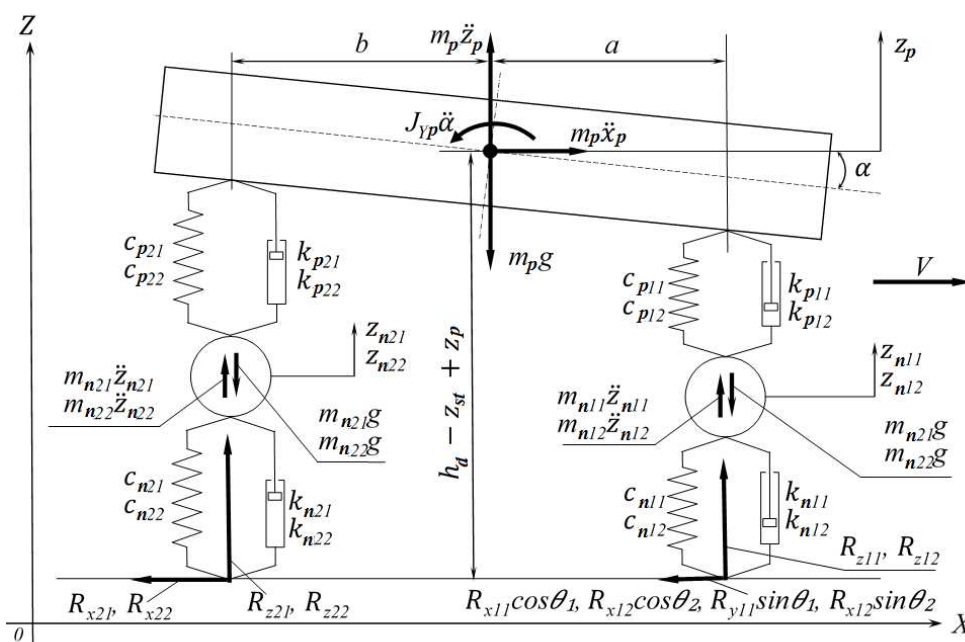


Figure 1. Diagram of vehicle movement along a circular path (X0Z plane)

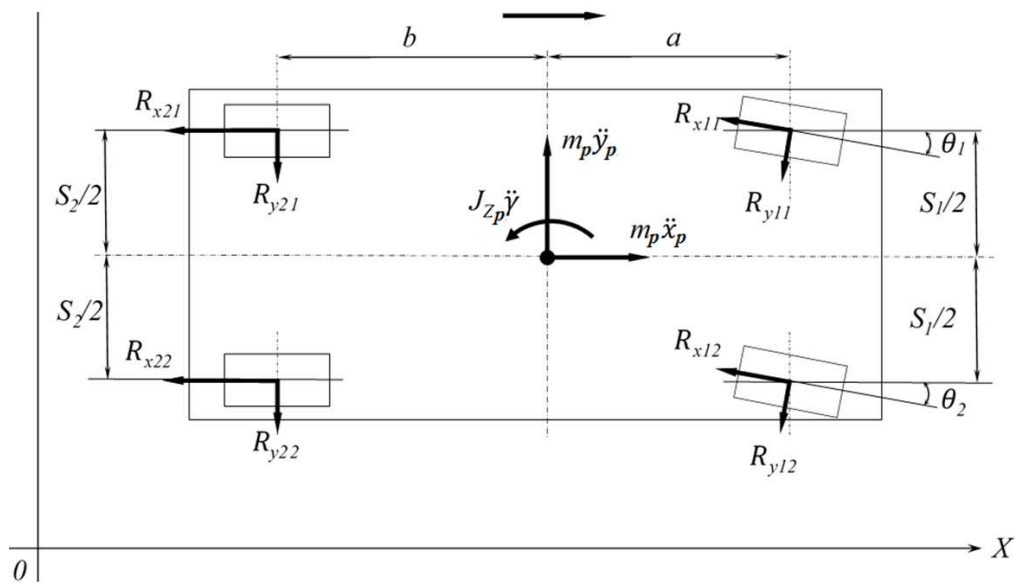


Figure 2. Diagram of vehicle movement along a circular path (XOY plane)

Let us assume that

1. Sprung mass of the vehicle is a solid body which is symmetrical in a longitudinal plane;
2. Mass center and vector of the longitudinal velocity of sprung mass movement are always in a longitudinal plane;
3. Sprung mass of the vehicle vibrates about a point which is in line perpendicular to the support surface and passing through the sprung mass center;
4. When the sprung mass turns about lateral and longitudinal axes, the distance between the mass center and front and rear axles of the car remains constant;
5. Axles of the suspension systems move in planes which are perpendicular to the plane of the car frame;
6. Moments of inertia and angles of turning around wheel spindles are equal to zero;
7. Disbalance and gyrotorque of rotating masses of car transmission and engine are neglected;
8. External effects of lateral and longitudinal road grades and air pressure forces are neglected.

