Reliability analysis of distribution network of mining enterprises electrical power supply based on measure of information uncertainty

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Abstract. In the reliability analysis of the distribution network, information entropy is considered as the measure of uncertainty. The simple mathematical model that allows determining an entropy of a network structure is offered. The model is based on Shannon's approach to the determination of information. Reliability analysis comes down to the comparison of two components of entropy: the first is the entropy of connection a source-electroreceiver in the structure; the second is a boundary entropy as information of observance of the requirement to support the required reliability level of electrical power supply of the electroreceiver. The example and the procedure how to perform the necessary calculations for the analysis are provided.

1. Introduction

The design and operation of systems of external power supply of mining enterprises are aimed to ensure the no-break operation of electroreceivers. One of the criteria of viability evaluation of the system is its reliability in electricity power supply of customers. The regular monitoring of system operation of electrical power supply, data processing and analysis allow designers and operators to develop effective solutions for compliance with a specified level of reliability.

There are many schemes of electrical power supply of mining enterprises. Let's address to the schemes of radial distributive networks, which are used to internal electrical power supply of quarries (mines). Such networks have a simple structure and successfully provide the given level of structural reliability. When analyzing structural reliability of the network, the statistical indices are considered to be the most important: failure flow frequency; the recovery time of damages; the states probabilities of the network elements, etc.

When performing reliability analysis of a distribution network of mining enterprises electrical power supply, then promising direction can be considered the estimation of information uncertainty about the state of the structure. The information uncertainty (its measure) is one of the criteria of the reliability evaluation of technical systems.

2. The role of information uncertainty in the assessment of the network structure state

The distribution network of sections is a complex technical object, which is working all year-round outdoors and is impacted by: atmospheric precipitation; temperature fluctuations of the environment;

dust count; airborne vapors of chemical reagents, etc. All of these factors (their accidental appearance) are connected with the information uncertainty of the network behavior [1]. Occurrence of an event, which can change an electric equipment operation mode (a network element) transferring it from operable to non-operable state is a consequence of influence of unwanted factors. The structure of the network varies that leads to damages of the production process. Monitoring and analysis of network conditions allow one to monitor its structural content (changing the state of elements) and then to make the decisions how to eliminate negative consequences. Therefore, the analysis of information about the state of the electrical equipment expands knowledge and allows one to make solutions for the removal of uncertainty in the network behavior.

When solving the reliability analysis issues, the difficulty of formalizing the problem of accounting the uncertainty forces us to restrict the only consideration of the structural content of distribution network. However, building the simplest network structure is not difficult. The structure can be represented as the graph, which has connections (edge) – the network elements and nodes (peaks) – junctions of elements. Network parameters are inherent only in graph edges: the states of elements, the frequency and probability of events occurrence.

In the reliability analysis of network functioning [2], the information value consists of the possibilities of using its uncertainty as a measure of information entropy [3]. This measure allows one to generalize the experience of the electricity system in a case when it is important to consider the impact of random factors on the variability of structure that expressed through an entropy value both in the quantitative and in the qualitative forms [4].

3. Problem definition of entropy measurement

Any element of the distribution network can be represented as a discrete device having one input and one output. From the standpoint of reliability theory at any given moment the element is in one of two opposite states: operable and non-operable states. Change of an element states or network in general – the physical process which is discretely changing in time and is reflected in the form of information that received by the control system and management system by means of message transmission.

Formally, the element is characterized by the ordered components of sets:

$A = (P, \delta, N)$,

where N is the set of internal states, $p \in P$ is the set of occurrence probabilities of random events, δ is a random transition function from one state to another (the opposite). Transition function reflects the following process in time: the energy discrete stream receives the input device, and the frequency and length of stream changes the output.

The reliability analysis problem of distribution network comes down to comparison of the amount of informational entropy (because of calculations) and the limits the amount of information that characterized support of the given level of reliability.

In turn, structural reliability is expressed through the probabilistic nature of network behavior or through requirements of reliability standards; therefore search of the finite decision is complicated. Difficulties in calculations can be overcome, though taking into account some assumptions. The authors published a number of works [2-5] in which the assessment of structural reliability is possible through the calculation of the information entropy [6-9].

