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# Transversal vibrations of elastic rod in magnetic field under simultaneous kinematic and force action

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**Abstract**. A model problem of transversal vibrations of an elastic conducting rod in the magnetic field is studied. Vibrations in the rod are excited due to kinematic and force factors. A partial differential equation of motion containing the integral term for the electromagnetic force was constructed. After applying the Fourier procedure, the problem is reduced to a set of ODEs. The condition for passive stabilization of the main vibrational mode's amplitude is derived. A method of active electromagnetic suppression of certain vibrational modes is proposed.

## 1. Introduction

Mechanical vibrations commonly appear in instruments and mechanisms of various purposes [1-3]. Such apparatus needs to be controlled in order to reach certain pre-chosen frequency response. In other cases, the unwanted vibrations have to be controlled and suppressed, or the systems need to be moved out of resonance [4-7]. This type of control in the case of conducting systems (a string or a rod) has some interesting properties. It can be used to suppress or excite certain vibrational modes, to selectively modify certain groups of partial frequencies, to affect nonlinear properties of the system or create conditions for parametric resonance.

The goal of the present work is to determine the conditions for passive and active electromagnetic suppression of transversal vibrations of the elastic rod, excited simultaneously by force and kinematic motion.

## 2. Results

Let us consider linear transversal vibrations of a homogeneous conducting rod made of non-magnetic material with its both ends attached to the base which is vibrating according to the law of motion $x = x(t)$. The ends of the rod are connected by an ideal electric circuit. The rod is placed in a homogeneous magnetic field which is stationary in the considered reference frame, with its $\vec{B}$ -vector directed along the $Oy$ axis (Fig. 1). The magnets that create the field rest on the same base as the rod. Reference frame $Oxyz$ connected with the base is non-inertial. Apart from the kinematic excitation, the rod is subject to the force excitation due to distributed load:

$$Q(z,t) = f(z)g(t),$$

where $f(z)$ and $g(t)$ are known functions.

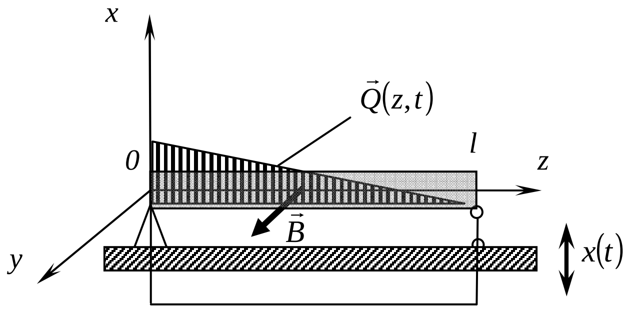


**Figure 1.** Scheme of vibrational system.

Let us construct the differential equation for the transversal vibrations of the rod introducing the following notations: $u(z,t)$ is the displacement function, $EJ$ is the stiffness of the rod, $\beta$ and $\beta^*$ are the coefficients of external and internal dissipation, respectively, $m_0$, $l$, and$A$ are linear mass density, length and cross-section of the rod, respectively, $\sigma$ is the conductivity of the rod, $B$ is the magnetic field:

$$EJ\left(\frac{\partial^4 u}{\partial z^4} + \beta^* \frac{\partial^5 u}{\partial z^4 \partial t}\right) + m_0\left(\frac{\partial^2 u}{\partial t^2} + \beta \frac{\partial u}{\partial t}\right) + \frac{\sigma B^2 A}{l}\int_0^l \frac{\partial u}{\partial t} dz = -m_0 \frac{\partial^2 x}{\partial t^2} + Q(z,t). \tag{1}$$

The last term in the left-hand side of (1) describes the electromagnetic force that appears when the conducting rod is moving in the external magnetic field. Let us apply the Fourier procedure to (1), expanding thedisplacement function into a series of amplitude eigenfunctions $X_r(z)$, using the generalized coordinates $q_r(t)$. The resulting system of ODEs reads:

$$\ddot{q}_r + \left(\beta + \beta^* p_r^2\right)\dot{q}_r + p_r^2 q_r = \frac{Q_r^* m_0}{m_r^*} - \frac{\Phi_r m_0}{m_r^*} - \frac{Q_r}{m_r^*}, \quad \{r = 1,2,...\}. \tag{2}$$

