

Differences in methods of quantifying the vertical hyporheic flow for streambank flow field

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Abstract. One-dimensional analytical methods are generally used to estimate the vertical hyporheic velocity, and inevitably yield values that differ from the true values when assumptions are not satisfied. In this study, numerical simulations were used to determine the amount of errors in the vertical hyporheic flow from several 1D analytical methods, including the solutions from amplitude ratio, phase shift of temperature, and hydraulic gradient with depth, under gaining and losing conditions. Results demonstrate that the velocity at the shallow layer is generally higher than the velocity at the deeper layer. Horizontal flow dominates the hyporheic exchange in the streambank-aquifer continuum at distances further away from the central channel. The amplitude ratio method and gradient method estimate the vertical velocity well when the vertical flow dominates the hyporheic flow and the flow is upwelling; otherwise, these two methods yield very inaccurate results. The amplitude ratio method and phase shift method yield better results compared with the gradient method for the streambank area. Our simulations indicate that two or more 1D analytical methods should be used in conjunction, and that the multi-dimensional hydraulic heads should be measured via an appropriate monitoring network to more accurately understand the hyporheic flow of a streambank.

1. Introduction

Heat transport is an important component in a variety of hydrologic and environmental processes. Diurnal and annual variations in hyporheic exchange, which means the water that flows from streams or rivers into shallow subsurface and then returns to the stream [1], have been directly linked to diurnal and annual variations in stream water temperature [2, 3]. Heat tracing is useful to depict the hyporheic flow path and quantify the associated flow [4]. Constantz impressed that advancements of temperature - based methods could produce a new “streambed science” [3].

Analytical solutions of the heat transport equation [5-8] have been built to evaluate surface water-groundwater interactions. However, the analytical methods are limited to environments with vertical 1D flow and sinusoidal temperature fluctuations [9-11]. In fact, the flow field in a natural stream is rather complicated, especially in the downstream reach which generally exhibits meanders. However, the stream channel within the mountainous area is rather straight and simple compared to the downstream reach. That is why 1D analytical model was widely used.



A generally simplification of the groundwater flow governing equations was originated from the assumption of the strictly vertical flow into or out of the streambed [2, 12-15]. The heat tracing method for hyporheic exchange was in a large part derived by Stallman [6]. A special solution of a 1D vertical hyporheic exchange was developed and applied [7, 8, 16, 17].

The hyporheic flow path is apt to tend toward a vertical direction near the riverbed interface and along a horizontal direction beneath the riverbed. For such cases, more sophisticated 2D numerical models have been used [18-20], but 2D numerical models generally require more information and therefore, can be difficult to apply. Analytical 1D models solving the heat transport equation have been popular for interpreting temperature data at the scale of a streambed due to their ease of use and cost-effectiveness, and because the heat flow parameters vary much less across a range of sediment textures than, for example, hydraulic conductivity [21].

Notably, the natural hyporheic exchange is more complicated than the assumed 1D flow solutions. These 1D solutions are based on simplified assumptions that always do not conform to the laws of nature. A non-vertical flow field, which is common in natural streams, can result in wrong result. Lautz [10] indicated that the deviation between the estimated and natural flux is mainly caused by non-vertical flow.

Analysis of non-vertical flow through heat tracing has been well documented by Jensen and Engesgaard [4], Cardenas and Wilson [9], Lautz [10], and Roshan et al [22]. Jensen and Engesgaard, Lautz, Shanafield et al and Swanson and Cardenas [4, 10, 19, 23,] documented that 1D temperature - based solutions inevitable make errors when estimating the vertical velocity with the flow dominated by horizontal flow. Cardenas and Wilson [9, 24] investigated the non-vertical flow induced by current-bed forms generated complex flow patterns when heat is used as a tracer. Additionally, Lautz [10] reported the deviations of the 1D vertical flow detailedly. Cuthbert and Mackay [25] inferred that convergent or divergent flow is occurring and that a 1-D analysis is inappropriate. Irvine et al [26] reported the best heat tracing method for a heterogeneous field.

Roshan et al [22] noted that horizontal flow is likely to occur in natural streams for not wide streams. In fact, a degree of horizontal flow always occurs beneath the streambed and especially near the stream banks. Different zones with identical hydraulics conditions have been recognized along the cross-section direction. The field works by Jensen and Engesgaard [4], and Shanafield et al [19] approved that temperature measurements are well dependent with its horizontal location. Moreover, Shanafield et al [19] also stated that 1D analytical solution can only fit the field temperature measurements at one side of the channel.

Shanafield et al [19] emphasized that horizontal movement of water through stream banks, as opposed to the streambed, may play a significant role in total infiltration; however, this idea has received little attention. To our knowledge, the impacts of non-vertical flow subject to a bank-aquifer continuum on thermal-based and gradient-based hyporheic flux estimates have not been studied in any detail. This type of hyporheic process could yield a highly variable flow field in the conditions of losing and gaining, which would result in severe deviations from the 1D field. A riverbank case is used herein to investigate the applicability of Heat-based analytical solutions and the Darcy-based gradient method.

The aim of this study is twofold: to characterize the flow paths within a riverbank under losing and gaining conditions and to assess the ability of analytical heat transport models and the gradient method for quantifying vertical fluid fluxes within a riverbank. For this, virtual experiments with numerical simulations were used. First, the 2D heat-mass transport equation was solved using VS2DH to generate hyporheic flow fields under both gaining and losing conditions and associated temperature distributions for a riverbank system. Secondly, the estimated streambed temperature was used to estimate the vertical velocity via these 1D analytical solutions. In addition, the vertical velocities were also calculated via the Darcy-based gradient method. Finally, comparison between these estimates and the simulated velocities was performed to identify the impact of horizontal flow component from these analytic methods.