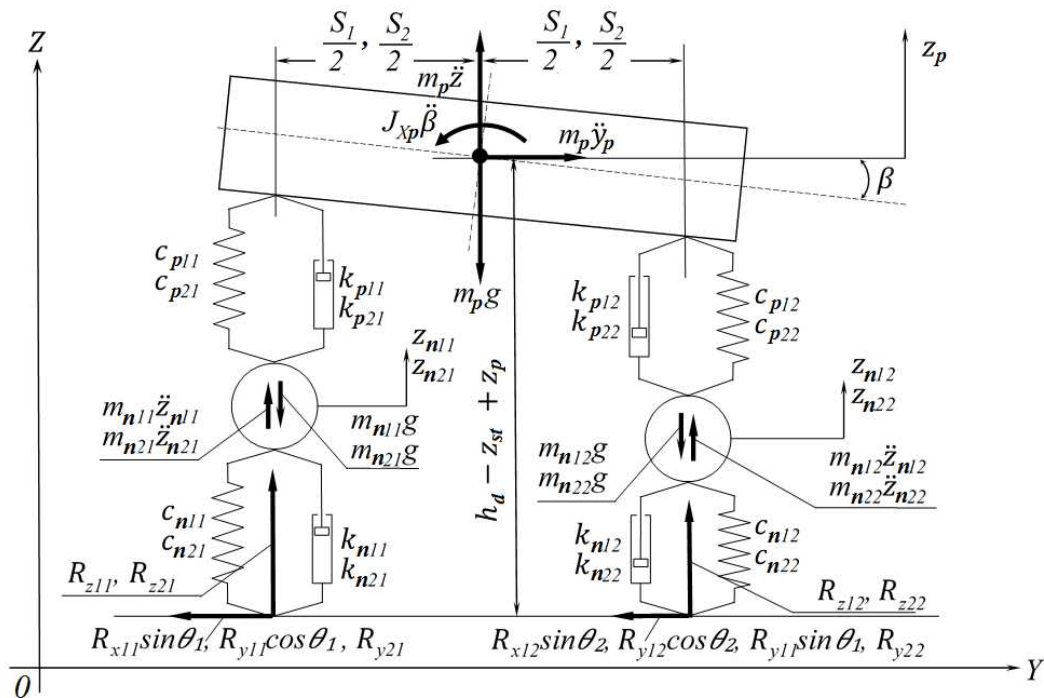


Figure 3. The diagram of vehicle movement along a circular path (ZOY plane)

The body (sprung mass of the vehicle) has six degrees of freedom, while the unsprung masses of the vehicle have only one – about the vertical z -axis.

Based on D’Alambert’s principle, let us create equations of the dynamic balance of sprung (1) and unsprung (2) masses of the vehicle moving along a circular path.

$$\begin{cases} m_p(\ddot{x}_p + \dot{\alpha}\dot{z}_p - \dot{\gamma}\dot{y}_p) = \Sigma F_x \\ m_p(\ddot{y}_p + \dot{\gamma}\dot{x}_p - \dot{\beta}\dot{z}_p) = \Sigma F_y \\ m_p(\ddot{z}_p + \dot{\beta}\dot{y}_p - \dot{\alpha}\dot{x}_p) = \Sigma F_z \\ J_X\ddot{\beta} + (J_Z - J_Y)\dot{\gamma}\dot{\alpha} = \Sigma M_x \\ J_Y\ddot{\alpha} + (J_X - J_Z)\dot{\beta}\dot{\gamma} = \Sigma M_y \\ J_Z\ddot{\gamma} + (J_Y - J_X)\dot{\alpha}\dot{\beta} = \Sigma M_z \end{cases}, \quad (1)$$

$$\begin{cases} m_{n11}\ddot{z}_{n11} = -m_{n11}g + c_{p11}(z_{p11} - z_{n11}) + k_{p11}(\dot{z}_{p11} - \dot{z}_{n11}) - \\ \quad - c_{n11}z_{n11} - \dot{z}_{n11}k_{n11} \\ m_{n12}\ddot{z}_{n12} = -m_{n12}g + c_{p12}(z_{p12} - z_{n12}) + k_{p12}(\dot{z}_{p12} - \dot{z}_{n12}) - \\ \quad - c_{n12}z_{n12} - \dot{z}_{n12}k_{n12} \\ m_{n21}\ddot{z}_{n21} = -m_{n21}g + c_{p21}(z_{p21} - z_{n21}) + k_{p21}(\dot{z}_{p21} - \dot{z}_{n21}) - \\ \quad - c_{n21}z_{n21} - \dot{z}_{n21}k_{n21} \\ m_{n22}\ddot{z}_{n22} = -m_{n22}g + c_{p22}(z_{p22} - z_{n22}) + k_{p22}(\dot{z}_{p22} - \dot{z}_{n22}) - \\ \quad - c_{n22}z_{n22} - \dot{z}_{n22}k_{n22} \end{cases}, \quad (2)$$

