

Optimization of operating costs in managing electrical submersible pumping systems

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Abstract. Using the methods of analysis, the task of minimizing the specific operating costs for oil production by submersible ESP units was formalized. The analysis of the multidimensional parameter space of the "ESP-well" system made it possible to isolate the vector of controlled parameters, determine the range of admissible parameters, and also to simplify the problem of finding an extremum up to the three-dimensional case. An application of the method of Lagrange multipliers to the solution of the problem is considered.

1. Introduction

The development of the world energy market is characterized by oil prices fluctuations [1-3] and reduces the oil fields profitability significantly. This fact determines the relevance of cost reduction and material losses targets in the oil fields development.

This task solution is possible due to the intensification and optimization of the oil field equipment usage at all stages of oil production and processing to transport. Moreover, it is the oil production stage that largely determines the oil-producing complex efficiency generally. Therefore, the optimal well equipment usage, the rational resources expenditure and the energy conservation compliance policy acquire particular relevance in this situation [4-7].

Currently, a large portion of wells are operated by means of electrical submersible pumping (ESP) systems, some of which are equipped with adjustable frequency converter, which significantly expands the ability to manage the fluid extraction process. Further the authors propose to consider the totality of the process equipment, control devices, and the well itself with the contiguous zone of the oil reservoir as "ESP-well" system [4].

The mathematical apparatus giving an analytical solution of the optimization problem has been thoroughly studied by such researchers as Lagrange Zh., Kantorovich L.V. and etc., is described in the classical literature and does not require any special comments. It is necessary to describe the task of controlling the object "ESP-well" in the formal language of mathematical analysis in the appendix, that means to make "formalization".

2. Optimization problem in managing ESP

Let there be a function $f_0 : X \rightarrow \bar{R}$ defined on some set of X parameters of the "ESP-well" system. \bar{R} is the expanded real line, quantitatively representing the optimality criterion. Also, let the constraint $C \subseteq X$ defining the working area of the system be given (the parameters are admissible by restriction). Then it is possible to formulate the problem in this way: to find the function extremum f



under the assumption that x is in the parameters range admissible by the restriction (1).

$$f(x) \rightarrow \min(mx), x \in C \quad (1)$$

Thus, to pose the task properly, it is necessary to describe x , f and C .

In general, the model of the "ESP-well" system establishes liaison with the main technological parameters in the following form (2):

$$q_i = P(u_i, g_i, \xi_i), \quad (2)$$

where q_i is the flow rate of the well fluid at the i -th control interval, which ensures the well performing the production volume target for the given control interval, P is the adopted model of the "ESP-well" system; u_i is the vector of the system controlled parameters at the i -th control interval; g_i – the vector of the system uncontrolled parameters at the i -th control interval; ξ_i is the vector of the system random effects at the i -th control interval.

At the same time, the interval of the well quasi-stationary operation which is limited on both sides by the facts of stopping production or the regime changing is defined as the control interval t_i . The totality of such intervals in the ESP installation lifecycle forms the interval of the overhauls wells exploitation.

It is possible to introduce the notion of specific operational costs for the i -th control interval $M_i = e_i/q_i$ and generalize it to the full interval of the overhauls wells exploitation within the framework of these designations:

$$M = e_{\Sigma} q_{\Sigma}^{-1} = f_0(u_i, g_i, \xi_i), i = 1, \dots, n \quad (3)$$

where e_{Σ} is the total operating costs at the overhaul interval; q_{Σ} is the total well production at the overhaul interval.

The system's objective function is assumed to take the minimization of specific operating costs (3).

The structure of the set X is represented as follows. In the general case, $X = R^n$, where R is the set of real numbers, and n is the number of the system parameters, each of which is the coordinate of the multidimensional vector in the space R^n .

The magnitude and frequency of the motor supply voltage at the i -th control interval are referred to the system controlled parameters of the u_i , which are usually normalized by nominal values and are subsequently used in the form of relative values f_i^* and u_i^* forming the coordinates of the vector U^* .

The system uncontrolled parameters include the parameters of the bottomhole formation zone, deep and ground technological equipment, etc. In general, they are divided into three categories:

- parameters not changing with time g_i^1 ;
- parameters varying with time g_i^2 ;
- indirectly controlled parameters g_i^3 .

Incidental impacts on the system include such factors as power outages, factory defects, accidents of deep equipment, etc. Obviously, the occurrence frequency of such events within the oil industry is unchanged with a sufficiently large control interval. Thus, these factors determine some constant component of the total operating costs (repair costs and losses of unprocessed oil).

It should be noted that the question of the correlation dependence presence $\xi_i = f(U^*)$ and the correlation coefficient magnitude remains open. Obviously, the correlation coefficient for a single ESP installation tends to zero (one well affects the field insignificantly).

A detailed analysis [4] shows that uncontrolled parameters g_i and random effects ξ_i at the overhauls exploitation interval are of a quasistatic nature and can be combined in the vector $G_i = \text{const}, \forall t \in t_i$.

The above allows us to formulate the optimization task:

$$M = f_0(U^*, G_i) \rightarrow \min, U^* \in C, G_i = \text{const}, i = 1, \dots, n \quad (4)$$

That is, for a given G_i it becomes not a multidimensional but a three-dimensional one. On the one hand, this fact simplifies its solution, but, on the other hand, it requires considerable efforts in determining the model M . Publications analysis allows us to conclude that at the present time, there is no such a model that gives a mapping $\{U^*, G\} \rightarrow M$, it requires additional investigation.