2. Methods

2.1. Mass and heat transfer equation

2.1.1. Energy transport equation. The energy transport equation, which is actually a form of the advection-dispersion equation, is derived by balancing the changes in energy stored within a volume of porous media. Such changes occur due to the flow of ambient water with different temperature into the volume, thermal conduction into or out of the volume, and energy dispersion into or out of the volume. The unsaturated zone is an essential part of the hydrologic continuum in a streambank-aquifer system. The advection-diffusion equation is shown as following:

$$\frac{\partial}{\partial t}[\theta C_w + (1 - \phi)C_s]T = \nabla K_T(\theta)\nabla T + \nabla \theta C_w D_h \nabla T - \nabla \theta C_w v T + q C_w T^* \quad (1)$$

where t refers to time, T refers to temperature, T^* is temperature of fluid source, ϕ is the sediment porosity, θ is the percentage of volume water content, K_T is the thermal conductivity of the bulk sediments, D_h is the thermal mechanical dispersion, v is the flow velocity, and q is the rate of flow source. C_s and C_w are the thermal capacities of the sediment and water, respectively. Further details on equation (1) can be obtained from Voss [27] and Kipp [28].

The thermal mechanical dispersion is a tensor defined as [29]

$$D_h = \alpha_t |v| \delta_{i,j} + \frac{(\alpha_l - \alpha_t) v_i v_j}{|v|} \quad (2)$$

where α_l and α_t are the longitudinal and transverse dispersivities, respectively, $\delta_{i,j}$ is the Kronecker delta function and equals 1 if $i=j$ and otherwise is equal to 0, v_i and v_j are the i^{th} and j^{th} component of the velocity vector, respectively, and $|v|$ is the magnitude of the velocity vector. Rau et al [30] and Roshan [22] indicated that the thermal dispersion induced less change compared to the hydraulic conductivity in the heat transport results using a range of sandy field velocities.

2.1.2. Fluid Flow Equation. The flow velocity within variably saturated material from equation (1) is generally described by Richards' equation. The hyporheic flow considered in this study is a 2D flow, including horizontal (x) and vertical (z) dimensions. The fluid flow is governed by equation (3) as follows:

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[K(h) \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial z} \left[K(h) \frac{\partial h}{\partial z} \right] + \frac{\partial K(h)}{\partial z} \quad (3)$$

where $C(h)$ is the specific moisture capacity, which is the slope of the retention curve = $d\theta/dh$, h is the pressure head, t is time, and K is the unsaturated hydraulic conductivity and is calculated by van Genuchten model.

2.2. Numerical model description

2.2.1. Model domain, initial condition (IC), and boundary conditions (BCs). The synthetic temperature series generated by numerical model were used to evaluate the impact of horizontal flow on vertical water velocity estimates. A streambank-aquifer continuum was conceptually modeled, in which hyporheic exchange occurs perpendicular to the channel. Exchange may occur in the 3D domain; however, the scenarios along the stream flow have been discussed by Lautz [10], which are regarded as constant in this case. Therefore, our numerical model was developed according to the 2D flow field. Simplifying the model extent to a streambank 2D cross-section in a lateral flow direction helps reduce model complexity for defining the boundary conditions and recognizing the hyporheic flow paths.

The solution of equations (1) and (3) requires a boundary condition within the model domain combined with the pressure head and temperature distribution:

$$\begin{cases} T(x, y, t) = T_0(x, y) \\ h(x, y, t) = h_0(x, y) \end{cases} \quad t = 0 \quad (4)$$

where the initial pressure head and temperature are set to 0 m and 15 °C, respectively. The top of the domain as a stream was set by Dirichlet-type boundary condition for temperature,

$$T(x, 0, t) = T_0 + \Delta T \cdot \cos\left[\frac{2\pi t}{Pr}\right] \quad (5)$$

where T is of given function on the specific boundary, and Pr refers to the period of temperature variation ($Pr = 1$ day). T_0 and ΔT refer to the mean temperature and the amplitude ratio on the surface ($T_0 = 15^\circ\text{C}$ and $\Delta T = 5^\circ\text{C}$), respectively. A constant head on the top stream boundary was set at 0 m. A water head with positive (0.3 m, 0.2 m, and 0.1 m) and negative values (-0.1 m, -0.2 m, and 0.1 m) was also applied on the left boundary for losing and gaining condition, respectively. The left side of the domain was head boundary, where different heads generated different average gradients (-0.1, -0.067, -0.033, 0.33, 0.67, and 0.1) as shown in table 1.

Table 1. Numerical scenarios of the flow and heat transfer model.

| Code of scenario | Total head of left side boundary | Total head of stream | Average gradient | Type of stream |
|------------------|----------------------------------|----------------------|------------------|----------------|
| L3 | -0.3 | 0 | -0.1 | <i>Losing</i> |
| L2 | -0.2 | | -0.067 | |
| L1 | -0.1 | | -0.033 | |
| G1 | 0.1 | | 0.033 | <i>Gaining</i> |
| G2 | 0.2 | | 0.067 | |
| G3 | 0.3 | | 0.1 | |

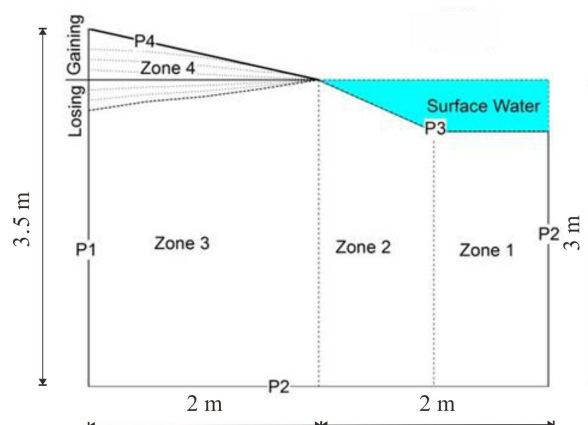


Figure 1. Schematic of the model domain including the boundary conditions using VS2DH.

The losing and gaining condition was achieved by modifying the pressure of the left boundary P1. P1 and P3 are total head boundary conditions, and P2 and P4 are the no-flow boundaries.