where m_p is a measure of inertia of the sprung mass; $\dot{x}_p, \dot{y}_p, \dot{z}_p$ and $\ddot{x}_p, \ddot{y}_p, \ddot{z}_p$ are projections of vectors of velocity and acceleration of the sprung mass in a moving coordinate system; $\dot{\alpha}, \dot{\beta}, \dot{\gamma}$ and $\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}$ are angular velocity and acceleration of the sprung mass; J_X, J_Y, J_Z are axial moments of inertia of the vehicle body; $\Sigma F_x, \Sigma F_y, \Sigma F_z$ are sums of the projection of forces in the axes of a moving coordinate system determined by expressions (3-5); $\Sigma M_x, \Sigma M_y, \Sigma M_z$ are sums of rotating moments

about the axes of a moving coordinate (6-8); m_{nij} is a measure of inertia of the unsprung mass (hereinafter $i=1$ – front axle, $i=2$ – rear axle, $j=1$ – left wheel, $j=2$ – right wheel); c_{pij} is a suspension stiffness; k_{pij} is an absorber damping rate; c_{nij} is a tire stiffness; k_{nij} is a tire damping rate; z_{pij} are vertical coordinates of the sprung mass above the point of contact of the wheel with a support surface determined by equations (9-12); \dot{z}_{pij} is a vertical velocity of the sprung mass; z_{nij} , \dot{z}_{nij} are vertical coordinates and vertical velocity of the unsprung mass, g is a gravity acceleration.

$$\Sigma F_x = R_{x11}\cos\theta_1 + R_{x12}\cos\theta_2 - R_{y11}\sin\theta_1 - R_{y12}\sin\theta_2 + R_{x21} + R_{x22} \quad (3)$$

$$\Sigma F_y = R_{y11}\cos\theta_1 + R_{y12}\cos\theta_2 + R_{x11}\sin\theta_1 + R_{x12}\sin\theta_2 + R_{y21} + R_{y22} \quad (4)$$

$$\begin{aligned} \Sigma F_z = & -m_p g + c_{p11}(z_{n11} - z_{p11}) + k_{p11}(\dot{z}_{n11} - \dot{z}_{p11}) + c_{p12}(z_{n12} - z_{p12}) + \\ & + k_{p12}(\dot{z}_{n12} - \dot{z}_{p12}) + c_{p21}(z_{n21} - z_{p21}) + k_{p21}(\dot{z}_{n21} - \dot{z}_{p21}) + \\ & + c_{p22}(z_{n22} - z_{p22}) + k_{p22}(\dot{z}_{n22} - \dot{z}_{p22}) \end{aligned} \quad (5)$$

$$\begin{aligned} \Sigma M_x = & \left((c_{p12}(z_{n12} - z_{p12}) + k_{p12}(\dot{z}_{n12} - \dot{z}_{p12})) - \right. \\ & \left. - (c_{p11}(z_{n11} - z_{p11}) + k_{p11}(\dot{z}_{n11} - \dot{z}_{p11})) \right) \frac{S_1}{2} + \\ & + \left((c_{p22}(z_{n22} - z_{p22}) + k_{p22}(\dot{z}_{n22} - \dot{z}_{p22})) - \right. \\ & \left. - (c_{p21}(z_{n21} - z_{p21}) + k_{p21}(\dot{z}_{n21} - \dot{z}_{p21})) \right) \frac{S_2}{2} - \\ & - (R_{x11}\sin\theta_1 + R_{y11}\cos\theta_1 + R_{x12}\sin\theta_2 + R_{y12}\cos\theta_2 + R_{y21} + \\ & + R_{y22})(h_d - z_{st} + z_p) \end{aligned} \quad (6)$$

