

# Design of distributed systems of hydrolithosphere processes management. A synthesis of distributed management systems

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**Abstract.** The paper considers an important problem of designing distributed systems of hydrolithosphere processes management. The control actions on the hydrolithosphere processes under consideration are implemented by a set of extractive wells. The article shows the method of defining the approximation links for description of the dynamic characteristics of hydrolithosphere processes. The structure of distributed regulators, used in the management systems by the considered processes, is presented. The paper analyses the results of the synthesis of the distributed management system and the results of modelling the closed-loop control system by the parameters of the hydrolithosphere process.

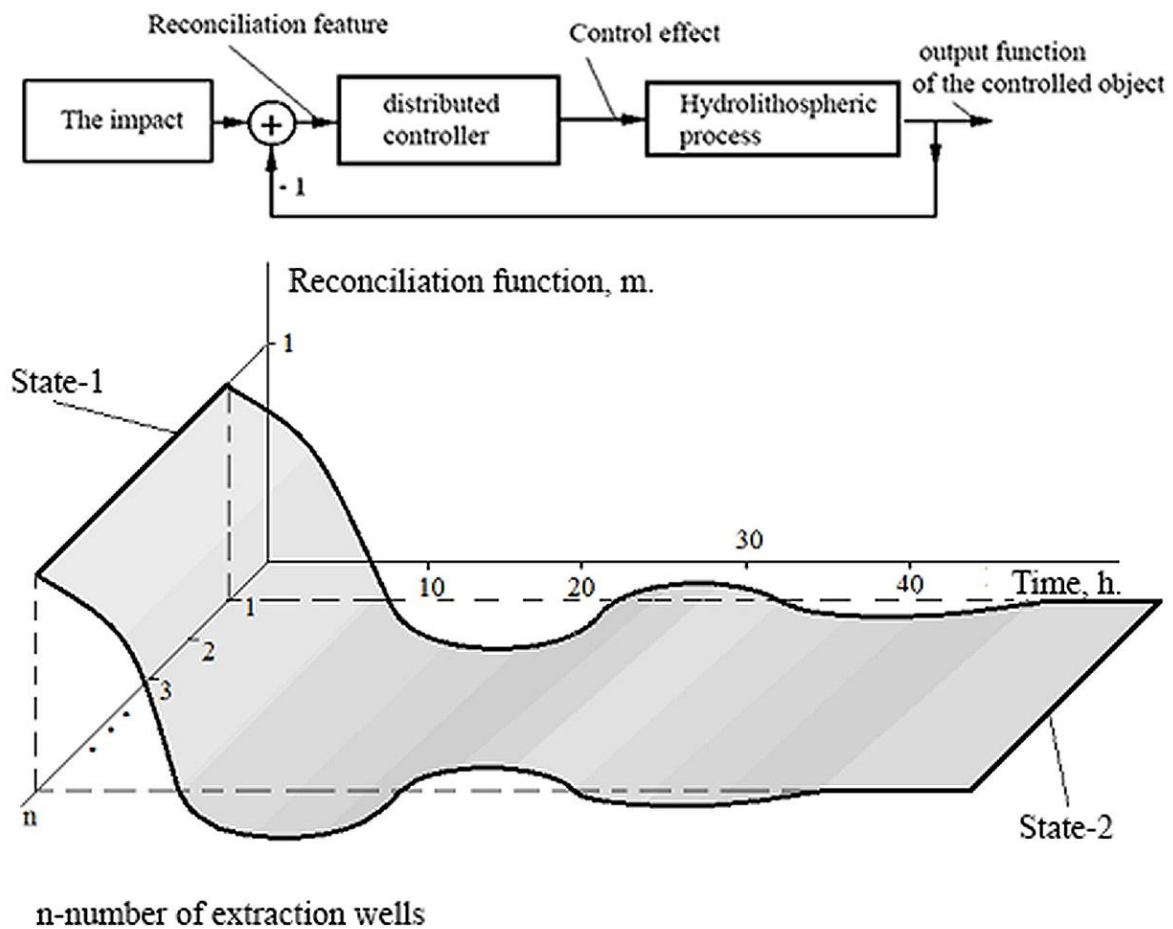
## Methods of synthesis of control systems

