

The Simulation Realization of Pavement Roughness in the Time Domain

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Abstract. As the needs for the dynamic study on the vehicle-pavement system and the simulated vibration table test, how to simulate the pavement roughness actually is important guarantee for whether calculation and test can reflect the actual situation or not. Using the power spectral density function, the simulation of pavement roughness can be realized by Fourier inverse transform. The main idea of this method was that the spectrum amplitude and random phase were obtained separately according to the power spectrum, and then the simulation of pavement roughness was obtained in the time domain through the Fourier inverse transform (IFFT). In the process, the sampling interval (Δt) was 0.1m, and the sampling points(N) was 4096, which satisfied the accuracy requirements. Using this method, the simulate results of pavement roughness (A~H grades) were obtain in the time domain.

1. Introduction

Pavement roughness was an important controlling indicator of pavement engineering design, pavement construction and operational maintenance. Pavement roughness was the excitation source for the vehicle-pavement system. In previous research, the pavement roughness was usually used as a simple deterministic function, in fact, the pavement roughness was a stochastic process [1].As the needs for the dynamic study on the vehicle-pavement system and the simulation of vibration table, it was an important guarantee that how to simulate the pavement roughness actually.

At present, the research on power spectral density function of the vehicle-pavement system usually used linear elastic model in the frequency domain. When analysis nonlinear random vibration, the most effective way is to obtain the sample of random excitation as input, then calculate the random response of system by using numerical integration. So the stochastic process of pavement roughness in the simulation of time domain becomes important problem. The simulation of time domain is required to express an indeterminist function of time. More concretely, it means obtaining a height-time model which excited by roughness to get a three-dimensional model about vehicle speed-space pavement. In the former case, the testing simulation of ride comfort about vibration on bench. In the latter case, the testing simulation of ride comfort about vehicle-pavement.

2. Power spectral density function of pavement roughness

The evaluation method of pavement roughness can be divided into two categories, which is the method of dynamic reaction; the other is cross section method. The method of dynamic reaction is based IRI (International Roughness Index) as the representative and the method of cross section is



based PSI (Present Serviceability Index) or power spectrum method as the representative. At present, in the dynamic analysis of road structure, the method of power spectrum is more applied. In this paper, the power spectrum density function of pavement roughness was used as the basis of time domain simulation.

In 1984, International Organization for Standardization proposes “the draft of pavement roughness representation method” (ISO/TC108/SC2N67). The method adopts the formula (1) to express [2] as follows:

$$G_q(\Omega) = G_q(\Omega_0) \left(\frac{\Omega}{\Omega_0} \right)^{-w} \quad (1)$$

In the formula (1), Ω_0 is the frequency of reference space, $\Omega_0 = 0.1\text{c/m}$. $G_q(\Omega_0)$ is density of surface power spectrum under reference space frequency, which is called road roughness coefficient. w is frequency index, power spectrum curve is a slash in double logarithmic coordinates. General it take $w=2$. In fitting the measured pavement power spectrum in order to reduce the error, it can be used different fitting coefficients to segment fitting in different spatial frequency range, which should not over four. In ISO/TC108/SC2N67, a new pavement power spectrum density is proposed, which is divided into eight levels.

3. Power spectral density function of pavement roughness

The power spectrum density function (PSD) of pavement roughness was a statistics of pavement roughness, whether it was the standard pavement spectrum or the measured pavement spectrum. So the pavement elevation was determined in the measurement range, the PSD was the only one. But when PSD was given, the analog-design elevation of the pavement was not unique. In other words, the statistical model and time-domain model has not one-on-one mapping. Therefore, the time-history functions derived from the statistical model of pavement roughness should be seen as a sample function that meeting the time-function of roughness in the given pavement spectrum. At present, the most commonly used main methods are: secondary filtering, triangle series method, white noise filtering and so on. But they also have some problems. Secondary Filtering was required to design the filter. We need to design a reasonable filter when pavement roughness has different power spectral density function, which is short of generality. The triangle series method and White noise filtering were considered to be a stationary Gaussian Process. That obviously did not match the actual situation. What's more, the calculation speed of those methods was slow, and has some error in the obtained irregularity samples.

In this paper, due to the shortage of the above methods, the frequency spectrum was constructed by the discrete sampling of the power spectrum density. Time domain simulation of track roughness excitation function is applied in the inverse Fourier transform of frequency spectrum. The main idea of this method was the spectrum amplitude and random phase were obtained according to the power spectrum, and then the simulation of pavement roughness was obtained in the time domain through the Fourier inverse transform (IFFT).