$$\begin{aligned} \Sigma M_y = & \left((c_{p11}(z_{n11} - z_{p11}) + k_{p11}(\dot{z}_{n11} - \dot{z}_{p11})) + \right. \\ & + (c_{p12}(z_{n12} - z_{p12}) + k_{p12}(\dot{z}_{n12} - \dot{z}_{p12})) \left. \right) a - \\ & - \left((c_{p21}(z_{n21} - z_{p21}) + k_{p21}(\dot{z}_{n21} - \dot{z}_{p21})) - \right. \\ & - (c_{p22}(z_{n22} - z_{p22}) + k_{p22}(\dot{z}_{n22} - \dot{z}_{p22})) \left. \right) b + \\ & + (R_{x11}\cos\theta_1 - R_{y11}\sin\theta_1 + R_{x12}\cos\theta_2 - R_{y12}\sin\theta_2 + R_{x21} + \\ & + R_{x22})(h_d - z_{st} + z_p) \end{aligned} \quad (7)$$

$$\begin{aligned} \Sigma M_z = & (R_{x11}\sin\theta_1 + R_{y11}\cos\theta_1 + R_{x12}\sin\theta_2 + R_{y12}\cos\theta_2)a - (R_{y21} + R_{y22})b + \\ & + (R_{x12}\cos\theta_2 + R_{y11}\sin\theta_1 - R_{x11}\cos\theta_1 - R_{y12}\sin\theta_2) \frac{S_1}{2} + \\ & + (R_{x22} - R_{x21}) \frac{S_2}{2} \end{aligned} \quad (8)$$

$$z_{p11} = z_p + a \sin \alpha - \frac{S_1}{2} \sin \beta, \quad (9)$$

$$z_{p12} = z_p + a \sin \alpha + \frac{S_1}{2} \sin \beta, \quad (10)$$

$$z_{p21} = z_p - a \sin \alpha - \frac{S_2}{2} \sin \beta, \quad (11)$$

$$z_{p22} = z_p - a \sin \alpha + \frac{S_2}{2} \sin \beta . \quad (12)$$

In equations (3-12): R_{xij} , R_{yij} are lateral and longitudinal responses of wheels; θ_1 is a turning angle of the front near-side wheel; θ_2 is a turning angle of the front outside wheel; a and b are distances between front and rear axes and the mass center; S_1 and S_2 are front and rear tracks, α and β are turning angles of the sprung mass about x-axis and y-axis correspondingly.

The values of lateral and longitudinal responses R_{xij} and R_{yij} are determined by a well-known method [4] using a normalized slip function:

$$f(s) = \sin[a \cdot \arctg(b \cdot S)] \quad (13)$$

They can be presented by the following functionals:

$$\begin{cases} R_{xij} = f_1(F_{zij}, \delta_{ij}, \omega_{ij}, \dot{x}_p, C_{sx}, C_{sy}, f_{bx}, f_{by}, n_{sx}, f(s), r_{k0}, \varphi_{max}, S_x) \\ R_{yij} = f_2(F_{zij}, \delta_{ij}, \omega_{ij}, \dot{x}_p, C_{sx}, C_{sy}, f_{bx}, f_{by}, n_{sx}, f(s), r_{k0}, \varphi_{max}, S_y) \end{cases} \quad (14)$$

where F_{zij} is a normal wheel load; U_{ij} δ_{ij} are elastic rubber tyred wheel slip angles; ω_{ij} are rotation velocities of wheels; C_{sx} is a stiffness rate of longitudinal slip; C_{sy} is a stiffness rate of lateral slip; f_{bx} is a longitudinal cohesion reduction rate; f_{by} is a lateral cohesion reduction rate; n_{sx} is a slip stiffness rate; $f(s)$ is a normalized slip function; r_{k0} is a force radius – a wheel rolling radius under driven conditions, φ_{max} is a maximum value of the tire grip rate; S_x and S_y are longitudinal and lateral slips of tires.

Velocities of movement of the sprung masses V_x and V_y about the axes of fixed coordinates $X_1; Y_1; Z_1$ are determined by equations (14, 15):

$$V_x = \dot{x}_p \cdot \cos \gamma - \dot{y}_p \cdot \sin \gamma , \quad (15)$$

$$V_y = \dot{x}_p \cdot \sin \gamma + \dot{y}_p \cdot \cos \gamma , \quad (16)$$

where γ is an angle for the sprung mass turning about the Z-axis.

Let us assume: $m_p = 1300$ kg; $J_X = 1800$ kg·m²; $J_Y = 2200$ kg·m²; $J_Z = 2000$ kg·m²; $m_{nij} = 30$ kg; $c_{pij} = 40$ kN/m; $k_{pij} = 2$ kN·sec/m (compression stroke); $k_{pij} = 4$ kN·sec/m (rebound stroke); $c_{n11} = 300$ kN/m; $k_{n11} = 2$ kN·sec/m; $\theta_1 = 0 - 20^\circ$; $a = 1.315$ m; $b = 1.3$ m; $S_1 = 1.489$ m; $S_2 = 1.477$ m; $h_d = 0.47$ m; $C_{sx} = 10.125$; $C_{sy} = 8.125$; $f_{bx} = 0.8$; $f_{by} = 0.8$; $n_{sx} = 10.95$; $f_x = 0.7$; $f_y = 0.7$; $r_{k0} = 0.297$ m.