Ronan et al [31] showed that the center vertical line can be regarded as a no-flow boundary for both gaining and losing streams at the steady state. Therefore, the right side and the bottom of the domain were modeled as a no-flow boundary. The settings of the model domain setting are shown in figure 1, which depicts the cross section of streambank-aquifer continuum. According to a general scale of mountainous stream, the depth of the domain was set at 3 m. And the widths of the surface water and model domain shown in figure 1 were set at 2 and 4 m, respectively. Moreover, the 3m depth was determined because there is not apparent changes in heat transfer when the depth surpassed 3m in a bank-aquifer flow system with the dimensions in this case.

For the flow system, P1 (different pressure at each simulation) and P3 (representative of surface water) are the total head boundary conditions, and P2 and P4 are the no-flow boundaries. A simple total head boundary of surface water was used for P3. In terms of energy transport, P2 is the no-energy flux boundary, and P1, P3, and P4 are specific temperature boundary conditions according to the temperatures of groundwater, surface water, and the ground, respectively.

Four zones are marked in figure 1. Zone 1 is the area beneath the central streambed, and zone 2 is the area beneath the side slope of the streambed. Zones 3 and 4 are beneath the river bank, and represent the saturated zone and vadose zone, respectively. The unsaturated zone was also included in the simulations because changes in the ground temperature could have an impact on the ground water temperature in a relatively shallow system. However, the unsaturated flow and energy transport are notably different in the saturated zone and this is inappropriate for heat tracing methods [7,8]. Therefore, the calculated temperature series data collected from zone 4 was not used to analyze the discrimination in 1D analytical solution. The thermal and hydraulic properties of water and sediment adopted in the numerical simulations are listed in table 2.

Table 2. Physical properties of water and sediments used in numerical model.

| Parameter | Unit | Symbol | Value |
|-----------------------------------|----------------------|------------|-----------------------|
| Solid Thermal Conductivity | W(mC) ⁻¹ | k_s | 2.4 |
| Water Thermal Conductivity | W(mC) ⁻¹ | k_f | 0.6 |
| Solid Specific Heat Capacity | J(kgC) ⁻¹ | c_s | 865 |
| Water Specific Heat Capacity | J(kgC) ⁻¹ | c_f | 4201 |
| Solid Density | kg m ⁻³ | ρ_s | 999.7 |
| Water Density | kg m ⁻³ | ρ_f | 2650 |
| Porosity | - | n | 0.375 |
| Longitudinal Thermal Dispersivity | m | α_l | 0.1 |
| Transverse Thermal Dispersivity | m | α_t | 0.1 |
| Saturated Hydraulic Conductivity | m/s | K | 4.05×10 ⁻⁵ |

2.2.2. Change in Peclet number and seepage velocity. The Peclet number (Pe) is a dimensionless number relevant in the study of transport phenomena in fluid flows. It is defined as the ratio of the rate of advection of a physical quantity by a flow to the rate of dispersion of the same quantity driven by an appropriate gradient. The hydraulic gradient changed in every simulation scenario, resulting in different seepage velocities and the associated Peclet number.

In the context of the transport of heat, the Peclet number is equivalent to the product of the Reynolds number (Re) and the Prandtl number (Pr).

For diffusion of heat (thermal diffusion), the Peclet number is defined as:

$$Pe = Re \cdot Pr = \frac{LU}{\alpha} \quad (6)$$

where L [m] is the characteristic length (considered as constant grid cell length), U [m/s] is the seepage velocity [m/s], and α [m²/s] is the thermal diffusivity,

$$\alpha = \frac{k}{\rho c_p} \quad (7)$$

where k [W/(m·K)] is the thermal conductivity, ρ [kg/m³] is the density, and c_p [J/(kg·K)] is the heat capacity.

2.2.3. Numerical code and simulation time. The USGS variably saturated 2D hydraulic model, VS2DH [29, 32, 33], is a 2D finite difference model that simulates heat and water flow in the hyporheic zone beneath a river. VS2DH has been successfully used to describe heat transport in variably saturated material at several sites near streams [31, 34]. The program provides numerical solutions of variably saturated porous media flow and heat transport problems, and is applicable to the study of unsaturated-zone riparian hyporheic exchange problems. Heat transport is simulated using the advection-diffusion equation and the flow of water is described by the Richards' equation, shown in equations (1) and (3).

VS2DH was used herein to generate synthetic time series temperature data for the streambed. The number of grids is 140×160, with the size of each grid equals to 25 mm × 25 mm. For each hydro-scenario, a three days period of the numerical model was simulated when a steady state flow field was attained. Each scenario was simulated for a period of three days. During the first two days (48 h) of each simulation, the flow field was influenced by the initial conditions, and therefore, the results of these two days were excluded from the analysis and only information from the last day was analyzed, as recommended by Lautz [10].

2.3. 1D analytical modeling of streambed heat transport and vertical gradient

Vertical velocities of hyporheic flow were estimated from the vertical pairs of modeled time series temperature based on analytical solutions to the 1D heat transfer equation and several assumptions [6-8]. This heat tracing method is advantageous because hyporheic flow can be obtained over time periods without calibrating two- or three-dimensional models. More details on the analytical solution can be found in Hatch et al. and Keery et al [7, 8].

The MATLAB-based VFLUX program solves analytical approximations [7, 8] of the 1D advection-diffusion equation to calculate vertical fluid flux in saturated porous media. First, VFLUX re-samples the measured data to achieve optimal sampling frequency (12-24 samples/d) for power-spectral analysis, which is used to identify the dominant diurnal component of the signal. This spectral analysis and subsequent extraction of the diurnal signal from each time series is performed using dynamic harmonic regression (DHR), as implemented by the Captain Toolbox [35, 36]. VFLUX then reports the amplitude ratio and phase shift of each temperature time series at the reduced sampling frequency (here, 2-hr time steps), and calculates the fluid flux based on the A_r and $\Delta\phi$ between the upper and lower diurnal signals. The same measured thermal parameters used in VS2DH were employed for the analytical models.

