

Spatial Distribution of Transverse Dispersion Coefficients at U-shaped Bend Confluence with 60° Junction Angle

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Abstract. The confluence of bend channel and the tributary is fairly common in nature. At present the study about the dispersion capacity of pollutants in the bend confluence is very limited. In this paper, a three-dimensional numerical model was established to explore the hydraulic characteristics such as velocity distribution and circulation structure of the different sections in the U-shaped bend confluence. Then the transverse dispersion coefficient was calculated using the dispersion tensor method and N-zone model on the base of flow structure data, and its spatial distribution along the bend is analyzed. The results show that there is a significant difference in the flow structure of each section at the U-shaped bend confluence, especially before and after the junction of tributary. The peak value location of the transverse dispersion coefficient at the cross section appears near the inner bank. Along the bend channel, the transverse dispersion coefficient tends to increase first and then decrease, and the maximum value of the section appears before the bend apex where the tributary feeding into the main channel.

1. Introduction

An understanding of the fate of pollutants is essential for the management of river environments. Numerous investigators have contributed to the understanding of mechanisms of transverse dispersion in rivers. In 1978, Fischer et al. [1] estimated the mixing coefficients in a regular straight channel. For a laboratory straight flume, the dimensionless transverse dispersion coefficient ranges from 0.1 to 0.26 [2]. For curved rivers, the transverse dispersion coefficient was positively correlated with the bending angle and the width-depth ratio, but negatively correlated with the radius of curvature [3-6]. Rutherford [7] reported that the dimensionless value of the transverse dispersion coefficient for a curved channel is 0.3-1.0 and that the value for a strongly curved channel is 1.0-3.0. Boxall et al. [8] demonstrated that the secondary circulation was the main factor affecting the transverse dispersion of a curved channel.

However, the dispersion coefficient of pollutants in a channel confluence differs from the straight and curved rivers due to the distinctive flow structure of a confluence. Confluences

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can be widely found in the natural river systems and artificial channels. When a tributary feeds into a mainstream, two flows meet and mix with each other, and the flow structure of the original main channel changes drastically. Thus the mixing process and the dispersion capacity of pollutants in the main channel will also change significantly. However, at present the major studies about the confluence focus on the flow structure of the junction, especially for the junction of two straight channels [9-12]. Best [10] proposed a generalized model of flow for the open channel junction, which was divided into six different zones, namely, regions of flow stagnation, flow deflection, flow separation, maximum velocity, flow recovery, and shear layers. Recently, some flume experiments about the flow structure for U-shaped bend confluence were carried out [13-15]. Moreover, many researchers used the numerical models to explore the complex flow structure for confluences [16-19]. Above all, the research about the dispersion capacity of pollutants in the bend confluence has not been reported.

In this study, a three-dimensional numerical model was established to investigate flow structures and dispersion characteristics of U-shaped bend confluences. The hydrodynamic model was validated using Gao et al. [14] laboratory experiments data. Then the flow structures, especially the circulation structure of the bend were investigated, and the transverse dispersion coefficient was calculated by the dispersion tensor method and the modified N-zone model. Finally, the spatial distribution characteristics of the transverse dispersion coefficient along the bend was analyzed.

2. Setting up of three-dimensional mathematical model for U-shaped bend confluence

The simulated model domain was based on Gao et al. [14] laboratory experiments. The widths of main channel and tributary are 1 m and 0.3 m, respectively. In order to facilitate the full development of flow, the upstream and downstream flumes will be extended to 6m in the numerical model. The simple layout of the model was shown in figure 1.

The bend has a turning angle of 180°. The inner wall radius is 1.5 m and the outer wall radius is 2.5 m. The bottom section of the bend is of partial V-shape. The mainstream discharge Q_1 is 0.03 m³/s, and tributary discharge Q_2 is 0.009 m³/s. Define the discharge ratio $Q_r=Q_2/Q_1$. In addition, the junction angle of two channels is 60°, and the downstream water depth is 0.18 m.