Let us consider the main techniques for synthesizing distributed regulators to manage hydrolithosphere processes. The distributed control effect on the controlled object is provided by a collection of extractive wells. The overall scheme of the management system is shown in Figure 1. The authors suggest that the state of the system is described by two variables: lowering the level in the  $i$ -th extractive well and time. The system from state - 1 (the unperturbed state of the  $H_0$ ) is transferred to state - 2 ( $H_0-1$ ). The diagrams of the system phase variable delineation function are shown in Figure 1. The schedule in question is a field of the disagreement functions that is formed by a combination of the disagreements at the point of installation of the extractive wells.

The synthesis task involves defining the control algorithms (the structure and parameters of the regulator) that reposition the system from the state-1 to the state - 2, and additional conditions are added to the disagreement functions. Depending on how the additional conditions are constrained, the following approaches to synthesis are identified:

-analytical design of the best regulators for systems with distributed parameters (AKOR for SPR), which is based on the Bellman optimum principle and the Pontriagin principle of maximum.





**Figure 1.** The division function

The works of Sirazetdinova T.K., Degtyarev G.L., Yegorova A.I., Rapoport J.I., Koval V.A., as well as the works of Eljai A., Khargonkar P.P. and Poolla K. etc. are devoted to the general issues of AKOR for the systems with distributed parameters. This method assumes that on the transition path from state 1 to state 2, the disagreement function provides a minimum of the best performance criterion (target function).

- *Synthesis methods that use calculations in an integrated area.* There are now two methods of synthesizing systems with distributed parameters that use calculations in the integrated area:

- the frequency method of synthesizing the regulator;
- synthesis of the regulators with the use of the qualitative theory of the distribution of modes, developed in the work of G. V. Vladimir [1].

Let us take a look at the use of the distributed regulator's frequency synthesis method for the hydrolithosphere process management system.

The mathematical model of the controlled object in question, consisting of hydrolithosphere processes describing the groundwater, the upper seam and the lower layer, is as follows:

$$\frac{\partial h_1(x, y, z, \tau)}{\partial \tau} = k_{1,x} \frac{\partial^2 h_1(x, y, z, \tau)}{\partial x^2} + k_{1,y} \frac{\partial^2 h_1(x, y, z, \tau)}{\partial y^2} + k_{1,z} \frac{\partial^2 h_1(x, y, z, \tau)}{\partial z_1^2};$$

$$0 < x < L_x; 0 < y < L_y; 0 < z < L_{z1},$$

$$\frac{\partial H_2(x, y, z, \tau)}{\partial \tau} = \frac{1}{\eta_2} \left( k_{2,x} \frac{\partial^2 H_2(x, y, z, \tau)}{\partial x^2} + k_{2,y} \frac{\partial^2 H_2(x, y, z, \tau)}{\partial y^2} + k_{2,z} \frac{\partial^2 H_2(x, y, z, \tau)}{\partial z^2} \right) -$$

$$- F_{2,x} \cdot \frac{\partial H_2(x, y, z, \tau)}{\partial x} + V_2(y, \tau) \cdot \delta(x_{0,j}, y_{0,j}, z_{0,j});$$

$$0 < x < L_x; 0 < y < L_y; 0 < z < L_{z2},$$

$$\frac{\partial H_3(x, y, z, \tau)}{\partial \tau} = \frac{1}{\eta_3} \left( k_{3,x} \frac{\partial^2 H_3(x, y, z, \tau)}{\partial x^2} + k_{3,y} \frac{\partial^2 H_3(x, y, z, \tau)}{\partial y^2} + k_{3,z} \frac{\partial^2 H_3(x, y, z, \tau)}{\partial z^2} \right) -$$

$$- F_{3,x} \cdot \frac{\partial H_3(x, y, z, \tau)}{\partial x};$$

$$0 < x < L_x; 0 < y < L_y; 0 < z < L_{z3}.$$

where:  $h_i$  - the head of the groundwater aquifer;  $H_i$  - the pressure of the first aquifer ( $i=2,3$ );