The analytical solutions to the 1D advection-diffusion equation, as presented by Hatch et al [7], predict A_r and $\Delta\phi$ for any given rate of vertical flux (q) based on v_t :

$$q = \frac{c}{c_w} \left(\frac{2k_e}{\Delta z} \ln A_r - \sqrt{\frac{\alpha + v_t^2}{2}} \right) \quad (8)$$

$$|q| = \frac{c}{c_w} \sqrt{\alpha - 2 \left(\frac{4\pi\Delta\phi k_e}{P\Delta z} \right)^2} \quad (9)$$

where C_w and C are the heat capacity of the water and sediment-water matrix, respectively; note that

only absolute values of vertical fluid flux can be determined with equation (9) due to similar $\Delta\phi$ reduction with both upwelling and downwelling. An upward velocity (gaining condition) and a downward velocity (losing condition) are defined as positive and negative, respectively.

Probe arrays T1, T2, and T3 were positioned at the central line of zone1, zone 2, and zone 3, respectively. Roshan et al [22] indicated that deeper probes were more probably to lead to significant errors. Therefore, two probes in every probe array were placed close to the sediment water interface (SWI) at depths of 0.1 and 0.3 m below the SWI. The derived flux can be apparently compared with the seepage velocity at a cell 0.2 m below the SWI. The average velocity from each zone was also compared with the estimated velocity via analytical solutions.

The numerical model listed the hydraulic head for each cell, which was used to calculate the vertical velocity through the Darcy-based gradient method. Under the general assumption of strictly vertical flow under the streambed, the vertical velocity can be estimated using Darcy's Law, the hydraulic heads, the distance between the two points, and the average hydraulic conductivity between the two points. The estimated vertical velocities from the gradient method and the heat transport method (Ar and $\Delta\phi$ methods) were compared with the 'true' velocity generated by the numerical model. The points located at the depths of 0.1 m and 0.3 m below the SWI were analyzed using the heat transport method and the gradient method.

3. Results

In general, the estimated velocity errors of stream bank-aquifer system is greater compared to the purely vertical flow. To generate a wider range of perturbations in the propagation of surface heat signals into the streambed, several horizontal gradients were applied in this study. These simulations could be used to reveal the effects of non-vertical flow on the vertical flow estimates using 1D analytical heat tracing and Darcy-based solutions for the riverbank cross-section.

3.1. Point and averaged V_z and V_x under different simulation scenarios

The flow velocity of each zone defined in figure 1 has its own spatial distribution. In the numerical model, the domain of zone 3 was changed and easily determined according to the saturation of each grid. Then, the average velocity of each zone was obtained. The observed points were assigned at the central line of each zone, and located 20 cm beneath the upper boundary of each zone. The temperatures in the shallow area, i.e. T1, T2, and T3, were used to analyze the vertical velocity through 1D analytic solution. Therefore, first the relationship between the point observed velocity and the average velocity was analyzed.

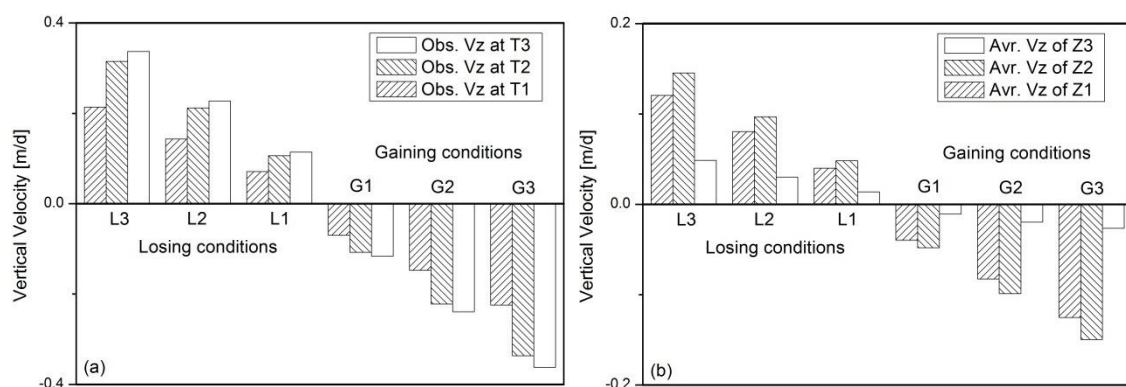


Figure 2. Measured point vertical velocities (a) and average vertical velocities (b) under different hydro-scenarios.

Positive and negative V_z indicate potential flow in the upwelling and downwelling directions, respectively.

Figure 2 illustrates the point vertical V_z (figure 2(a)) as well as the average V_z for each zone

(figure 2(b)) under six different conditions; L1, L2, and L3 for losing, which means the surface water recharge the groundwater. G1, G2, and G3 for gaining conditions, which indicates the groundwater recharge the surface water. The relationship between the point vertical velocities at the three zones can be explained using figure 2(a). For the point vertical observation, the velocities observed at T3 were the largest among the three points. The T1 point velocities were the smallest, and T2 had medium-level point velocities. Similar patterns were observed under gaining and losing conditions.

Interestingly, the average velocities show a different pattern (figure 2(b)). With respect to the average V_z values, zone 2 had the largest value, while zones 1 and 3 had the second largest and smallest values, respectively. This pattern is different from that shown in figure 2(a).

The point V_z values at zones 1 and 2 are around 1.77 times and 2.22 times larger than the corresponding average V_z values. However, the point V_z values at zone 3 ranged from 6.91 to 13.75 times greater than the average values of the six simulation scenarios. The hydraulic gradient of these scenarios decreased from L3 (or G3) to L1 (or G1). Both the point V_z and average V_z showed a consistent decreasing trend with the hydraulic gradient.