2.1. Control equation and solution

The mass conservation equation and the momentum equations of Reynolds-averaged Navier-Stokes in three dimensions are as follows (in tensor form):

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \{ (\mu + \mu_t) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij} \} + (\rho - \rho_a) g_i \quad (2)$$

Where ρ is the fluid density, ρ_a is the density of air, p is the static pressure, g is the gravitational force, k is the turbulent kinetic energy, δ_{ij} is the Kronecker delta, μ is the molecular viscosity of fluid, and μ_t is the turbulence eddy viscosity of fluid.

In order to simulate the flow structure of U-shaped bend confluence more accurately, the three-dimensional RNG k - ε turbulence model of water-gas two-phase flow is used, and the free surface is tracked by VOF. In this study, the phases of air and water were represented by using the subscripts a and w respectively. Accordingly, the volume fractions of air α_a and water α_w sum to unity. Other details can be referred to Dai [20].

2.2. Boundary conditions

(a) Inlet and outlet boundary conditions: The velocity value and direction was specified at the upstream water inlet of mainstream and tributary. The pressure was specified at the upstream air inlet and downstream outlet.

(b) Wall boundary conditions: Non-slip boundaries were applied at all sidewalls and channel floor. The channel floor roughness height was 0.009.

2.3. Mesh generation

The total number of grids is about 600,000 and the local computational grids of bend is shown in figure 2. The mesh is refined near bend and near the interface of water and air.

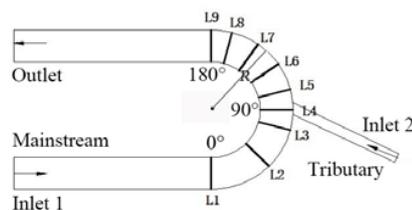


Figure 1. Sketch map of a simple U-shaped bend confluence.

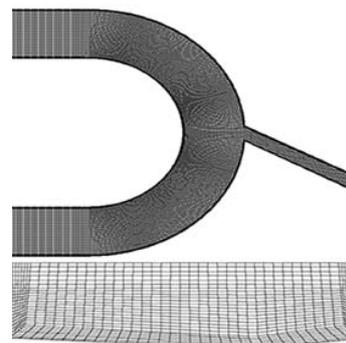


Figure 2. Local computational grids.

2.4. Model validation

Experimental results of Gao et al.[14] were used to validate the numerical model. The measured values and calculated values of the water surface elevation and longitudinal velocity were compared. The locations of some typical cross sections were shown in figure 1 and Table 1. All the section plots in this paper were looked downstream

Table 1. The cross section corresponds to the center angle.

Cross-section	L1	L2	L3	L4	L5	L6	L7	L8	L9
Central angle	0°	45°	75°	90°	105°	125°	145°	165°	180°

2.4.1. Surface elevation. The surface elevation of the measured and simulated values was shown in figure 3. It was apparent that water surface elevation of outer bank was significantly higher than the inner bank before the tributary into the U-shaped bend. The water surface elevation will rapidly decline after the confluence, and then slowly rising in the downstream. The water surface elevation was relatively low near the inner bank. It can be seen that the simulated surface elevation coincides with the measured value basically, and the deviation was less than 5%.

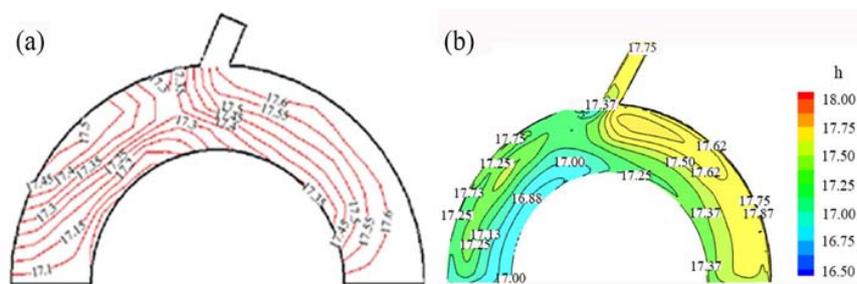


Figure 3. The surface elevation of the measured and simulated value, (a) measured value (b) simulated value (the unit is m).