$k_{i,x}, k_{i,y}, k_{i,z}$  - spatial coordinates filtering factors, which are defined as:

$$k_{1,x} = 0.198 \text{ m./day}, k_{1,y} = 0.191 \text{ m./day}, k_{1,z} = 0.0199 \text{ m./day},$$

$$k_{2,x} = 0.196 \text{ m./day}, k_{2,y} = 0.191 \text{ m./day}, k_{2,z} = 0.0195 \text{ m./day},$$

$$k_{3,x} = 0.146 \text{ m./day}, k_{3,y} = 0.149 \text{ m./day}, k_{3,z} = 0.018 \text{ m./day},$$

$\eta_i$  - compressibility of the  $i$ -st layer ( $\eta_2 = 0.0012$  1/m.,  $\eta_3 = 0.0006$  1/m.);  $F_{i,x}$  - velocity of the current in the aquifer ( $F_{2,x} = 0.6$  m./day;  $F_{3,x} = 0.5$  m./day);  $V_2(y, \tau)$  - the lowering of pressure due to the impact of the  $j$ -mining wells (in the case of  $j=1..9$ , and the input effect is only on the upper layer);  $\delta(x_{0,j}, y_{0,j}, z_{0,j})$  - function equal to one if  $x = x_{0,j}$ ,  $y = y_{0,j}$ ,  $z = z_{0,j}$  and equal to zero in other cases;  $x, y, z$  - spatial coordinates;  $\tau$  - time.

The boundary conditions between the layers are recorded in the form:

$$\begin{aligned} & \text{groundwater - upper layer} \\ h_1(x, y, L_{z1}, \tau) &= h_1(x, y, L_{z1}, \tau) + b_1 \cdot (H_2(x, y, 0, \tau) - h_1(x, y, L_{z1}, \tau)), \\ H_2(x, y, 0, \tau) &= H_2(x, y, 0, \tau) - b_1 \cdot (H_2(x, y, 0, \tau) - h_1(x, y, L_{z1}, \tau)); \end{aligned}$$

$$\begin{aligned} & \text{upper layer - medium} \\ H_2(x, y, L_{z2}, \tau) &= H_2(x, y, L_{z2}, \tau) + b_2 \cdot (H_3(x, y, 0, \tau) - H_2(x, y, L_{z2}, \tau)), \\ H_3(x, y, 0, \tau) &= H_3(x, y, 0, \tau) - b_2 \cdot (H_3(x, y, 0, \tau) - H_2(x, y, L_{z2}, \tau)), \end{aligned}$$

where  $b_i$  is the parameters of the  $i$ -th layer ( $b_1 = 0.000035 \text{ days}^{-1}$ ,  $b_2 = 0.000039 \text{ days}^{-1}$ ); the lower boundary of the lower layer -  $\partial H_3(x, y, L_{z3}, \tau) / \partial z = 0$ ;

$$\begin{aligned} & \text{side faces} \\ h_1(0, y, z, \tau) &= h_{1,0}; H_2(0, y, z, \tau) = H_{2,0}; H_3(0, y, z, \tau) = H_{3,0};, \\ \partial h_1(L_x, y, z, \tau) / \partial x &= 0; \partial H_2(L_x, y, z, \tau) / \partial x = 0; \partial H_3(L_x, y, z, \tau) / \partial x = 0 \end{aligned}$$

