

Registration of reactive power for case of distortions in electric grid

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Abstract. Nowadays, virtually everywhere worldwide, the electricity is paid for based on active and reactive power meters. Higher harmonics appeared to influence the electric power meter readings, and this specifically relates to reactive power. Incorrect power meter readings occur due to different so-called “measuring” protection levels in device circuitry in various devices. The paper analyzes reactive power meter readings of devices from various manufacturers. Based on the analysis, the authors introduce a new approach to evaluating the costs in connection to reactive power generation, transmission and consumption, with current/voltage distortions in power supply lines.

1. Introduction

Efficient power usage depends on correct determination of costs for its generation and usage. A significant factor in connection therewith is distinguishing between power generation and power transmission costs.

Electricity is usually paid for with both the active and the reactive components accounted for. Until recently, measuring of electric power was mostly conducted by electro-dynamic electric power meters. Now electronic meters are used. The latter have many advantages, including good stability, accuracy, improved measurement functions and remote measuring features.

Electronic meter operation is based on “measuring” control as it is embedded into the device’s microprocessor for the device network connection defined.

It is the common practice to issue electricity bills based on power meter readings, regardless of the power meter type. The only accuracy that may differ depending on the class of a device is electricity power line with sine currents and voltages [1-3]. Therefore, traditional active and reactive power components in sine modes satisfy both consumers and suppliers from this point of view. Problems begin when the power in the mains gets changed, where both voltage and current comprise higher harmonics. The active power matches the power consumed by load with all harmonics taken into account, and it may be described with the well-known expression:

$$W = \int_0^t p dt = \int_0^t u(t)i(t)dt,$$

where $p = u(t)i(t)$ is the instant power; $u(t)$ and $i(t)$ are, correspondingly, instant values of voltage and current.

Reactive power is something else. So far, reactive power in non-sine periodic modes has not been defined well [4-7].



The authors analyzed readings from three types of meters, with the same measurement parameters. Meters of the first type implement the math expression for reactive power, looking like

$$Q = \frac{1}{T} \int_0^T u(t) i \left(t + \frac{T}{4} \right) dt = Q_1 - P_2 - Q_3 + P_4 + \dots,$$

where $u(t)$, $i(t)$ are instant values of voltage and current, correspondingly; T is the first harmonic period; t is the current time; Q_k , P_j are reactive and active powers, corresponding to the k and the j harmonic.

As for the second type of the meters, the math expression for calculating the power looks like

$$Q = \frac{1}{T} \int_0^T \omega \left(- \int u(t) i(t) dt \right) dt = Q_1 + \frac{Q_2}{2} + \frac{Q_3}{3} \dots$$

As for the third type of the meters, the following math expression is relevant:

$$Q = \left\{ \sum_0^T \left[\frac{nT}{\omega} \int_0^T i_t(t) \left(\int u_n(\tau) d\tau \right) dt \right]^2 \right\}^{\frac{1}{2}} = (Q_1^2 + Q_2^2 + Q_3^2 + \dots)^{\frac{1}{2}},$$

where n is the number of the corresponding current and voltage harmonics.

Determining the law for reading changes for various types of meters depending on the harmonic composition of current and voltage is set to be based on the statement that dependencies would be built under the following conditions:

First current and voltage harmonic remains unchanged and is equal to initial ones.

The change in sequence is the multiplication $U_5 I_5$ under constant $U_7 I_7$ or vice versa. Therewith, the phase difference between current and voltage remains unchanged.

With constant acting values of all harmonic current/voltage components, the change is done in sequence of the fifth harmonic's phase shift angle between voltage and current within the range of 0-90°. The same change is done in sequence of the seventh harmonic's phase shift angle between voltage and current.

The following was concluded based on the results obtained.

It has been revealed that higher harmonics in current/voltage of the power lines visibly affect meter readings; the relative error of measuring with different types of meters, under a particular ratio between harmonics and the phase shift value between voltage and current at the initial harmonic may greatly exceed the allowed error of the device and reach 6.35%. This is a significant factor to be accounted for when paying for electricity.