3. Results and discussion

Having solved the equations by the Euler method, the authors obtained calculation change dependences for a turning angle γ , roll β and trim α on the time of a car movement along a straight-line trajectory and a circular path. The process of a car crossing a single obstruction was simulated. Comparison of calculation and experiment [8] dependences (fig.4-6) shows that the model describes changes in a turning angle, but there are some quantitative differences between calculation and experiment results in roll and trim angles.

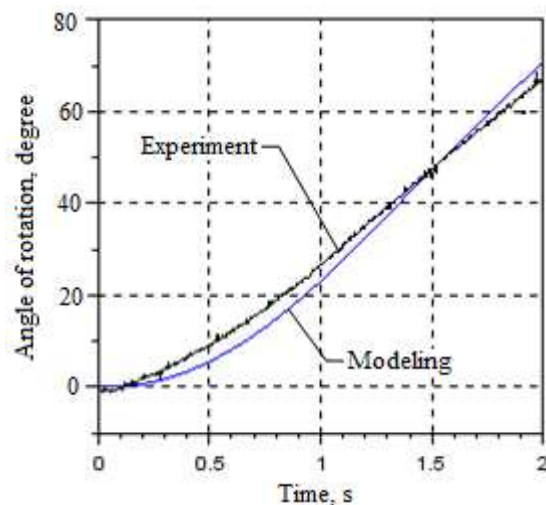


Figure 4. Change in a turning angle of the vehicle

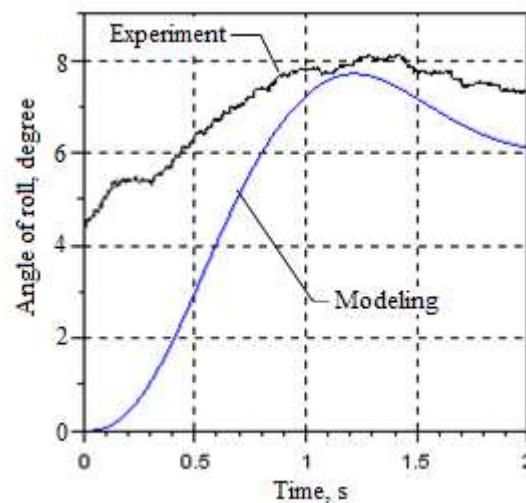


Figure 5. Change in a roll angle of the vehicle

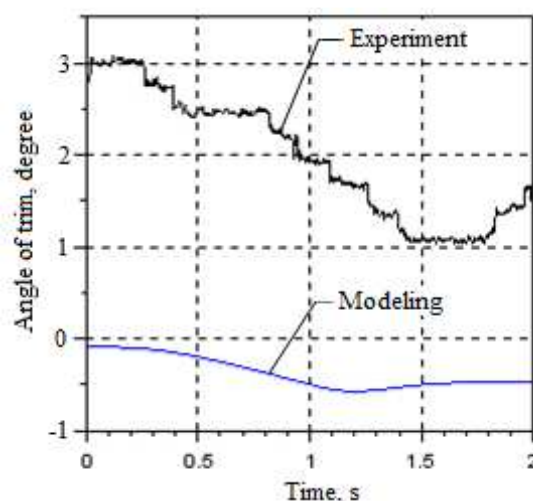


Figure 6. Change in a trim angle of the vehicle

Differences in calculation and experiment results are due to the fact that developed mathematical tools do not take into account all the factors existing in real tests, for example, a slope of roadway, unbalanced loading upon wheels, stabilizer bar operation, differences between mass center coordinates and roll center coordinates. Qualitative calculations of roll and trim angles are the same as the experimental ones [8-10]. Figure 5 shows that in real tests when a vehicle enters a turn, one can observe an increase in rolling for 0-1.4 sec. which later decreases. Calculation simulation also shows an increase in a roll angle for 0-1.2 sec. which later decreases. The trim angle changes in the same way.

4. Conclusion

Despite quantitative differences between calculation and experimental results, the mathematical model describes competently the process of hitting a single road irregularity by a motor vehicle moving along a circular path. The equations describe the contact of a tyre with a road irregularity. An improved model can be used to analyze the impact of suspension system performance on vehicle stability and behavior.

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