Believing that the perturbations from the extraction wells do not affect the state of the layer at the  $y$ -coordinate points, let us write the conditions in the form of:

$$h_1(x, 0, z, \tau) = h_1(x, L_y, z, \tau) = h_{1,0}, H_i(x, 0, z, \tau) = H_i(x, L_y, z, \tau) = H_{i,0}, (i=2,3),$$

where:  $h_{1,0}(z)=z$ ,  $(0<z<L_{z1})m$ ,  $H_{2,0}=100m$ ,  $H_{3,0}=160m$ . - initial conditions of undisturbed groundwater and layers.  $L_x:=480m$ ;  $L_y:=400m$ .  $L_{z1}:=25m$ ;  $L_{z2}:=72m$ ;  $L_{z3}:=90m$ ;  $\Delta y=L_y/24$ . At that, the distance between the wells equals  $2\cdot\Delta y$ .

**Table1.** Wells in the upper layer

№ of extractive well	x	y	z
1	240	$4\cdot\Delta y$	36
2	240	$6\cdot\Delta y$	36
3	240	$8\cdot\Delta y$	36
4	240	$10\cdot\Delta y$	36
5	240	$12\cdot\Delta y$	36
6	240	$14\cdot\Delta y$	36
7	240	$16\cdot\Delta y$	36
8	240	$18\cdot\Delta y$	36
9	240	$20\cdot\Delta y$	36

The methodology for carrying out the numerical research to determine the dynamic and static characteristics of the control is shown in [2-4,6]. In accordance with the methodology, let us form the input impact on the control object as  $Q_2(y_j, \tau) = I \cdot \sin(\psi_i \cdot y_j)$ ,  $\psi_i = \pi \cdot i / L_y$ , where  $y_j = y$  – coordinate value  $y$  for  $j$ -th well,  $i$  – number of the mode. The debit of extractive wells  $Q_2$  is associated with the function  $V_2$  by the following ratio  $V_2 = K^* \cdot Q_2$ . Value  $K^*$  is defined based on the hydrodynamic parameters of the deposit in question. In this case,  $K^* = 0.0001$ . Figure 2 shows modeling the spasmodic input impact on the control object on the graphs of the transition processes.

Calculated static reinforcement ratios for the considered spatial modes are equal to the entry effects:  $K_1 = 11.909$ ,  $K_6 = 8.645$ . Figure 2 shows the response of the object to the harmonic input effect  $Q_2(y_j, \tau) = I \cdot \sin(\psi_i \cdot y_j) \cdot \sin(\omega \cdot \tau)$ ,  $\psi_i = \pi \cdot i / L_y$ , when  $\omega = 0.000015$ .