It has been found that the measurement error of harmonics amplitude exceeds 6%, and this is the strongest influence in connection therewith. Phase difference results in the error of up to 2%.

2. Methods

Following facts are based on the non-symmetric three-phase system operation mode. In the sine mode, it is convenient to use artificial terms of full (S) and reactive (Q) power. Then the value of S is convenient for determining the load current and power coefficient:

$$I = \frac{S}{\sqrt{3}U}; K_{PWR} = \frac{P}{S}.$$

Higher harmonics change the picture radically. Let us obtain the reactive power value on every harmonic: $Q_1 = P_1 \tan \varphi_1$; $Q_2 = P_2 \tan \varphi_2$ and etc. In this case, the actual value of current would be defined in accordance with the following expression:

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots} = \frac{1}{\sqrt{3} \prod_{i=1}^{\infty} U_i} \sqrt{\sum_{i=1}^{\infty} \left(\left(\prod_{j=1, j \neq i}^{\infty} U_j^2 \right) P_i^2 \right) + \sum_{i=1}^{\infty} \left(\prod_{j=1, j \neq i}^{\infty} U_j^2 Q_i^2 \right)}, \quad (1)$$

where U_i and U_j are voltages of the i and j harmonics, P_i and Q_i represent, correspondingly, the active and the reactive power of the i harmonic.

The expression (1) can not be associated with the commonly used one:

$$I = \frac{\sqrt{(\sum_{i=1}^{\infty} P_i)^2 + (\sum_{i=1}^{\infty} Q_i)^2}}{3U_N}, \quad (2)$$

where U_N is the nominal voltage in the mains.

The expression (2) would give an incorrect actual value of current. Therefore, defining reactive power as the sum of its values on every harmonic is useless, carrying no information. Multiple publications stated the same [1, 2]. The method provided below is good for metering and paying for electric power, with higher harmonics presented in the main lines.

Different types of electric power meters are known [1]:

Electromechanical meters that determine the power of only the main harmonics for current and voltage, based on the expression $Q_1 = \sqrt{3}U_1 I_1 \sin \varphi_1$, where Q_1, U_1, I_1, φ_1 represent, correspondingly, reactive power, voltage, current and phase shift at the first harmonic.

Electronic meters determine power in accordance with the expression

$$Q = \frac{1}{T} \int_0^t u(t) i \left(t - \frac{T}{4} \right) dt.$$

Electronic meters determine power in accordance with the expression

$$Q = \frac{1}{T} \int_0^t \omega_1 \left(-i(t) \int_0^t u(t) dt \right) dt.$$

Electronic meters determine reactive power as follows:

$$Q = \sqrt{S^2 - P^2}.$$

It is worth noting that when using any of the above-mentioned meters, payment is made not for the electric energy expended by the supplier (except the first half-period), but for its transmission [8-9].

What does the consumer pay for, paying electricity bills?

The sine current brings electric power, and it can be represented as the sum of active and reactive components:

$$\dot{I} = I_a + jI_p,$$

where I_a is the active and I_p is the reactive current components.

With the constant voltage at the three-phase power network $I_a = \frac{P}{\sqrt{3}U}$;

$$I_p = \frac{Q}{\sqrt{3}U}, \text{ or } I_a = \frac{S}{\sqrt{3}U} \cos \varphi; I_p = \frac{S}{\sqrt{3}U} \sin \varphi.$$

On the other hand, provided the left and the right parts of the expression

$$P + jQ = \dot{S} \quad (3)$$

being divided by $\sqrt{3}U$ (which in this case is just a constant coefficient), then

$$\frac{\dot{S}}{\sqrt{3}U} = \frac{S}{\sqrt{3}U} \cos\varphi + j \frac{S}{\sqrt{3}U} \sin\varphi = I_a + jI_p.$$

Therefore, paying for the value of the expression's first component (3) suggests paying for the active power throughout the active component of the entire current. The error resulting from the voltage changes and the accuracy class of devices should be taken into account. The latter cannot exceed $\pm 5\%$ accuracy boundaries.