Here,

$$Q_r^* = g(t)\int_0^l X_r f(z)dz = g(t)\delta_r^*$$

are generalized forces of the distributed load, with

$$\delta_r^* = \int_0^l X_r(z) f(z) dz; \quad m_r^* = m_0 \int_0^l X_r^2 dz = m_0 Y_r^*$$

being generalized masses, with

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$$Y_r{*} = \int_0^l X_r^2 dz; \qquad Q_r = -\frac{\sigma B^2 A}{l} \int_0^l X_r dz \cdot \sum_{n=1}^{\infty} \left( \dot{q}_n \int_0^l X_n dz \right) = -\frac{\sigma B^2 A \gamma_r}{l \alpha_r} \cdot \sum_{n=1}^{\infty} \left( \dot{q}_n \frac{\gamma_n}{\alpha_n} \right)$$

being generalized electromagnetic forces, with

$$\gamma_n = \int_0^l X_n dz, \qquad \gamma_r = \int_0^l X_r dz; \; \alpha_n = \sqrt[4]{\frac{p_n^2 m_0}{EJ}}$$

being the wave numbers, $p_n$ are partial eigenfrequencies.

$$\Phi_r = \ddot{x} \int_0^l X_r dz = \ddot{x} \gamma_r$$

are generalized inertial forces.

Equations (2) can be rewritten in the dimensionless form:

$$\ddot{q}_r + \left(\beta + \beta{*} p_r^2\right) \dot{q}_r + p_r^2 q_r = \frac{g(t) \delta_r^*}{Y_r{*}} - \frac{\Phi_r \gamma_r}{Y_r{*}} - \boldsymbol{St} \frac{\gamma_r}{\alpha_r Y_r{*}} \sum_{n=1}^{\infty} \frac{\gamma_n}{\alpha_n} \dot{q}_n, \{r = 1,2,...\}. \qquad (3)$$

Here, $\boldsymbol{St} = \dfrac{\sigma B^2}{\rho p_1}$ is the Stuart number, $\rho$ is the volumetric density of the rod, $p_1$ is the principal vibrational frequency.

Analysis of (3) leads to several conclusions:

1) modes that satisfy condition $\gamma_r = 0$ are isolated from the action of electromagnetic field and kinematic excitation; 2) $r$-th partial vibration is not affected by the distributed load if $\delta_r^* = 0$.

Electromagnetic force can be controlled by varying the width of the active area. The problem can be generalized by assuming that the field acts on interval $\Delta z = z_2 - z_1$, and load $Q(z,t)$ is distributed over interval $\Delta' z = z_2' - z_1'$. In this case the problem is reduced to the system of ODEs:

$$\ddot{q}_r + \left(\beta + \beta{*} p_r^2\right) \dot{q}_r + p_r^2 q_r = \frac{Q_r^* m_0}{m_r^*} - \frac{\Phi_r m_0}{m_r^*} - \frac{Q_r}{m_r^*}, \; \{r = 1,2,...\}, \qquad (4)$$

where $Q_r^* = g(t) \int_{z_1'}^{z_2'} X_r(z) f(z) dz$ are generalized forces of the distributed load. In dimensionless form, the system becomes:

$$\ddot{q}_r + \left(\beta + \beta{*} p_r^2\right) \dot{q}_r + p_r^2 q_r = \frac{g(t) \delta_r'^*}{Y_r{*}} - \frac{\Phi_r \gamma_r}{Y_r{*}} - \boldsymbol{St} \frac{\gamma_r^*}{\alpha_r Y_r{*}} \sum_{n=1}^{\infty} \frac{\gamma_n^*}{\alpha_n} \dot{q}_n, \qquad \{r = 1,2,...\}. \qquad (5)$$

with the following notations:

$$Q_r = -\frac{\sigma B^2 A}{l} \int_{z_1}^{z_2} X_r dz \cdot \sum_{n=1}^{\infty} \left( \dot{q}_n \int_{z_1}^{z_2} X_n dz \right) = -\frac{\sigma B^2 A \gamma_r^*}{l \alpha_r} \cdot \sum_{n=1}^{\infty} \left( \dot{q}_n \frac{\gamma_n}{\alpha_n} \right)$$

being generalized electromagnetic forces, with

$$\gamma_n = \int_0^l X_n dz\,, \quad \text{and } \gamma_r^{\,*} = \int_{z_1}^{z_2} X_r dz\,; \quad \Phi_r = \ddot{x}\int_0^l X_r dz = \ddot{x}\gamma_r\,; \quad Q_r^* = g(t)\int_{z_1'}^{z_1'} X_r(z) f(z) dz = q(t)\delta_r'^*$$

being generalized forces from the distributed load, with $\delta_r'^* = \int_{z_1'}^{z_2'} X_r(z) f((z) dz$.