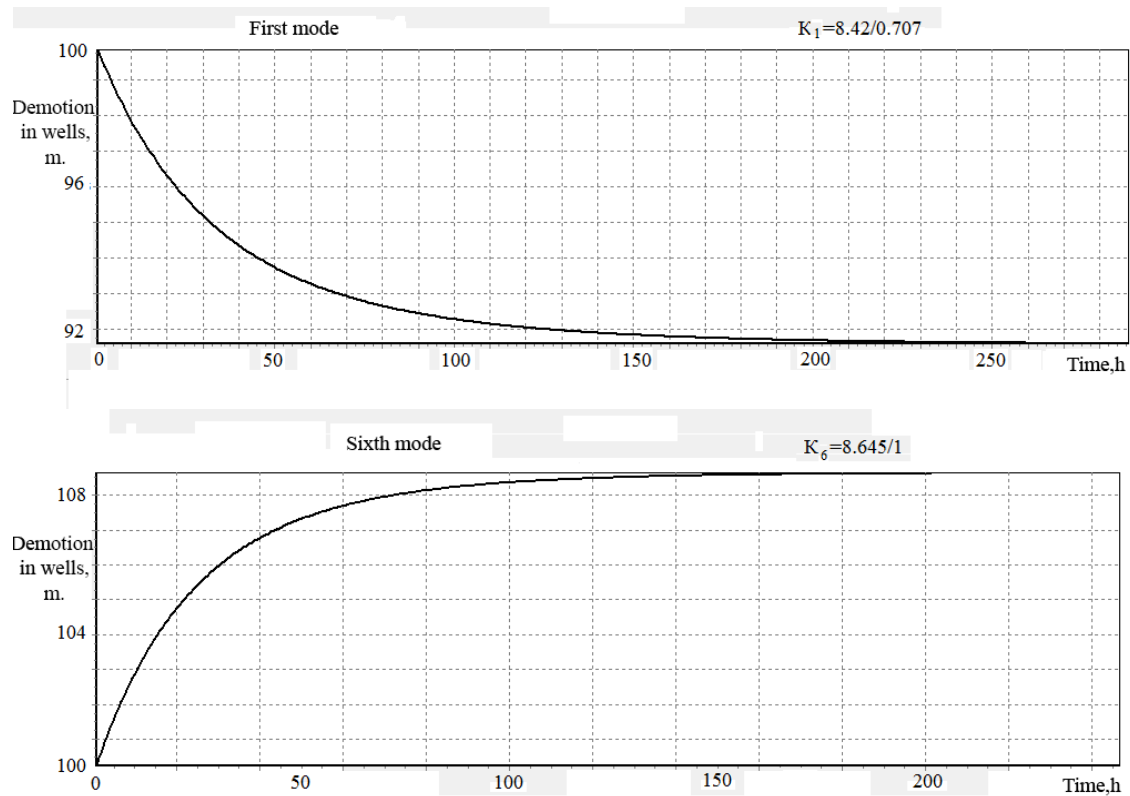
The phase shift of the signal output is determined from the ratio of  $\Delta\varphi_1 = -2 \cdot \pi \cdot 19 / 117 = 1.02$ .

Let us approximate the static and dynamic characteristics of an object in the selected input mode of the link in form [2]:

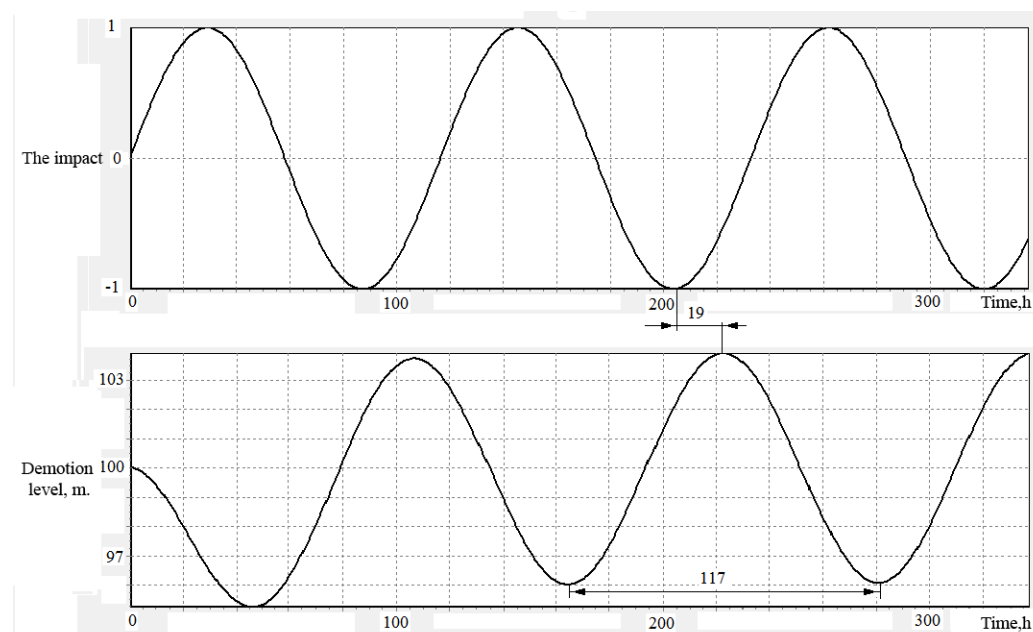
$$W_i(s) = \frac{K}{\beta_i} \exp(-\beta_i \cdot \Delta z), \quad \beta_i = \left( \frac{s}{a^*} + \psi_i^2 \right)^{1/2}, \quad (1)$$