The second component in (3) relates to reactive power that is directly proportional to the reactive component of the entire current.

With reactive power being changed, the full current changes not in direct proportion to the reactive power, because $I = \sqrt{I_a^2 + I_p^2}$. Directly proportional to the reactive power, the current might be in a very rare case: if all the load is the reactive one.

Paying for active power is in direct proportion with costs for power generation, but what does payment for reactive power reflect? The costs may only relate to power transmission. Those cost grow to the smaller extent, than the reactive power would be. All this relates to well-established sine mode.

The active power is defined by the expression

$$P = \frac{1}{T} \int_0^T P dt = \frac{1}{T} \int_0^T u(t)i(t)dt = P_1 + P_2 + \dots + P_\infty.$$

Therefore, the active power, i.e. electric power, is defined by the sum of all harmonics components. This is what any system of meter determines. In this case things correspond to the physics of the process of power generation, transmission and consumption.

The full power is defined by the expression

$$S_1 = UI = \sqrt{U_1^2 + U_2^2 + \dots + U_\infty^2} \times \sqrt{I_1^2 + I_2^2 + \dots + I_\infty^2}, \quad (4)$$

and the full power $S_{11} = \sqrt{P^2 + Q^2}$ do not match. Multiple papers disclose it [2,3]. Any reactive power meter may determine the value of Q. But what is the information behind this value? The total current in the mains cannot be calculated based on Q, neither the add-on of the reactive (or non-active) current component. Therefore, in this case the current should be defined by the expression

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots}$$

There is no sense to calculate S_{11} just to compare it to the earlier value of S_1 to find out the so-called distortion power [10-11] $T = \sqrt{S_1^2 - P^2 - Q^2}$, for T only relates to qualitative information about higher harmonics presence. As for quantitative, not only qualitative, information, the current/voltage non-THD coefficients and spectra might be relevant. All artificial coefficients related to distortion power or do not bring no useful information.

The only usable coefficient would be power coefficient

$$K_{PWR} = \frac{P}{S_1},$$

wherein S_1 value should be defined in accordance with (4). This coefficient characterizes active power reserve unused both in sine mode and in any other mode.

3. Results

Apparently, with $S_1^2 - P^2$ equal to zero, higher harmonics and reactive power are absent. Also, if reactive power at each harmonic becomes compensated, the value would be zero, too.

The problem cannot be solved easily; therefore, the presence of higher harmonics would not allow power coefficient to reach the value of 1, except when costly active filters would be used.

With reactive power having no physical sense, using it to charge consumers considering non-sine modes seems incorrect.

Based on the abovementioned, two payment options might be proposed to pay for electricity:

Option 1 – paying by parts – for active power and the full current. Paying for the active power is common practice that corresponds to the physical principles of power transmission. Paying for full current (1) accounts for costs of full power transmission via power lines. Implementing this option would need active power meters and integral current meter.

Option 2 – paying for power the same way as it is done when sine mode is taken into consideration. In this case, full power is to be defined in accordance with (4) to calculate non-active power [12]:

$$M = \sqrt{S_1^2 - P^2}.$$

Then the current component that is not related to the active power would correspond to non-active power M . Therefore, payment would be for the active and the non-active powers. This corresponds to paying for the active and the non-active electric powers in the sine mode.

4. Conclusion

Based on theory and practice studies, electric power meters were found to count the power consumption in an incorrect way, resulting from missing reactive power readings in presence of current/voltage distortions. In addition, the difference was discovered in the readings of metering devices from various manufacturers. Due to certain conventionalism of reactive power value, a method was proposed for counting it by full current or non-active power. The first case suggests paying for consumed power basing on active power and full current values. The second one is based on active and non-active power values.

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