2.4.2. Longitudinal velocity distribution. The longitudinal velocity comparison of the inner bank ($R = 1.55$), centerline ($R = 2.05$), and outer bank ($R = 2.45$) for cross-section L4 was

shown in figure 4. z^* , u^* was the dimensionless depth and velocity. Obviously, the simulated value had a good agreement with measured value.

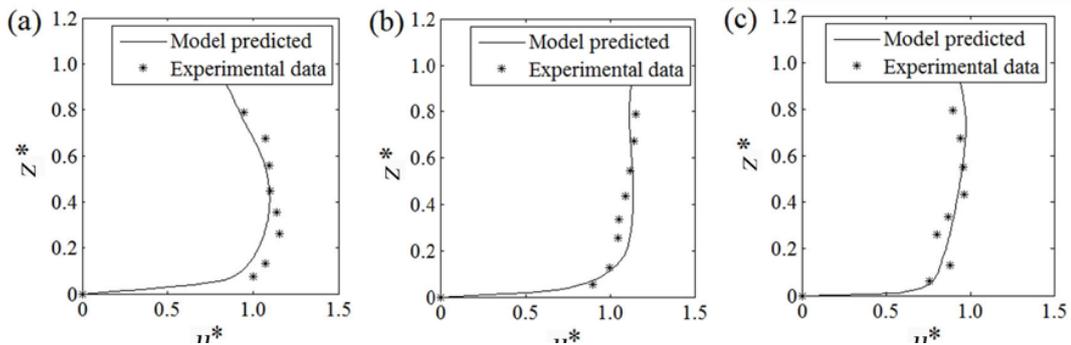


Figure 4. Comparison of measured and calculated longitudinal velocity for cross-section L4, (a),(b),(c) are the verticals at $R=1.55\text{m}$, $R=2.05\text{m}$ and $R=2.45\text{m}$, respectively; R is the distance from the center of the bend to the measuring point.

3. Spatial distribution of transverse dispersion coefficients

The detailed three-dimensional flow structure of the whole U-shaped bend confluence was obtained by the above mathematical model for the test with the junction angle 60° and discharge ratio $Q_2=0.3$. Then, the transverse dispersion coefficient of the U-shaped bend confluence was calculated by the two-dimensional dispersion tensor method proposed by Fischer [2], and its spatial distribution characteristics along the bend was analyzed.

3.1. Calculation Method of Dispersion Coefficient

A dispersion tensor method by Fischer (1979) [2] was used to calculate the transverse dispersion coefficient at the typical verticals of the cross-section, where the equation for transverse dispersion coefficient was as follows,

$$D_{yy} = -\frac{1}{h} \int_0^h v' \int_0^z \frac{1}{\varepsilon} \int_0^z v' dz dz dz \quad (3)$$

where D_{yy} is a component of the dispersion tensor in the y direction, h is the local depth of flow, v' denotes the vertical deviations of point velocities with respect to the depth-averaged velocity v , and ε is a vertical turbulent diffusion coefficient.

Then the modified N-zone model [21] was used to calculate a cross-sectional dispersion coefficient value by Fourier transform of the point values at every vertical line. The modified N-zone model [21] was based on the model of Chickwendu [22]. However the transverse velocity was divided into N different regions in the lateral direction, which is different from the model of Chickwendu [22]. Combined with the dispersion tensor method and the modified N-zone model, the variation of transverse dispersion coefficient was studied in the lateral and longitudinal direction, respectively. The specific calculation method can be referred to Jiao [21] and Gu et al. [23].