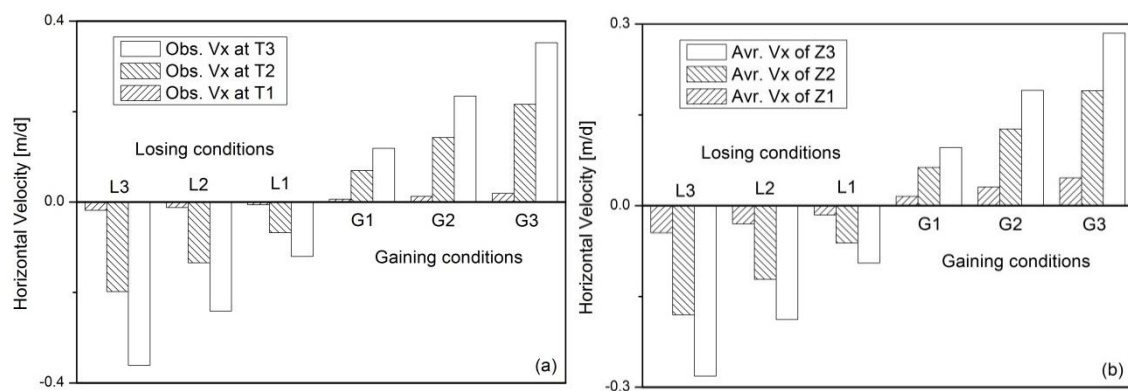


Figure 3. Measured point horizontal velocities (a) and average horizontal velocities (b) under different hydro-scenarios.

Although only the vertical velocity can be estimated using equations (8) and (9), the natural movement of flow is 2D even within a 3D flow field. In the cross-sectional numerical model in this study, the horizontal velocity can be easily obtained. The point horizontal velocity and average V_x are shown in figure 3 under different scenarios. The relationship among the average V_x of the three zones is similar to that of the point V_x among the three observed points, and is relatively more straightforward as compared with the V_z pattern. The order of magnitude of the V_x of the three points was $T1 > T2 > T3$. Similarly, the order of average V_x magnitude was Zone 1 > Zone 2 > Zone 3. The point V_x shows the same pattern as the average values, but the magnitude of the point V_x is significantly different than the zonal V_x . The V_x at T1 for the L3 scenario is -0.018 m/d, and the average V_x of Zone1 for L3 is -0.045 m/d. The ratio of the average V_x to the point value for zone 1 is generally around 2.46. This ratio decreases to 0.90 and 0.80 for zone2 and zone 3, respectively. This shows that the superficial horizontal velocity is much higher than the horizontal velocity at deeper locations in zone 1, and that this pattern is opposite for zones 2 and 3.

Both the point V_x and the average V_x show an increasing trend with the general horizontal gradient from L1 (G1) to L3 (G3). This trend is the same as that of the V_z in figure 2. This indicates that both the V_x and V_z are closely controlled by the hydraulic gradient. However, the ratio of V_x to V_z at each location did not vary significantly with the hydraulic gradient in our results. Therefore, the hydraulic gradient between groundwater and surface water varied the magnitude of V_x and V_z spatially, but the flow path and direction did not change significantly with the hydraulic gradient.

The 1D heat tracing analytic method requires diurnal temperature variations. The observed diurnal temperature variations at the points were used to estimate the hyporheic flux. The 2D bank-aquifer

flow fields, which is obtained by the numerical simulations under losing (L3) and gaining (G3) conditions, are presented in figures 4(a) and 4(b), respectively. Figures 4(a) and 4(b) show that the velocity over the cross-section is not consistent; the velocity vector directions are almost opposite, and the magnitude for any point within zones 1, 2, and 3 are almost equal.

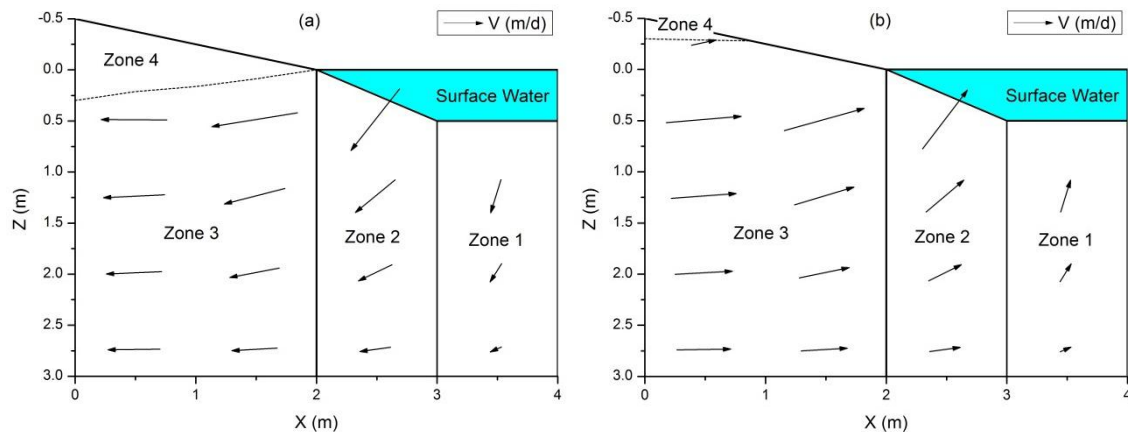


Figure 4. Flow vectors in the cross-section on the third day of scenarios L3 (a) and G3 (b).

The vertical velocity in real is not always constant with depth for most field settings [22]. The simulation results show that the velocity distribution can be described by a function, which depends on depth and location if the hydraulic heads of the boundaries are unchanged. For zone 1, which is beneath the center of the streambed, the shallower part has a larger velocity, and the horizontal component is relatively smaller than the vertical component. With increasing depth, the magnitude becomes smaller whereas the ratio of the horizontal component to the vertical component of flow velocity increases to become larger than one.

Jensen and Engesgaard [4] suggested that a higher degree of hyporheic flow is promoted at the middle of a stream than at the two banks due to the presence of more loose and coarse-grained sediments in the middle of the stream. This finding was not verified in this study, but the area near the SWI has greater velocity. The spatial pattern of velocity of zone 2 beneath the riverbank is similar to that of zone 1, although the magnitude of the velocity at the same specific elevation in zone 2 is greater than the velocity in zone 1. The ratio of the horizontal component to the vertical component is slightly different between the two zones. The zone 3 is the farthest one from the stream among the three zones. The total velocity at the shallow part of zone 3 was relatively larger than that at the deeper part and the horizontal component of velocity was always larger the vertical component, indicating that horizontal flow dominated the hyporheic flow in zone 3.

