

# Comparing of cogging torque reduction methods in permanent magnet machines with fractional slot windings

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**Abstract.** The results of the investigation of the cogging torque in permanent magnet synchronous machines, which is caused by the stator slotting and the rotor eccentricity, are presented in the paper. A new design of the machine has been developed in the course of the investigation, and the value of the cogging torque in this construction is less considerably compared to other constructions. In contrast to the available methods of the cogging torque reduction, the solution suggested not only decreases the level of the cogging torque but also has negligibly small influence on characteristics of the machine with the rotor eccentricity which is typical of the mass production and long-term usage.

## 1. Introduction

Permanent magnet synchronous machines (PMSM) with fractional slot concentrated windings are used widely in the ever growing number of fields because of their high level of technological effectiveness, power density, low value of inertia and copper consumption and capability to provide high torque. Electrical machines for precision systems and other fields requiring low noises and vibrations must have minimum of the cogging torque. This problem can be solved both by construction improvement [1] and by the use of special control algorithms.

The cogging torque is caused by variation of the magnetic field energy during the rotor rotation according to the interaction of the magnetic field with the varying permeance of the air-gap. The variation of the air-gap permeance can be produced not only by stator slotting but by some manufacturing faults as well. The cogging torque leads to additional noises and vibrations in working modes, moreover it induces magnetic sticking in the no-current state.

Moreover the cogging torque is a strongly parasitic and undesirable effect in such fields like electric power steering [2]. Such drive rotor of the machine is coupled with steering column permanently in every mode. So irrespective of the stator, winding supplying the cogging torque is available and it can be felt on the steering wheel. So a driver takes the sticking and the road sense degrades. Because of that, the cogging torque must be taken into account during the design stage and limited according to an application field. For example, the cogging torque value for electric power steering motor is restricted to 1% and less of maximal electromagnetic torque by most of automotive manufacturers.

The physical origin of the cogging torque in permanent magnet machines should be described before the review of several reduction methods.



## 2. Cogging torque problem

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### 2.1. Cogging torque origins

The investigations [3] show that there are two common reasons of cogging torque in permanent magnet machines in the no-current state. The first one is the interaction between excitation field and harmonics of the air-gap permeance caused by stator slotting. The second reason is the interactive process between excitation field and harmonics of the air-gap permeance due to the rotor eccentricity and other manufacturing faults. In the work [4] the way of the cogging torque investigation based on the Ampere's law is presented. According to the Ampere's law, a torque can be presented as a result of the interaction between the air-gap flux density and surface current density:

$$T(\theta) = l_\delta R_\delta^2 \int_0^{2\pi} B_\delta(\alpha, \theta) j(\alpha, \theta) d\alpha, \quad (1)$$

here  $l_\delta$  is the active machine length,  $R_\delta$  is middle air-gap radius,  $B_\delta$  is air-gap flux density,  $j$  is surface current density,  $\alpha$  is angle coordinate,  $\theta$  is rotor angle position.

If there is no current in a stator winding, the air-gap flux density is the product of the air-gap magneto-motive force (MMF)  $F_\delta$  and air-gap permeance  $\Lambda_\delta$ . In general an air-gap MMF and air-gap permeance can be presented as infinite harmonic series because of various reasons such as an air-gap teeth-slots structure, magnetic saturation of cores areas, magnetic flux asymmetry etc.:

$$\Lambda_\delta(\alpha) = \Lambda_0 + \sum_{v=1}^{\infty} \Lambda_{mv} \cos v z_1 \alpha, \quad (2)$$

$$F_\delta(\alpha, \theta) = \sum_{k=1}^{\infty} F_{mk} \sin kp(\theta - \alpha), \quad (3)$$

here  $\Lambda_0$  is the permeance constant component,  $\Lambda_{mv}$  is an amplitude of a permeance  $v$ 's harmonic,  $z_1$  is slots number,  $F_{mk}$  is an amplitude of a MMF  $k$ 's harmonic,  $p$  is pole pairs number.