3.2. Result analysis

The circulation structure and lateral distribution of transverse dispersion coefficient of cross section L2, L4 and L6 were shown in figure 5. At section L2, the transverse dispersion coefficient increases first and then decreases, and the maximum value appears at $y=0.45\text{ m}$ near the centerline of the section (figure 5(b)). The flow structure of this section was shown in figure 5(a), where a big clockwise circulation exists at the whole section and the big transverse velocity gradient of the vertical is at the center of the circulation. This strong velocity shear leads to the increase of the transverse dispersion coefficient, and thus the peak value occurs near the middle of the section (figure 5(b)).

L4 were the cross-section where the tributary feeds into the mainstream. It was obvious from figure 5(d) that the transverse dispersion coefficient has two peak values in this section,

and the peak near the inner bank was larger than the one near the outer bank. The reason was that a larger clockwise circulation occurs near the inner bank, and the larger lateral velocity difference of the verticals makes the transverse dispersion coefficient increase. On outer bank, due to the inflow of tributary, lateral velocity was big, however the difference between the lateral velocity was not big. Thus there was a smaller peak on outer bank (figure 5(c)).

On the section L6, the clockwise circulation and counterclockwise circulation appear in the inner bank and the outer bank, respectively. Due to the two circulation structures, the transverse dispersion coefficient appears two peaks at cross section L6, and the value of the outer bank was significantly smaller than the inner bank (figure 5(e) and (f)). The peaks value of transverse dispersion coefficient for cross section L2, L4 and L6 all appear near the inner bank. However the transverse dispersion coefficient of L6 with the double circulation structure was significantly smaller than that of L2 and L4 with a single clockwise circulation. (figure 5(b),(d) and (f)).

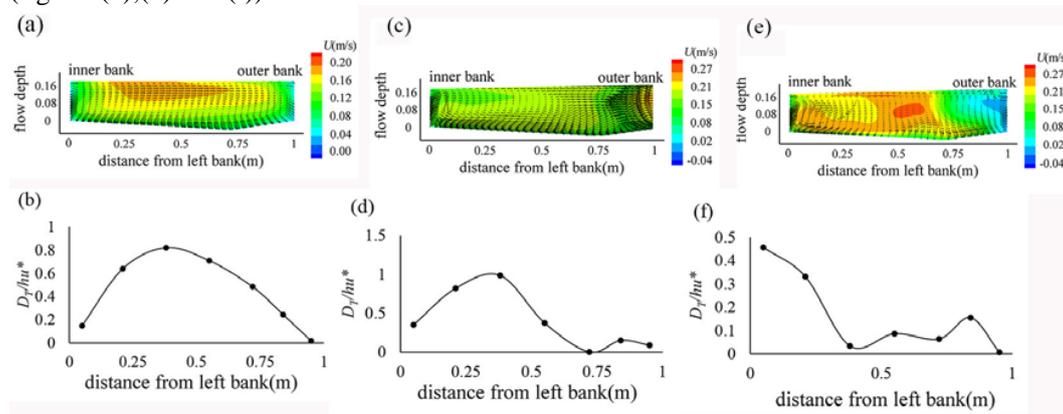


Figure 5. The circulation structure and lateral distribution of transverse dispersion coefficient of cross section L2, L4 and L6: (a) and (b), (c) and (d), (e) and (f) are corresponding to L2, L4, L6, respectively.

The longitudinal distribution of the transverse dispersion coefficients of different sections was depicted in figure 6. It can be seen that the transverse dispersion coefficient increases first and then decreases along channel. The maximum value appears at the positions of the cross section L2 and L3, whose values were a little bigger than the bend apex L4 section. The flow structures of cross sections L2 and L3 were similar to L4, where a single clockwise circulation exists at the whole cross section, and a large lateral velocity difference results in a larger transverse dispersion coefficient value.