First, introduce the period map method of power spectral density, it was estimated by time series. We assume that time series is $\{x_s\}$, $s = 0, 1, \dots, (N-1)$, record length is $T = N\Delta$, Δ means time interval.

The time delay of related functions $\tau = r\Delta$ is also discrete values. So we can get equations as follows:

$$R_{xx}(t) = \frac{1}{T} \int_0^T x(t)x(t+\tau)dt \quad (2)$$

$$R_r = R_{xx}(\tau = r\Delta) = \frac{1}{N} \sum_{s=0}^{N-1} x_s x_{s+r} \quad (3)$$

In the formula: $r = 0, 1, \dots, (N-1)$, $x_s \equiv x(s\Delta)$.

We can be obtained by $S_{xx}(f) \Leftrightarrow R_{xx}(\tau)$

$$S_{xx}(k) = \frac{1}{T} S_{xx}(f = k \frac{1}{T}) = \frac{1}{N} \sum_{r=0}^{N-1} R_r \exp[-i(k \frac{2\pi}{N})r] \quad (4)$$

so

$$\begin{aligned} S_{xx}(k) &= \frac{1}{N} \sum_{r=0}^{N-1} \left\{ \frac{1}{N} \sum_{s=0}^{N-1} x_s x_{s+r} \right\} \exp[-i(k \frac{2\pi}{N})r] \\ &= \left\{ \frac{1}{N} \sum_{s=0}^{N-1} x_s \exp[i(k \frac{2\pi}{N})s] \right\} \left\{ \frac{1}{N} \sum_{r=0}^{N-1} x_{s+r} \exp[-i(k \frac{2\pi}{N})(s+r)] \right\} \end{aligned} \quad (5)$$

We can make $j = r + s$, so we can get as follows:

$$\sum_{r=0}^{N-1} x_{s+r} \exp[-i(k \frac{2\pi}{N})(s+r)] = \sum_{j=s}^{(N-1)+s} x_j \exp[-i(k \frac{2\pi}{N})j] \quad (6)$$

Time series $\{x_s\}$ has been discrete cycled because of discrete Fourier transform, N as period. So we can get as follows:

$$\sum_{j=s}^{(N-1)+s} x_j \exp[-i(k \frac{2\pi}{N})j] = \sum_{j=0}^{N-1} x_j \exp[-i(k \frac{2\pi}{N})j] \quad (7)$$

Put formula (4) and formula (5) into formula (3)

$$\begin{aligned} S_{xx}(k) &= \left\{ \frac{1}{N} \sum_{s=0}^{N-1} x_s \exp[i(k \frac{2\pi}{N})s] \right\} \left\{ \frac{1}{N} \sum_{j=0}^{N-1} x_j \exp[-i(k \frac{2\pi}{N})j] \right\} \\ &= \frac{1}{N^2} |DFT[x_s]|^2 = \frac{1}{N^2} [X^*(k)X(k)] \end{aligned} \quad (8)$$

In the formula, $X(k)$ is spectrum of time series $\{x_s\}$, $k = 0, 1, \dots, (N-1)$

Modulus of spectrum in time series by formula (8) as follows:

$$\begin{aligned} |X(k)| &= |DFT[x(n)]| = \sqrt{N_r^2 \times S_k(k)} = N_r \sqrt{S_k(k)} \\ &= N_r \sqrt{S_x(f = k\Delta f)\Delta f} \quad k = 0, 1, \dots, N_r - 1 \end{aligned} \quad (9)$$

Modulus of Spectrum $X(k)$ in series $x(n)$ is concluded by formula (9). The spectrum phase must be random because the time series $x(n)$ was a stochastic process.