The subset C defining the values range of the vector U^* , which is admissible under the constraint, organically follows from the requirements of the regulations and instructions for the operation of such installations. These constraints can be represented by the following considerations in a general form [4].

Solving the task of ESP control is carried out in difficult conditions caused by the need to take into account a large number of different factors that impose restrictions on some of the indirectly controlled technological parameters of the ESP-well system g_i^3 . Exit from the technological parameters field permissible by restrictions leads either to a violation of the oil extraction technology, or to the stopping of the selection process by automatic protection systems of the control station. The main factors limiting the range of permissible values of technological parameters are outlined below.

The liquid flow rate limitation is:

$$q_{min} \leq q(U^*) \leq q_{max}, \quad (5)$$

where q_{min} is the minimum permissible liquid flow rate, due to the need to ensure cooling the SEM and the implementation of the planned production target; q_{max} is the maximum permissible liquid flow rate due to oil reservoir capacity (avoidance of pump failure) and the gas factor (avoidance of reservoir fluid degassing);

The SEM temperature limitation is:

$$T_{SEM}(U^*) \leq t_{SEM\ max}, \quad (6)$$

where $t_{SEM\ max}$ is maximum temperature, determined by the necessity to avoid the avalanche thermal destruction of insulation;

Supply frequency limitation is:

$$f^*_{min} \leq f^* \leq f^*_{max}, \quad (7)$$

where $f^*_{min}; f^*_{max}$ are minimum and maximum permissible frequencies of the frequency converter (in relative values);

The SEM current and voltage limitation is:

$$u^*_{min} \leq u^* \leq u^*_{max}, \quad (8)$$

$$i^*_{min} \leq i^*(U^*) \leq i^*_{max}, \quad (9)$$

where $u^*_{min}; u^*_{max}; i^*_{min}; i^*_{max}$ are the maximum and minimum values of the voltage and supply current of the SEM, determined by the settings of the control station protection system for overload, short-circuit, voltage drops and other similar events (in relative values);

The radial and axial vibrations level limitation is:

$$\lambda_z(U^*) \leq \lambda_{z\ max} \text{ and } \lambda_r(U^*) \leq \lambda_{r\ max}, \quad (10)$$

where $\lambda_{z\ max}$ and $\lambda_{r\ max}$ are maximum permissible values of axial and radial vibrations, respectively.

The limitation of the deep equipment wear rate is. In this case, gradient $\nabla \vec{r}$ of the multidimensional vector resource function, representing the dynamics of the ESP resource is of practical importance:

$$|-\nabla \vec{r}_i(U^*)| \leq |-\nabla \vec{r}_{max}|, \quad (11)$$

where $|-\nabla \vec{r}_{max}|$ is the threshold providing the ESP trouble-free operation during the overhaul well operation period specified by the capital repair plan.

For further reasoning, the above-mentioned inequalities (5-11) must be reduced to a single form:

$$f_k(U^*) \leq 0; k = 1, \dots, m' \quad (12)$$

where m' is the coordinates number of the vector g_i^3 on which constraints are imposed.

The adopted constraints (12) define control area C in the two-dimensional space of the controllable “ESP-well” system parameters, in which the optimal value of control vector U^*_i will be searched. It should be noted that further the domain boundaries C can be refined by adding new components to the system of inequalities (12) (including the form equality $f_k(U^*) = 0$).

The aggregates (4) and (12) are a formalized description of the optimal control problem for the ESP installation with constraints such as equalities and inequalities.

3. Solution of optimal control problem by method of Lagrange multipliers

The analytical solution of this problem is possible by the classical method of Lagrange multipliers [8,9,10] using the following algorithm:

1) To construct Lagrange function $\Lambda(U^*, \lambda) = \sum_{k=0}^m \lambda_k f_k(U^*)$ in section G_i , for the i -th interval (sub-interval) of control t_i so that $G_i = \text{const}, \forall t \in t_i$.

2) To write down the necessary 1st-order extremum condition for local minimum points \hat{U}^* :

- stationarity condition:

$$\Lambda_{U^*}(\hat{U}^*, \lambda) = 0 \Leftrightarrow \left\{ \frac{\partial \Lambda(\hat{U}^*, \lambda)}{\partial u^*} = 0, \frac{\partial \Lambda(\hat{U}^*, \lambda)}{\partial f^*} = 0 \right\}; \quad (13)$$

- complementary rigidity condition:

$$\lambda_k f_k(\hat{U}^*) = 0, k = 1, \dots, m'; \quad (14)$$

- nonnegativity condition:

$$\lambda_k = 0, k = 0, 1, \dots, m'. \quad (15)$$

3) To determine and investigate the critical points of the local and absolute extremum (minimum). Undoubtedly, the absolute minimum of function $M = f_0(U^*, G_i)$ is of practical interest in the considerable problem context.

4. Conclusion

The task of minimizing the specific operating costs for oil production by submersible ESP units was formalized by using the methods of analysis. The analysis of the multidimensional parameter space of the ESP-well system made it possible to isolate the vector of controlled parameters U^* , determine the parameters range admissible with respect to constraint C , and simplify the problem of finding an extremum up to the three-dimensional case as well. An application of Lagrange multipliers method was considered to be the problem solution.

The study of the global minimum point location in the parameters range admissible with respect to restriction C , as well as its behavior in the evolution of vector G in the overhaul well exploitation interval, can be the subject of a separate investigation.

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