The problem involves preparation of input data (building structure, determining of set indicators, which characterize the states of the network elements). The decision should be looked for based on the mathematical model offered further.

In the process of determining the information amount, it is necessary to observe the set of conditions:

- only statistical data about the state of the network structure are considered;
- the simplest discrete states are considered;
- information is subject to accumulation;
- information must be connected with its uncertainty;
- the logarithmic measure having additivity is used;

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- in case of information determination the K. Shannon's classical approach is used.

4. The entropy distribution model in the network structure

The determination of the entropy amount of a distributive network is possible based on K. Shannon's classical model [9]. It allows applying the probability of occurrence of events that determined based on long-standing registration and statistical processing of parameters of elements states: frequency of failure; time to failure; recovery time etc. The connection between the probabilities of states and amount of information (obtained during the occurrence of the events) can be expressed by the Shannon's formula:

$$I = -\sum_{i=1}^{N} p_i \log_2 p_i, \text{ by } \sum_{i=1}^{N} p_i = 1, \quad (1)$$

where I – the information amount; N – the number of possible events; p_i – occurrence probability of i-th event.

The Shannon's formula (1), in our opinion, allows one to differentiate information according to a qualitative character [4]:

$$I = -(\sum_{i=1}^{N_1} p_i \log p_i + \sum_{j=1}^{N_0} q_j \log q_j), \text{ by } \sum_{i=1}^{N_1} p_i + \sum_{j=1}^{N_0} q_j = 1, \quad (2)$$

where p_i and q_j=1-p_i – the probabilities of operable and non-operable states of the element, N_1 and N_2 – the number of operable and non-operable states of the element.

Practically for all electrical power supply systems, statistical data indicate that $p_i >> q_j$. Therefore, the value of the left term in the formula (2) exceeds the value of the right term. In addition, equality of these terms will be reached when $p_1 = q_1 = \ldots = p_i = q_j = \ldots$, that indicates the equilibrium of the opposite states or the presence of chaos (disorder) in the network system. This equilibrium (lack of the order) is unacceptable in case of the networks operation, because it does not satisfies reliability requirements.

The occurrence of many events is randomly and most of them are subordinated to distribution laws of random variables, therefore, according to (2), it is possible to determine informational entropy for one element i, and also the total entropy of all network elements without structural connection between them:

$$H_\Sigma = \sum_{i=1}^{N} [H(p_i) + H(q_i)] = -\sum_{i=1}^{N} (p_i \log_2 p_i + q_i \log_2 q_i), \quad (3)$$

where $H(p_i) = -p_i \log_2 p_i$ and $H(q_i) = -q_i \log_2 q_i$ – the entropy operable and non-operable states of the network elements, N – the number of elements in the network structure. Here the logarithm base equal 2 demonstrates two qualitatively different opposite states. The pi + qi = 1.0 expression is observed for all elements i. If for all elements: $p_i = q_i = 0.5$, then the maximum entropy $H_\Sigma = N$.

The expression (3) allows one to obtain the total entropy H_Σ under the assumption of independence of the elements functioning, i.e. without taking into account structural connection between elements. The obtained value of H_Σ will remain constant in time, if not dealt situations are not associated with the arrival or disposal of information according to the given network structure. For example, if any element was not included in the network or not excluded from the network.

Reliability analysis of network functioning requires the accounting of correlations between elements. It is required to determine the entropy amount of operable $H(P_{0-j})$ and non-operable $H(Q_{0-j})$ states according to each selected section 0-j on the graph of a distributive network. Where 0 – the peak of a graph that indicates a network electric power source (for example, the principal step-down substation of a pit), j – number of graph peak that indicates the electroreceiver which is associated with a network. When calculating these values the probable joint states of elements should be taken into account (e.g., according to mathematical expressions in [4]).