We assume that ξ_n as independent phase series, so the mean value of each component is zero. Because the real sequence FFT is a complex sequence (real part is even symmetric, imaginary part is odd symmetry). So ξ_n as plural and $|\xi_n| = 1$. We assume that as follows:

$$\xi_n = \cos \varphi_n + i \sin \varphi_n = \exp(i\varphi_n) \quad (10)$$

In the formula, φ_n obey the uniform distribution of $0 \sim 2\pi$

Because the real part of $X(k)$ about $N_r/2$ even symmetry, the imaginary part about $N_r/2$ odd symmetry. So we only need to calculate frequency spectrum $X(k)$, $k = 0, 1, \dots, N_r/2$. We can be obtained by the formula (7) as follows:

$$X(k) = \xi_k |X(k)| = N_r \xi_k \sqrt{S_n(f = k\Delta f)\Delta f} \quad k = 0, 1, \dots, N_r/2 \quad (11)$$

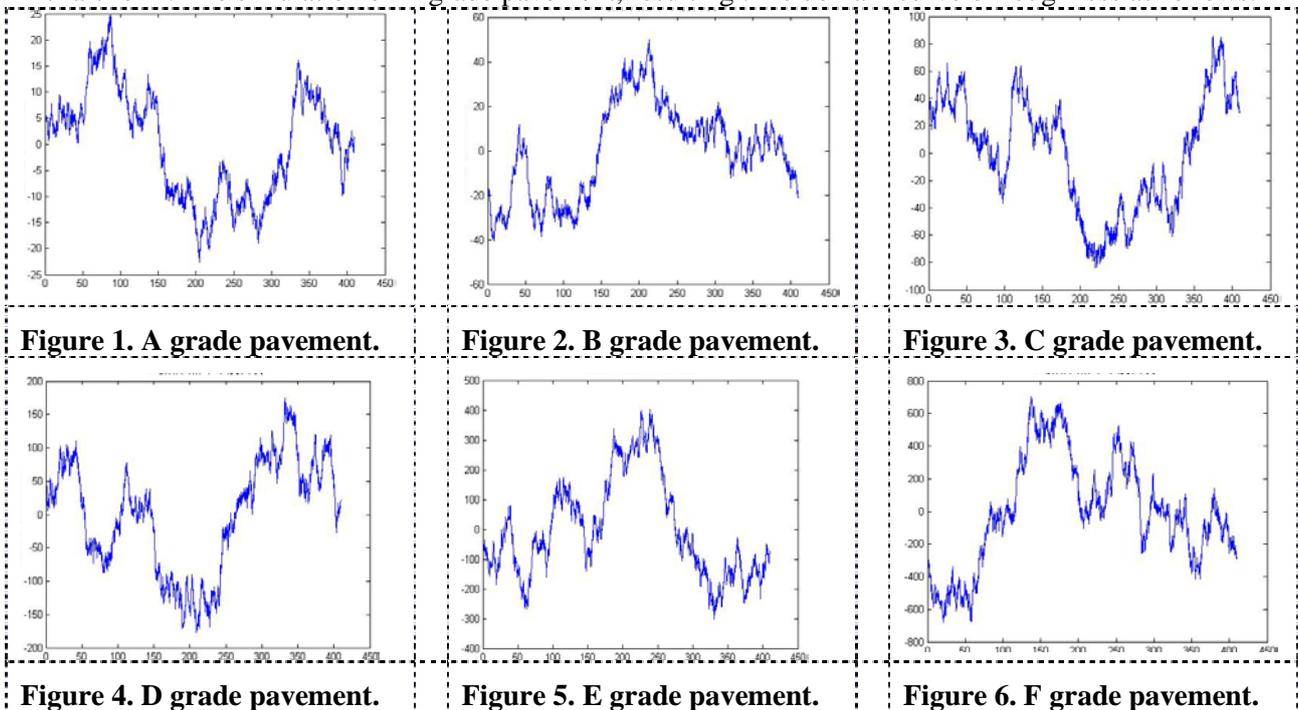
Under the symmetric condition, it obviously that we easy to get $X(k)$, $k = 0, 1, \dots, N_r - 1$.

The simulation of pavement roughness would be obtained in the time domain through the Fourier inverse transform (IFFT) of complex sequence $X(k)$.

$$x(n) = \frac{1}{N_r} \sum_{k=0}^{N_r-1} X(k) \exp\left[\frac{i2\pi kn}{N_r}\right] \quad n = 0, 1, \dots, N_r - 1 \quad (12)$$

4. The simulation results of each pavement roughness were obtain in time-domain

According to the above method, the corresponding mathematical calculation program was written for analyzing and calculating the given power spectral density function, the pavement roughness values list can be given. In terms of power spectral density grade, the simulate results of pavement roughness (A~H grades) were obtained in the time domain. The results were intuitive to express with chart form. The simulation of A grade pavement, resulting time domain curve of roughness as follows:



References

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