3.2. Deviations of the V_z estimated by the 1D heat tracing and gradient methods from the numerical model's values

The velocity, head, and temperature of any grid can be easily extracted from numerical results. Although the velocity varies spatially, the values from the shallow observation points are usually used to calculate the vertical velocity via the heat tracing and gradient methods. In this study, shallow observation points, at a depth of 20 cm beneath the SWI, were positioned for the three zones. Since only vertical velocity can be obtained using the heat tracing methods, the vertical velocity from numerical models, and the estimated velocities from the phase shift, amplitude ratio, and gradient methods were compared and are shown in figure 5.

For zone 1 (figure 5(a)), the amplitude ratio method estimates the downwelling vertical velocity well. The maximum error in the three scenarios (L1, L2, and L3) is $<0.96\%$ in L1. When the flow is upwelling, the amplitude ratio method gives the worst results; the estimated velocity has the correct

direction, but the relative error is up to 90%. The error in the estimated velocity from the phase shift method is generally considerable in the six scenarios. The estimated error diminished with the magnitude of the velocity. The results of this study demonstrate that the error is no more than 20% when the true velocity is larger than 0.1 m/d. The gradient method also estimates the velocity under both losing and gaining conditions well. The relative errors range from 12.75% to 13.07% in the six scenarios.

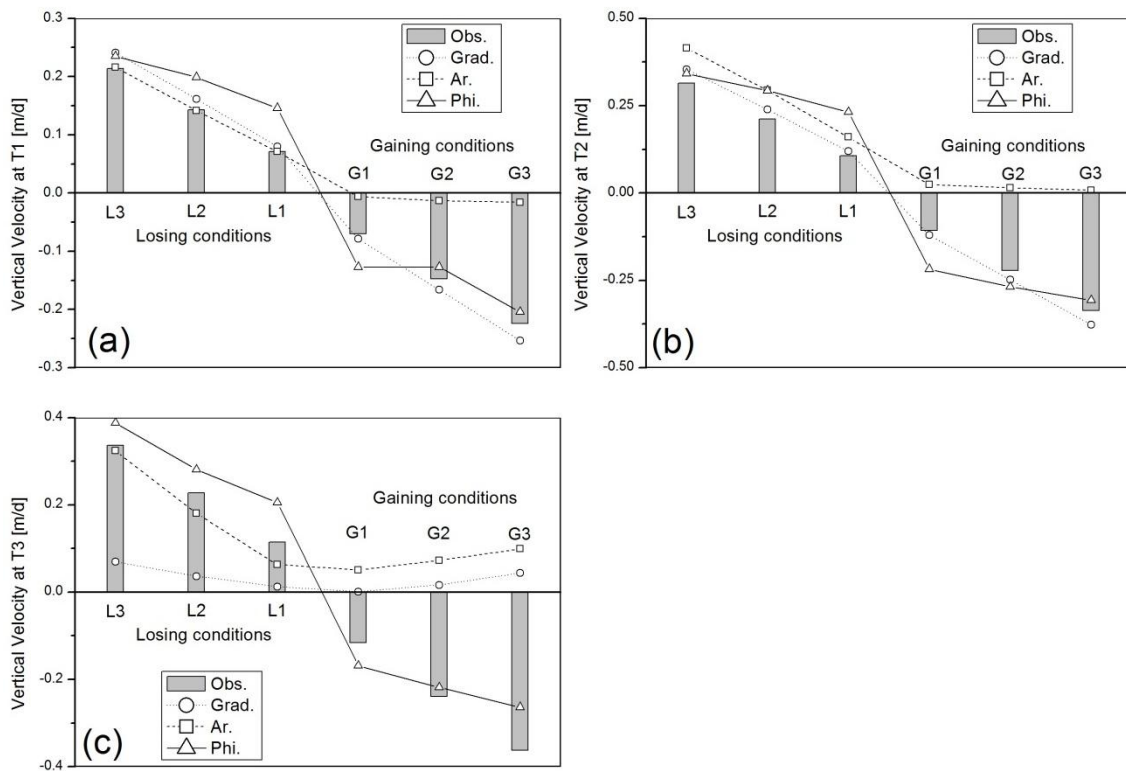


Figure 5. Comparison of the vertical velocity estimated from the 1D heat transfer analytical solution, the Darcy-based gradient method, and the numerical solution.

Horizontal flow dominated the flow field in zone 3 (figure 4), resulting in quite different estimated velocity results for zone 1 (figure 5(c)). Roshan et al [22] suggests that large variations from one side of the stream to the other exists in velocity estimates, which could be an indicator of potential errors resulted from significant horizontal flow components. For zone 3, the relative error of the downward estimated velocity from the amplitude ratio method was no more than 50%, and the relative error decreased with increasing velocity. For upward flow, to a certain extent, the direction and magnitude of the velocity could not be precisely estimated via the amplitude ratio method. In terms of accuracy, the phase shift method produced results comparable to that for zone 1. Figure 5(c) shows that the gradient method provided a low-quality estimation of the velocity. The estimated velocities from the gradient method have relative errors of -79.39 % to -89.02% and -100.75 to -112.07% for losing and gaining conditions, respectively. At the moment, it is not possible to explain why the methods provided an inaccurate flux except that there was probably a significant horizontal flow component that caused errors in both the amplitude ratio and gradient methods.

The flow field of zone 2 is a transition area between zone 1 and zone 3. The true velocity and the estimated values from the three methods are shown in figure 5(b). The estimated errors for this zone lie in between those for zone 1 and zone 3. No more detailed descriptions for zone 2 are provided in the remainder of this paper.

Analysis of the methods provides some key points. When the hyporheic flow is downward and vertical, the amplitude ratio method generates the most accurate vertical velocity estimates followed by the gradient and the phase shift methods. In all, the three methods estimate the vertical velocity well when the true velocity is almost vertical and downward. When the flow is vertical but upward, the gradient method shows the highest accuracy followed by the phase shift method. However, the amplitude ratio method has poor accuracy under gaining conditions. The error from the phase shift method did not vary significant with the flow condition. Although the phase shift method can yield reasonable estimates with a small error in all types of flow conditions, a disadvantage is that the flow direction is unknown and needs to be determined in advance. Moreover, the gradient method performs inadequately under non-vertical flow conditions, as does the amplitude ratio method.