A surface current density should be interpreted as derivative of an air-gap MMF with respect to angle coordinate:

$$j(\alpha, \theta) = -\frac{dF(\alpha, \theta)}{R_\delta d\alpha} = \frac{p}{R_\delta} \sum_{k=1}^{\infty} k F_{mk} \cos kp(\theta - \alpha). \quad (4)$$

According to the (2) and (3) an air-gap flux density can be expressed as:

$$B_\delta(\alpha, \theta) = \sum_{k=1}^{\infty} F_{mk} \sin kp(\theta - \alpha) \left( \Lambda_0 + \sum_{v=1}^{\infty} \Lambda_{mv} \cos v z_1 \alpha \right). \quad (5)$$

After the equations (4) and (5) are inserted in (1) and mathematical manipulations are done, the torque has following expression:

$$M(\theta) = \frac{\pi l_\delta R_\delta p}{2} \left[ \sum_{v=1}^{\infty} \sum_{k=1}^{\infty} k F_{mk}^2 \Lambda_{mv} \sin v z_1 \theta + 2 \sum_{v=1}^{\infty} \sum_{k=1}^{\infty} \sum_{\substack{i=1 \\ i \neq k}}^{\infty} k F_{mk} F_{mi} \Lambda_{mv} \sin v z_1 \theta \right]. \quad (6)$$

The first component is not equal to zero under the condition of:

$$\frac{k}{v} = \frac{z_1}{2p}.$$

The second component does not equal to zero under the condition of:

$$\frac{(k \pm i)}{v} = \frac{z_1}{p}.$$

So it is seen that cogging torque is a result of the interactions between a specific air-gap MMF and air-gap permeance harmonics. In the work [4] the formulas permitting one to calculate these harmonics numbers are developed. It is also shown that the number of the cogging torque pulsations is determined by the product of the slots number and the pole pair number:

$$n_{ct} = j \cdot z_1 \cdot p,$$

here  $j = 1, 2, \dots$  is positive integer.

The rotor eccentricity results in additional harmonics of the cogging torque and their order is due to the kind of the eccentricity. In the case of dynamic eccentricity, the harmonics order is multiple to the stator teeth number  $n_{atq} = j \cdot z_1$ , and the static eccentricity leads to the pole-tuple harmonics order  $n_{atp} = j \cdot p$ .

Therefore, the results obtained in the work [4] show that the cogging torque arises because of main and specific high air-gap MMF and air-gap permeance harmonic components. In order to reduce the cogging torque, these components amplitudes must be eliminated. From this point of view, all cogging torque minimization ways are the attempts of these harmonics reduction. So there are three groups of these ways. The first group is the methods eliminating high air-gap MMF harmonics. The second group methods are based on the air-gap permeance reduction. The third group is the combination of the first two. Most of the cogging torque reducing approaches lead to the decreasing of the main electromagnetic torque of a machine. Moreover some of them are followed by the complication of a machine construction and appreciation. So during the cogging torque optimization, the main electromagnetic torque value and a machine complexity and cost must be taken into account.

## 2.2. Cogging torque minimization ways

The cogging torque can be reduced both by the construction improvement of a motor and by using of special control algorithms. There are several conventional constructional optimization methods for the cogging torque reducing. In [5] recommendations for minimization of the cogging torque by the way of slots and poles number optimization, slot opening width, pole arc width and slot skewing are presented. The recommendations are given based on investigation of machines with the number of slots per pole per phase equal to 1/2 and 1/3.

The slot skewing is ineffective way for machines with fractional slot concentrated windings because of the slot pitch, which is particularly equal to the pole pitch, consequently the skewing decreases the torque generated significantly.

As it is shown in Reference [6, 7] the rotor packs skewing has the similar impact on the cogging torque. This skewing is implemented by circular displacement of the rotor packs or by the use of one-piece resin-bonded magnet which is magnetized in a special way. However, these ways mean additional manufacturing procedures resulting in cost increasing, or low flux density in the air-gap.

In [8] optimization of the pole shape for reducing the excitation field high harmonics is suggested. In the case of the ideal machine manufactured with high accuracy, such way decreases the cogging torque considerably.

One of the most utilizable ways is the reduction of the cogging torque cycle through the seeming increasing of a slot number. Small slots are implemented on the teeth crowns in this solution [9].

Another approach is the creation of the asymmetric distribution of excitation field. This elegant method is actualized by variation of the pole arc width on the rotor [10].

If the motor has conventional three-phase distribution winding and surface mounted magnets, acceptable results can be achieved by the magnets segmentation. Using such method, the optimal segmentation step must be obtained previously [11].