where:  $s$  - laplace operator;  $K$ ,  $a^*$ ,  $\Delta z$  - parameters of approximating transfer functions, which are calculated using the pilot studies of the controlled object. Believing that  $s=0$  and equating static coefficient of link reinforcement (1) to the static reinforcement ratios, calculated above, the following will be obtained:

$$\begin{cases} K_1 = \frac{K}{\beta_1} \cdot \exp(-\beta_1 \cdot \Delta z) \\ K_6 = \frac{K}{\beta_6} \cdot \exp(-\beta_6 \cdot \Delta z) \end{cases}$$



**Figure 2.** The transition process at the location of the second working well in the first and sixth spatial modes of input



**Figure 3.** Dynamic characteristics of the controlled object at the point of the second work well in the first spatial mode of input

Substituting the initial data and solving the system of equations, it is possible to obtain the following result:  $K=0.06969$ ;  $\Delta z=37.471701$ .

Believing that (1)  $s=j\omega$ , let us write the relationship to determine the phase of the approximating function

$$\Delta\phi = -\Delta z \cdot \text{Im}(\beta_1) - \arctan(\text{Im}(\beta_1)/\text{Re}(\beta_1)) \quad (2)$$

Substituting the source data  $\omega=0.000015$ ,  $\Delta\phi=-1.020000$  in (2), let us evaluate the value of the  $a^*=0.06269$ . The resulting mathematical model of the approximating link is as follows:

$$W_i(s) = \frac{0.06969}{\beta_i} \exp(-\beta_i \cdot 37.471701), \quad \beta_i = \left( \frac{s}{0.06269} + \psi_i^2 \right)^{1/2}. \quad (3)$$

**Problem statement of the regulator synthesis:** for the control system of the object, the transmission function of which is recorded in the form (3), let us synthesize the distributed regulator (CMI), the transmission function of which is recorded as:

$$W(y, s) = E_1 \cdot \left[ \frac{n_1 - 1}{n_1} - \frac{1}{n_1} \nabla^2 \right] + E_4 \cdot \left[ \frac{n_4 - 1}{n_4} - \frac{1}{n_4} \nabla^2 \right] \cdot \frac{1}{s} + E_2 \cdot \left[ \frac{n_2 - 1}{n_2} - \frac{1}{n_2} \nabla^2 \right] \cdot s. \quad (4)$$

The following restrictions are applied to the  $\Delta f$  phase of the open system and the  $\Delta$  parameter:  $\Delta f \geq \pi/6$ ;  $\Delta=50$ , where:  $x, y$  – spatial coordinates;  $\nabla^2$  – laplacian;  $s$  – Laplace operator;

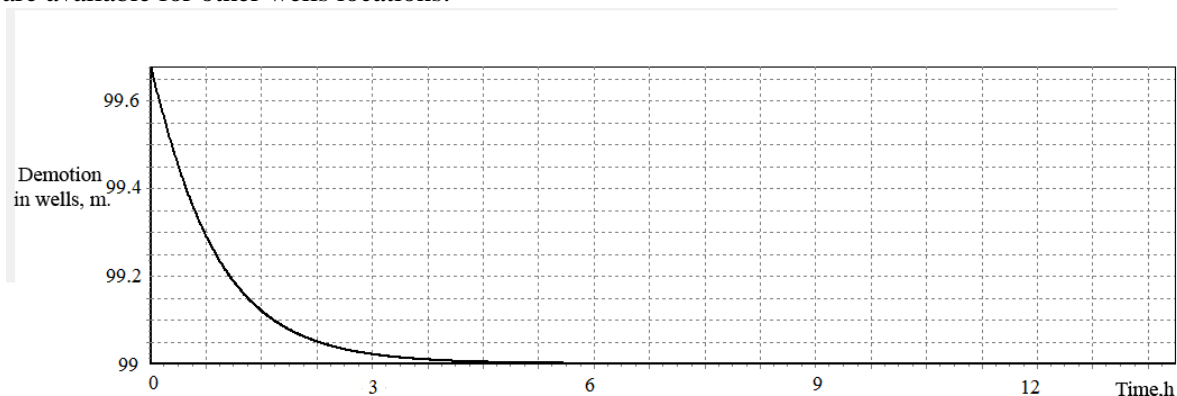
Synthesis of the regulator in question (definition of  $E_i$  and weighting factors  $n_i$  ( $i=1,2,4$ )) is set out in [2, 5, 6] and implemented in a standard package of programmes of the mining University. Using the standard package, for the object (3), the slider was synthesized, which is written as follows:

$$W(y, s) = 5326.879 \cdot \left[ \frac{0.001151}{1.001151} - \frac{1}{1.001151} \nabla^2 \right] + 0.02692 \cdot \left[ \frac{0.001547}{1.001547} - \frac{1}{1.001547} \nabla^2 \right] \cdot \frac{1}{s} + 462.462046 \cdot s. \quad (5)$$

**Selection of the impact** (see Figure 1). The studies in [7-11] show that a rational exploitation regime is one that ensures the same dynamic-level ratio to the maximum permissible in all producing wells at any given time. In this case, the limit position of the level in all the extractive wells will be reached at the same time.

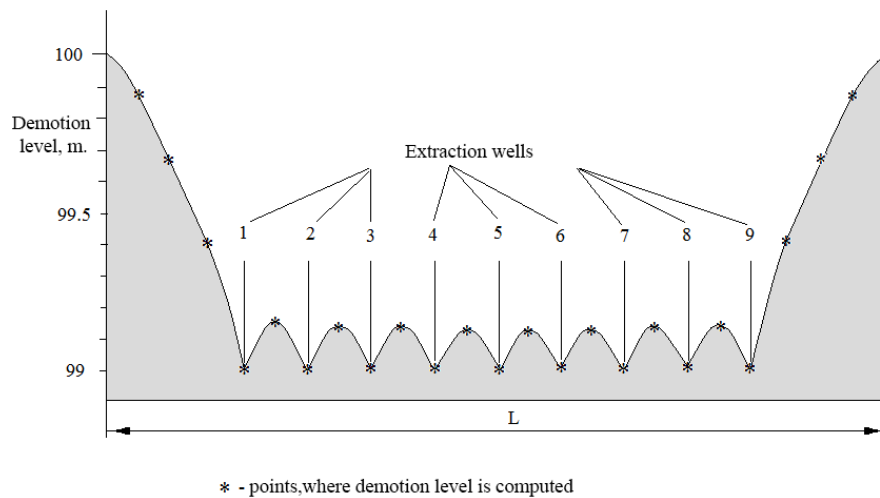
In the case under review, let us choose for the in pact on the control system  $H_{2,0}-l=99m$ .

When modeling the work of a closed management system, the regulator was using the ratio (5) and the object using discrete analogue equations in the private derivatives above. The simulation resulted in a timetable for the transition process at the location of the second working well. Similar schedules are available for other wells locations.



**Figure 4.** The graph of the transition process at the second working well location

Figure 5 shows the level graph  $H_2$  ( $x=240$  m.,  $y, z=36, \tau=20$  days).



**Figure 5.** Level distribution profile with  $\tau = 20$  days

It follows from the graph, shown in Figure 4, that the system has good dynamic properties and put the object smoothly into the specified mode.

### Conclusion

The use of distributed input impacts (several extraction wells) reduces the pressure on the reservoir and ensures the technological safety of the deposit.

Application of distributed regulators, taking into account the mutual influence of output wells, allows obtaining enhanced dynamic characteristics of the distributed management process.

The methodology, shown for the design of distributed management systems, can be applied in different industries.

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