According to [5], when the considered sections of structure consist of consistently connected elements, then expressions for determination the entropy value, taking into account the division of the entropy into components, will be:

- for operable state of a section:

$$H(P_{0-j}) = \sum_{i=1}^{m} [(\prod_{\substack{k=1 \\ k \neq i}}^{m} p_k) H(p_i)]; \quad (4)$$

- for non-operable state of a section:

$$H(Q_{0-j}) = \sum_{i=1}^{m} [H(q_i) + (1 - \prod_{\substack{k=1 \\ k \neq i}}^{m} p_k) H(p_i)]. \quad (5)$$

In (4) and (5) k – sequence element number of a section 0-j, m – elements number of section, $i = \overline{1,m}$ – sequence element number of a section.

The following condition is kept for any section 0-j:

$$H_{0-j} = H(P_{0-j}) + H(Q_{0-j}) = \sum_{i=1}^{m} (p_i \log_2 p_i + q_i \log_2 q_i). \quad (6)$$

Component of the structural reliability analysis of a distribution network is observance of a condition about an assumption of a possible break in electrical power supply of group or separate electroreceivers. Therefore, it is necessary both to determine ($H(P_{0-j})$ and $H(Q_{0-j})$), but also to compare them with boundary (critical) values:

- $H^0(P_j)$ – entropy as information to ensure the no-break power supply of electroreceiver j on a section 0-j;
- $H^0(Q_j)$ – entropy as information of allowable failure in electrical power supply of the electroreceiver j on a section 0-j.

Here the superscript 0 means that the value of the boundary entropy of the electroreceiver j is determined based on the requirements to reliability of consumers power supply.

Thereby, in the course of the analysis the following conditions are considered:

– for operable state of the section:

$$H(P_{0-j}) \le H^0(P_j); \quad (7)$$

– for non-operable state of the section:

$$H(Q_{0-j}) \le H^0(Q_j). \quad (8)$$

To determine values of $H^0(P_j)$ and $H^0(Q_j)$ using the boundary values of time operable and non-operable states of the network section. The parameters of the network: M_{pj} – the average time during which no-break power supply of the electroreceiver j on the section 0-j will be provided; M_{qj} – the average time during which the interruption of power supply of the electroreceiver j on the section 0-j is allowed; the probability of no-break power supply of the electroreceiver is j – $p_j^0 = M_{pj}/T$; the probability of an admissible failure in electrical power supply – $q_j^0 = M_{pj}/T$; $T = M_{pj} + M_{qj}$ – number of hours in a year.

The boundary values of entropy are determined by the expressions:

$$H^0(P_j) = p_j^0 \log_2 p_j^0; \quad (9)$$

$$H^0(Q_j) = q_j^0 \log_2 q_j^0. \quad (10)$$

The entropy calculation and the reliability analysis of the network are performed in the following order:

1. The structure of the distributive network in the form of the graph is built. Its elements: branches – network elements (transmission lines, transformers); peaks – electroreceivers.

2. The probabilities p_i and q_i are determined based on statistical data (failure rate, recovery time) about the state of the network elements.

3. An entropy of each element is calculated.

4. Values of a boundary entropy for the selected electroreceivers are determined.

5. The calculation of the entropy of the operable or non-operable states for the selected network branch is executed.

6. Entropy values of a branch state are compared with a boundary entropy of the electroreceiver for each of the selected branches.

5. Example of determination of entropy amount

There is distribution network in the form of a graph in Fig 1. It is necessary to determine of entropy value for sections 0-3 (branches 0-1 and 1-3) and 0-2 and to compare the received results with the boundary value of entropy of the electroreceivers at peaks 2 and 3. The source data of the task: the probabilities of operable state of elements – p_1= 0.999, p_2=0.95, p_3= 0.995 (p_4 is not considered in the calculations).

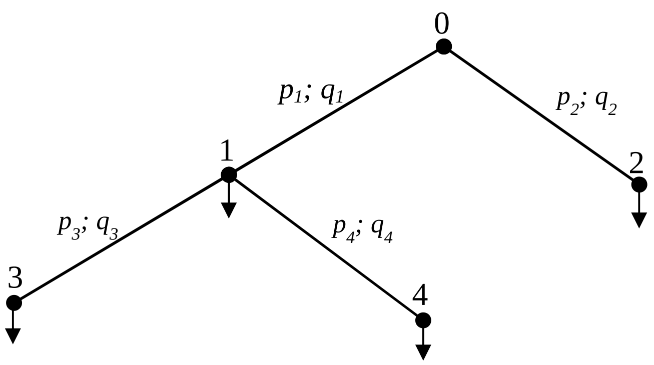

Figure 1. The graph of the distribution network

Solution. Let us determine the probabilities of non-operable states of the q_j=1-p_i elements: q_1=0.001, q_2=0.05, q_3= 0.005.