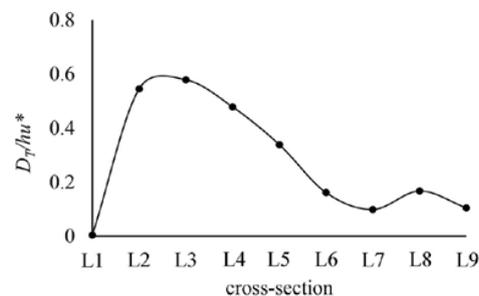


Figure 6. The longitudinal distribution of the transverse dispersion coefficients.

4. Conclusion

The aim of this research was to investigate the spatial distribution of transverse dispersion coefficient in the U-shaped bend confluence. By analysis, it was found that there is a significant difference in the flow structure of each section at the U-shaped bend confluence. Before the tributary into bend, only a single clockwise circulation was formed in the cross

section, and after the junction, the reverse double circulation structure is formed in the cross section with the clockwise cell at inner bank and counterclockwise cell at outer bank. The circulation structure makes the spatial distribution of transverse dispersion coefficients change. In the lateral direction, the peak value position of the transverse dispersion coefficient appears near the inner bank. In the longitudinal direction, the transverse dispersion coefficient tends to increase first and then decrease along channel, and the maximum value of the section appears before the bend apex with the tributary inflow.

5. References

- [1] Fischer H B 1978 *J. Geophys. Res.* **83** 2373-75
- [2] Fischer H B, List E J, Koh R C Y, Ouml, Omberger R G and Brooks N H 1979 *Mixing in inland and coastal waters* (London: Academic Press)
- [3] Beltaos S 1979 *Can. J. Civ. Eng.* **6** 575-591
- [4] Xing Z and Jiang N 1994 *Chongqing. Environ. Sci* 38-43 (In Chinese)
- [5] Albers C and Steffler P 2007 *J. Hydraul. Eng.* **133** 186-196
- [6] Garutti E 2011 *J. Hydraul. Eng.* **137** 1126-34
- [7] Rutherford J C 1994 *River mixing* (England: Wiley)
- [8] Boxall J B, Guymer I and Marion A 2003 *J. Hydraul. Res.* **41** 153-165
- [9] Best J L and Reid I 1984 *J. Hydraul. Eng.* **110** 1588-94
- [10] Best J L 1987 *Special Publications* **39** 27-35
- [11] Weber LJ, Schumate E D and Mawer N 2001 *J. Hydraul. Eng.* **127** 340-350
- [12] Szupiany R N, Amsler M L, Parsons DR and Best JL 2009 *Water Resour. Res.* **45** 641-8
- [13] Li W Q 2012 *Water transport engineering* 62-68 (In Chinese)
- [14] Gao Y, Huang S H and Li Q 2012 *Hydropower and Energy Science* **30** 90-93 (In Chinese)
- [15] Fu Z M, Gu Z P, Zheng J T and Lie G P 2013 *Water transport engineering* 46-51 (In Chinese)
- [16] Biron P M, Ramamurthy A S and Han S 2004 *J. Hydraul. Eng.* **130** 243-253
- [17] Constantinescu G, Miyawaki S, Rhoads B, Sukhodolov A and Kirkil G 2011 *Water Resour. Res.* **47** 5507
- [18] Shakibainia A, Tabatabai M R M and Zarrati A R 2010 *Can. J. Civ. Eng.* **37** 772-781
- [19] Wang X and Cheng L 2000 *Computers & Fluids* **29** 415-433
- [20] Dai B 2014 Master Thesis Nanjing: Hohai University (In Chinese)
- [21] Jiao Z N 2015 Master Thesis Nanjing: Hohai University (In Chinese)
- [22] Chikwendu S 1986 *J. Fluid Mech.* **167** 19-30
- [23] Gu L, Jiao Z N and Hua Z L 2014 *Water Sci. Technol.* **70** 256-264