The investigation carried out shows that mentioned methods may reduce the cogging torque but have following disadvantages: complicating of manufacturing process and the cost increase. Moreover, the rotor eccentricity caused either by manufacturing faults or by bearing degradation leads to heavy growing of the cogging torque.

### 3. The subject and methods of study

The influence of the rotor eccentricity on the cogging torque in the permanent magnet machine with fractional slot concentrated winding for the electric power steering is investigated. The machine has a number of slots per pole per phase equal to  $2/5$ , 12 teeth, 5 pole pairs and the spoke magnets structure on the rotor. The technical information of the machine is presented in Tab. 1.

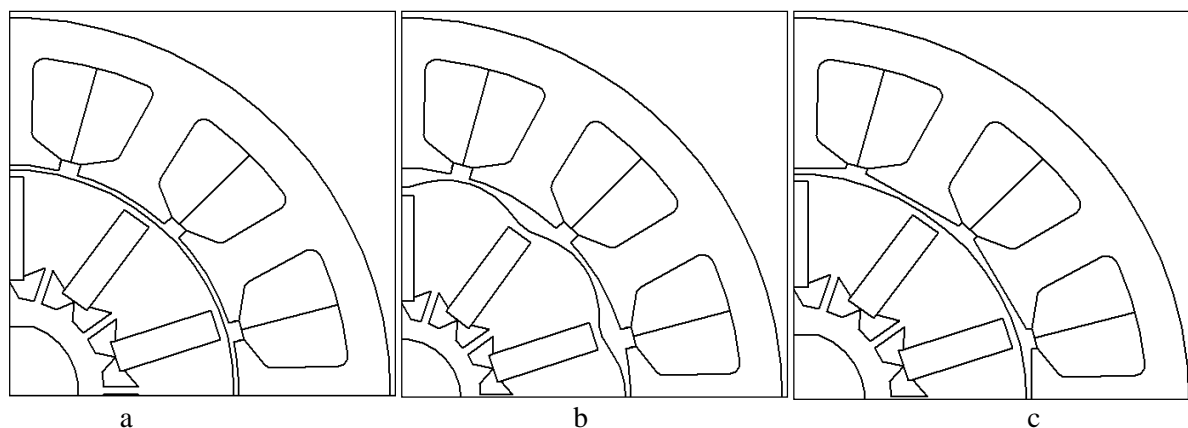
**Table 1.** Technical information of the machine

Parameter	Value
torque (N·m)	3
rotational speed (rpm)	1080
number of phases	3
voltage (V)	10
current (A)	50
external diameter (mm)	76
frame length (mm)	74

The complex air-gap configuration and high saturation of some core regions necessitates using the Finite Elements Analysis (FEA). The cogging torque is calculated basing on the results of two-dimensional magnetic field distribution in the machine under the various rotor rotation angle and no-current state by using the stress tensor. Three types of the machine structure are studied (Fig. 1).

### 4. Results

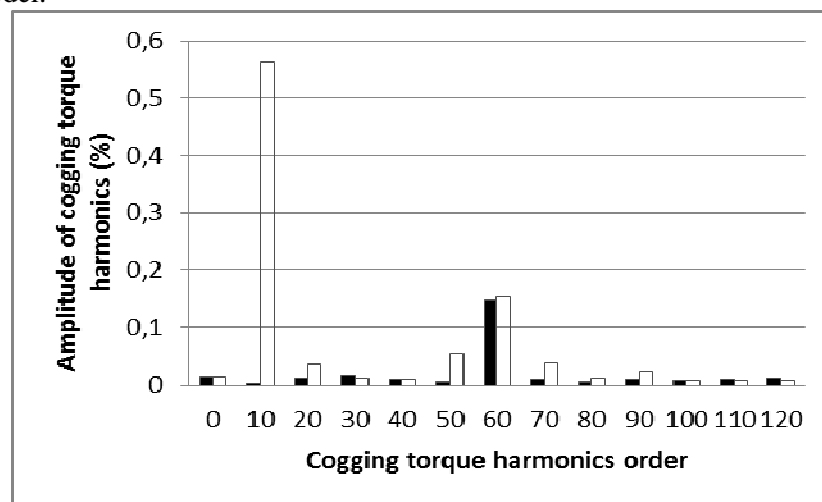
As it was expected, there are only main cogging torque harmonics having multiple to the product of teeth number and pole pairs number orders in the case without the rotor eccentricity. If the static eccentricity is available, the additional harmonics having multiple to the pole number orders appears. For example, Fig. 2 shows the cogging torque spectrum for the machine with the lobe rotor structure (Fig. 1b).