The results of Roshan et al [22] also indicate that the amplitude ratio method do not perform well under gaining conditions, while the phase shift method performs well. For a losing stream, both methods can be applied but the amplitude ratio method achieves a better estimation than the phase shift method. Lautz [10] concluded that analytical methods using the amplitude ratio to derive flux are less prone to error than methods using phase shift under non-ideal field conditions. Swanson and Cardenas [23] indicated that flux estimation methods using a temperature time-series amplitude ratio analysis more closely matched field measurements than the phase shift methods at their study site. In fact, the differences between the velocity estimates from the amplitude ratio and phase shift methods only roughly indicate probable errors in the velocity estimates and it seems as if there is no obvious answer with respect to the extent to which the field velocity data can be suffered the impact of the horizontal flow. In this study, horizontal flow became more dominant when going from zone 1 to zone 3, and the error in the estimation increased significantly.

Lautz [10] stated that in the case of non-vertical flow fields (losing condition), the estimated vertical velocity from a 1D analytical solution is always higher than the true vertical velocity. However, the study of Roshan et al. do not support this conclusion [22]. In fact, our results (figure 5) show that the relationship between the estimated vertical velocity and the true velocity changes among the three zones.

3.3. Effect of Pe number on the error of the estimated flux

Jesen et al. [4] emphasized that the Pe number is key to the temperature distribution of a steady-state hyporheic exchange obtained via the analytical solution of Bredehoeft and Papaopulos [37]. Cardenas and Wilson [24] used a global Pe number reflecting the relative dominance of heat conduction versus advection in a dune geometry-induced hyporheic heat transfer. Moret [38] also stated that thermal Pe affects the numerical solution of the advection-diffusion heat transport significantly.

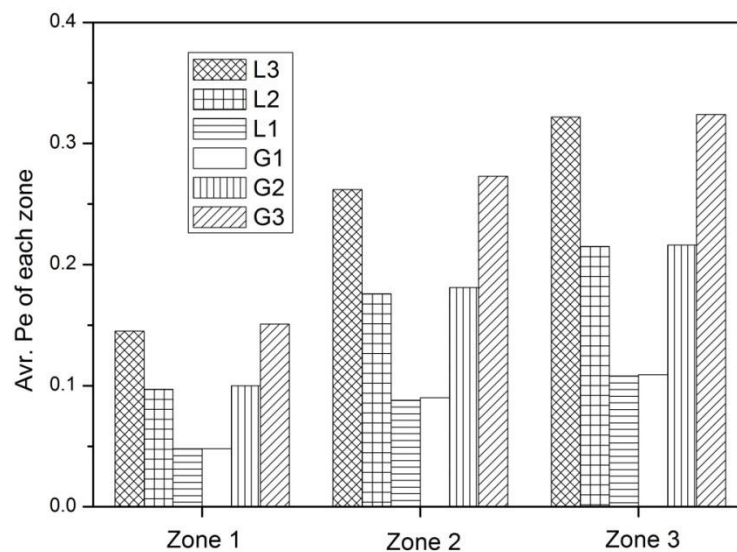


Figure 6. The Pe numbers of T1, T2, and T3 under different hydro-scenarios.

The Pe can be calculated using equations (6) and (7). For any location in this study, the Pe only changed with the seepage velocity while the other parameters were constant during each simulation. The hydraulic gradient varied in the six different scenarios due to the change in the hydraulic head at the left boundary as discussed previously. The losing and gaining conditions were also altered by the change in the hydraulic head at the boundary. With the decrease in the hydraulic gradient from L3 (G3) to L1 (G1), the corresponding Pe (figure 6) showed a linear correlating decrease with the decrease in the boundary head.

Though the average hydraulic gradient of L1 (L2, L3) was designed to be equal to the gradient of G1 (G2, G3), the calculated Pe for the same location shows little difference with the simulated velocity, and the gaining scenarios generally have a slightly larger Pe value, as shown in figure 6. The Pe also varied among the three zones. The Pe of zone 3 (river bank) always had the greatest value, and that of zone 1 (streambed below the center stream) the least. As the 2D flow field in figure 4 shows, zone 1 has the greatest vertical velocity component, but the overall velocity is the least among the three zones. Since water flows from zone 1 to zone 3, the horizontal velocity component gradually increases and the vertical velocity finally approximates to zero; in addition, the overall velocity becomes higher.

Ronan et al [31] studied the relationship between the FSD (full-scale deflection) error and Pe, and concluded that the FSD error in the vertical velocity estimates decreases with increasing Pe. The parameter that varies with a constant Peclet number is the stream width in Ronan et al [31]. However, the seepage velocity was the variable in this study. The results from the different zones show that the effects of Pe on the error of the estimates varied among the zones.

Because the Pe number is significantly affected by the boundary conditions of the numerical model, this spatial pattern of Pe is not explained further. The Pe induced error of the vertical velocity estimation is shown in figure 7. For the losing condition (figure 7(a)), the estimated error from the phase shift method exhibited a dramatic decreasing trend with increasing Pe; the amplitude ratio method gave relatively stable and accurate estimates even with changing Pe. The gradient method provided very close estimates of the vertical velocity except for the horizontal flow dominated zone 3.

For the gaining condition (figure 7(b)), the results are somewhat different from that under the losing condition. The error in the vertical velocity estimates increases with decreasing Pe in the phase shift method. The velocity estimation from the phase shift method also shows a decreasing trend with the Pe. The estimated velocity from the phase shift method overestimates the velocity at a low Pe. With the increasing of Pe, the estimated error decreases and then remains at a relatively low level. As shown in figure 7(b), the amplitude ratio method is highly inaccurate under gaining conditions. The

gradient method provides quite good results when the horizontal flow is weaker than the vertical flow, but otherwise, provides quite bad results when the Pe becomes larger.