Careful analysis of (5), shows the following:

1) there is a special set of modes for which $\gamma_r = 0$ and which are not excited kinematically;
2) the modes for which $\gamma_r^* = 0$ are not susceptible to electromagnetic field;
3) condition $\delta_r'^* = 0$ defines the group of modes that are not affected by the force coming from the distributed load.

## 3. Discussion

In general, all three sets of modes defined this way are different. When analyzing a given problem, it is important to determine the intersection of these sets. Let us assume, for example, that $\delta_1'^* \neq 0$, $\gamma_1 \neq 0$, $\gamma_1^{\,*} \neq 0$ for the principal partial vibration, i.e. all the force factors are in play simultaneously. In this case there is a possibility to stabilize the amplitude of vibrations by applying electromagnetic field. Invoking the method of energy balance, let us determine the Stuart number for which the energy of kinematic and force excitation is compensated by dissipation. The principal vibration in the dimensionless form reads:

$$q_1 = c_1 \sin t, \tag{6}$$

where $c_1$ is its steady-state amplitude. For two fixed points, the corresponding amplitude eigenfunction has the form:

$$X_1(\alpha_1 z) = \sin\frac{\pi}{l} z\,. \tag{7}$$

In a one-mode approximation the principal vibration is described by the following ODE:

$$\ddot{q}_1 + \left(\beta + \beta^* + \boldsymbol{St}\frac{\gamma_1^{2^*}}{\alpha_1^2 Y_1^*}\right)\dot{q}_1 + q_1 = \frac{g(t)\delta_1'^*}{Y_1^*} - \frac{\Phi_1\gamma_1}{Y_1^*}\,. \tag{8}$$

After equaling the work done by inertial and external forces to the work of dissipative force, one gets:

$$\int_0^{T_1}\left(\beta + \beta^* + \boldsymbol{St}\frac{8\gamma_1^{2^*}}{\pi^2}\right)c_1 \cos t \cdot \sin t dt = \int_0^{T_1}\left(\frac{g(t)\delta_1'^*}{Y_1^*} - \frac{\Phi_1(t)\gamma_1}{Y_1^*}\right)\sin t dt\,, \tag{9}$$

where $T_1 = 2\pi$.

The definite integral in the right-hand side contains the known functions of time. Renaming:

$$N_1 = \int_0^{T_1}\left(\frac{g(t)\delta_1'^*}{Y_1^*} - \frac{\Phi_1(t)\gamma_1}{Y_1^*}\right)\sin t dt \tag{10}$$

and utilizing (9) and (10), one gets the Stuart number that grants passive stabilization of the principal vibration with given amplitude $c_1$:

$$\mathbf{St} = \frac{\alpha_1^2 Y_1^*}{\gamma_1^2}\left(\frac{2N_1}{c_1} - \beta - \beta^*\right). \tag{11}$$

One can see that electromagnetic force is limited by the value of induced electric current. The damping can be increased many times by introducing the source of electromotive force (EMF) into the electric circuit connecting the ends of the rod, which creates the AC current $j(t)$. This current can be generally expressed as a harmonic series with any number of frequencies and initial phases. In particular, function $j(t)$ can be expanded over damped eigenfrequencies. The generalized forces (all or some of them) will then be the same as the corresponding dissipative forces, so increasing the damping of all or some vibrations. In other words, with the help of amplifier with the electronic frequency analyzer, it is possible to increase the induced currents by using external EMF source. In contrast to a passive magnetic stabilization, the proposed method of electromagnetic damping can conveniently be called an active one.

## 4.Conclusion

The obtained results allow construction of mechatronic system of vibration control for various purposes. They could be applied when the traditional methods of vibration isolation are not sufficient. Industrial mining could serve as an example of such situation, when in addition to significant ground vibrations (kinematic excitation) there exist sound waves that create force excitation. These factors negatively affect the registration apparatus and thus have to be compensated.

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