**Figure 1.** Cross sections of motor configurations: a – with constant air-gap; b – with lobe rotor structure; c – with aligned stator teeth.

As it is evident, the static rotor eccentricity leads to appearance of additional cogging torque harmonics having the amplitudes larger than main harmonic amplitude. Maximum values of the cogging torque calculated by FEA are summarized in Tab. 2. Data presented in the Tab. 2 show that the machine with aligned stator teeth (Fig. 1c) has the minimal value of the cogging torque and insignificant influence of eccentricity on the cogging torque.

Basing on the results, the obtained structure with aligned stator teeth is accepted to produce an experimental model.



**Figure 2.** Cogging torque spectrum ■ – without eccentricity; □ – eccentricity available.

**Table 2.** Cogging torque maximum value as a percentage of rated torque

Structure	Cogging torque maximum value	
	without eccentricity	eccentricity available
Fig. 1a	2.37	2.77
Fig. 1b	0.19	0.7
Fig. 1c	0.14	0.19

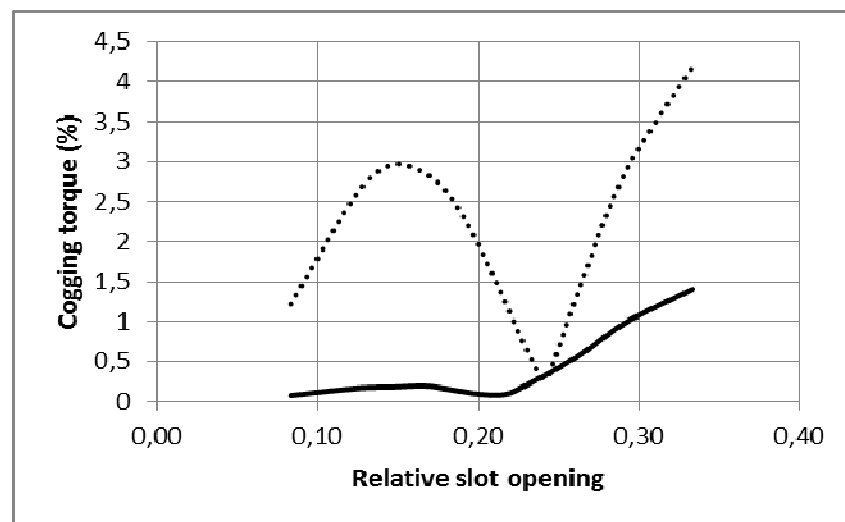
Optimization of the stator slot opening is an additional way for the cogging torque minimization in the machines with fractional slot concentrated winding. The impact of slot opening on the air-gap permeance spectrum is similar to influence the winding pitch reduction on the electromotive force spectrum. Therefore optimization of slot opening suppresses air-gap permeance harmonics responsible for the cogging torque appearance. The relative value of the cogging torque as a function of relative slot opening of machines with constant air-gap (curve 1) and with aligned stator teeth structure (curve 2) is presented in Fig. 3.

It is seen that there are very pure regions of the optimal slot opening value in the machine with the constant air-gap width. The range of optimal slot opening values providing minimum of the cogging torque is wider in the machine with aligned stator teeth. Therefore machine structure with aligned stator teeth allows a designer to choose slot opening value from the wide range and provides minimum of the main cogging torque harmonics and harmonics caused by the rotor eccentricity.

## 5. Conclusion

The cogging torque in three structure types of the permanent magnet synchronous machines with the fractional slot concentrated winding for the electric power steering having 2/5 slots per pole per phase and 5 pole pairs are investigated using FEA. The study shows that the machine with the constant air-

gap width has the maximum value of the cogging torque. The lobe rotor structure reduces the cogging torque significantly. However the maximum value of the cogging torque increases drastically in the case of the rotor eccentricity available. Minimum of the cogging torque and insignificant influence of eccentricity on the cogging torque are provided by the aligned stator teeth structure. Moreover this type of construction has a wide range of the optimal stator slot opening value. It is recommended to choose slot opening between 0 to 0.23 of the tooth pitch.



**Figure 3.** Cogging torque as a function of relative stator slot opening: .... 1;— 2.

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