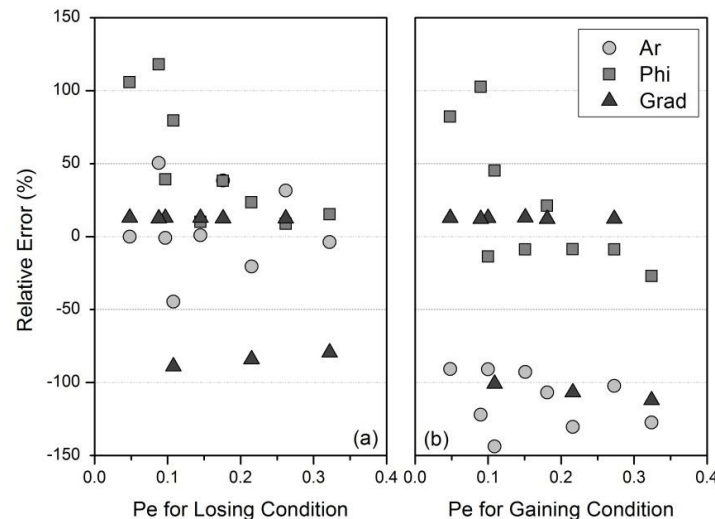


Figure 7. Estimated error against Peclet number in the three analytical methods.

4. Discussion

Since temperature is a primary variable for many physical, chemical, and biological processes occurring in streams, lakes, estuaries, and oceans, and in their bottom sediments, the ramifications of our simulation results on choosing an appropriate method for describing a hyporheic field and the use of the gradient method are briefly discussed here.

Generally, the impacts of horizontal flow on vertical velocity estimates are not straightforward and require comprehensive analyses of the flow field. Nevertheless, our current analysis provides some guidance, and these observations are overall agreed with and extended from the findings of Jensen and Engesgaard, Lautz, Shanafield et al, and Ronan et al [4, 10, 11, 31]. The 1D analytic solution of the heat advection-diffusion equation is obtained under the assumption of vertical flow. Therefore, both the amplitude ratio and the phase shift methods basically inherit the advantages of the analytic solution for the hyporheic zone right beneath the streambed. In particular, these methods estimate the vertical velocity for a losing stream quite well. On the other hand, for a gaining stream, the phase shift method also yields fair results, but the amplitude ratio method's results are very poor. The Darcy-based gradient method estimates the velocity quite well under both losing and gaining conditions.

In our simulations, horizontal flow became dominant with increasing distance from the center of the stream. The estimated error in the velocity from the three methods differed by a large amount, whose maximum value can reach nearly 200%. As discussed in section 3.2, the results of the three methods for the hyporheic zone right below the streambed are similar. However, the differences became larger for the hyporheic zone at a greater distance from the stream (zone 3 in this study). The phase shift method can be used for a gaining stream, and the amplitude ratio method can only be used for a losing stream. The gradient method provides very error-prone results for the hyporheic zone at a greater distance from the stream; a possible explanation for this is that the flow here is not strictly vertical. Lautz [10] stated that the use of temperature data to quantify flux across the streambed is a promising alternative to the more commonly used approaches, such as the Darcy flux calculations. However, it is known that the amplitude ratio method and phase shift method have their own advantages and perform better than the gradient method for areas far from the center of a stream.

Roshan et al [22] also noticed that using only one analytic heat transport solution could cause a large uncertainty in the estimation, and that using both methods in combination was preferable. However, one shortcoming of the phase shift method is that it does not provide the flow path and the amplitude ratio method cannot capture the direction and magnitude of the vertical velocity for a

gaining stream reasonably. A combination of the two methods may not result in a credible estimate as shown by Kalbus et al [39], who demonstrated that the heat-tracing methods should not be applied along. Information on the hydraulics of a system in the flow direction is required to guide the choice of analytic methods. The simple measurement of hydraulic heads at different depths for a single location will yield erroneous flow direction, and the Darcy-based gradient method will provide inaccurate estimates of the vertical velocity.

5. Conclusions

Through numerical simulation of streambank-aquifer flow and heat transport, a general flow pattern was determined. The heat transport provided information on the spatial and temporal variations in temperature, which were used to estimate the vertical velocity via the amplitude ratio and phase lag methods. The vertical hydraulic gradient was also used to obtain the vertical velocity. The true vertical velocity obtained from numerical simulation was compared with the indirect methods. Certain indicative points were determined in our study.

For a streambank-aquifer continuum, the velocity at the shallower part is generally higher than the velocity at the deeper part. The vertical component dominates the velocity vector beneath the center of a stream. At large distances from the center of the stream, horizontal flow dominates the hyporheic exchange in the stream bank-aquifer continuum. Although the hydraulic gradient can affect the magnitude of the vertical and horizontal velocities, the ratio of vertical velocity to horizontal velocity did not change significantly with the hydraulic gradient in both losing and gaining conditions in the present study.

Our results indicate that the relationship between the estimated vertical velocity and the true velocity is location and method dependent. The amplitude ratio method and gradient method estimate the vertical velocity well when vertical flow dominates the hyporheic flow and the flow is upwelling. For the other hydro-scenarios, these two methods provide inaccurate results. The phase shift method estimates the vertical velocity relatively well whatever the location and flow conditions, but requires the direction of the hyporheic flow to be known beforehand. The amplitude ratio method and phase shift method yield better results compared with the gradient method for the streambank area.

The Peclet number was affected by the hydraulic gradient in our simulations, with losing and gaining conditions affecting the average Pe insignificantly. The estimated error in the phase shift method showed a dramatic decreasing trend with increasing Pe . However, the amplitude ratio method was not sensitive to the Pe . The relationship between the estimates from the gradient method and the Pe is location-dependent, but at each location, no increasing or decreasing trend was found.

As discussed above, non-vertical flow could be the factor most responsible for the error in the estimates by the amplitude ratio method, phase method, and gradient method. Each analytic method has its own advantages and two or more methods should be used in conjunction to more accurately understand the true flow field; the hydraulic heads should be obtained from the appropriate monitoring network.

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