Let us determine values of an entropy for the selected elements: $H(p_i) = -p_i \log_2 p_i$; $H(q_i) = -q_i \log_2 q_i$. Result: $H(p_1) = 0.0014$; $H(q_1) = 0.01$; $H(p_2) = 0.0703$; $H(q_2) = 0.2161$; $H(p_3) = 0.0072$; $H(q_3) = 0.0382$.

Let us determine the boundary values of entropy for 2 and 3 electroreceiver according to expressions (9) and (10):

• let us accept boundary values of annual average duration for non-operable state: for the electroreceiver 2 – M_{q2} = 50 hours; for the electroreceiver 3 – M_{q3} = 100 hours. Also for operable state: M_{p2}=T–M_{q2} = 8760–50 = 8710 hours; M_{p3} = 8660 hours;

• let us determine probabilities of no-break electrical power supply and an admissible failure in electrical power supply:

- for electroreceiver 2: $p_2^0 = M_{p2} / T$ =8710/8760=0.9943; $q_2^0 = M_{q2} / T$ =50/8760=0.0057;

- for electroreceiver 3: $p_3^0 = M_{p3} / T$ =8660/8760=0.9886; $q_3^0 = M_{q3} / T$ =100/8760=0.0114;

• boundary values of an entropy:

- for electroreceiver 2: $H^0(P_2) = -p_2^0 \log_2 p_2^0 = -0{,}9943 \log_2 0.9943 = 0.0082$;

- for electroreceiver 3: $H^0(Q_2) = -q_2^0 \log_2 q_2^0 = -0.0057 \log_2 0.0057 = 0.0425$.

Let us calculate an entropy of operable and non-operable states for the selected network sections according to (4) and (5):

- for operable state of section 0-2: $H(P_{0-2}) = H(p_2) = 0.0703$;
- for non-operable state of section 0-2: $H(Q_{0-2}) = H(q_2) = 0.2161$;
- for operable state of section 0-3:

$$H(P_{0-3}) = p_3 H(p_1) + p_1 H(p_3) = 0.995 \cdot 0.0014 + 0.999 \cdot 0.0072 = 0.0086;$$

- for non-operable state of section 0-3:

$$H(Q_{0-3}) = H(q_1) + H(q_3) + (1 - p_1)H(p_3) + (1 - p_3)H(p_1) =$$

$$= 0.01 + 0.0382 + (1 - 0.999) \cdot 0.0072 + (1 - 0.995) \cdot 0.0014 = 0.0482.$$

The check of the calculation for section 0-3:

Comparison of entropy values:

- for section 0-2: $H(P_{0-2}) > H^0(P_2)$, $H(Q_{0-2}) > H^0(Q_2)$;
- for section 0-3: $H(P_{0-3}) < H^0(P_3)$; $H(Q_{0-3}) < H^0(Q_3)$.

When comparing the values of entropy the conditions (7) and (8) for the section 0-3 of the distribution network are fulfilled, but for – 0-2 are not fulfilled. On the 0-3 section the work to ensure a specified level of reliability is satisfactory, but on the 0-2 section - is unsatisfactory. On this section, it is necessary to take some steps to prevent emergency situations and eliminate unwanted consequences.

6. Conclusion

The reliability analysis problem of distribution network of mining enterprises can be solved taking into account the information uncertainty. The offered mathematical model is quite simple and applicable for networks of a tree structure of any complexity. The important place in model is assigned to calculation of informational entropy as the measure of information uncertainty. Reliability analysis of the network structure is based on the comparison of the entropy calculated value of the source-electroreceiver connection with a boundary entropy value that is determined on the basis of requirements to comply with the required reliability level of electrical power supply of the electroreceiver. The comparison of the calculated and boundary values of the entropy allows making the conclusion that the possibility of the distribution network can provide given levels of reliability in electricity supply of quarries.

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