

Comparison of three-dimensional photoacoustic effect with different Gaussian radii

Huipeng Du¹ and Hanping Hu¹

¹Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui 230027, P. R. China

E-mail: hphu@ustc.edu.cn

Abstract. It is simulated that the photoacoustic effect in three-dimensional model to compare photoacoustic effect with attenuated different Gaussian radii distribution light sources by the finite element method (FEM). We motivate the enclosed photoacoustic cell at a range of frequencies and compare the results of the amplitude of acoustic pressure frequency curve, the distribution of acoustic pressure and the variation of temperature within the gas layer.

1. Introduction

The photoacoustic effect was found in 1880 by A.G.Bell [1], the famous telephone inventor. However, due to lack of light source of good performance and acoustic transducer of high sensitivity, the study of photoacoustic effect had been stagnant. At that time, the photoacoustic phenomenon in gases had been satisfactorily explained, but liquid and solid were not. With the advent of laser device and microphone, the photoacoustic effect get people's attention again. In 1976, the basic theory of photoacoustic effect in condensed matter was proposed by Rosencwaig and Gersho [2]: a periodically modulated laser beam is focused onto the material, and part or all of the absorbed light energy is transformed to heat through non-radiative de-excitation process. Owing to alternate expansion and contraction of gas layer adjacent to the solid surface, the photoacoustic signal which has the same frequency as the modulated frequency and amplitude, phase depending on the material characters is generated. This theory is known as the "RG" theory which is the basis of modern photoacoustic. From then on, many scholars have studied the theory of photoacoustic effect. McDonald and Wetsel [3] established the thermoelastic coupled equations and given the first order approximate solution. Hanping Hu [4, 5] considered photoacoustic coupled equations of multilayer materials and derived the analytical solution. All of these sound is generated within the gas layer and detected by acoustic transducer in the surrounding gas, but Gu Liu [6] considered both heat and photoacoustic signal are generated and detected within the same solid or liquid. There are also some simulated research on gas photoacoustic cell [7, 8]. But there are few discussion about comparison of three-dimensional photoacoustic cell with attenuated different Gaussian radii distribution light source using the finite element method(FEM).

2. Theory

Photoacoustic effect is governed by conservation equations which are written:



$$\begin{cases} \frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{u} = 0 \\ \rho_0 \frac{\partial \mathbf{u}}{\partial t} = -\nabla p \\ \rho_0 C_p \frac{\partial T}{\partial t} - T_0 \alpha_0 \frac{\partial p}{\partial t} = -\nabla \cdot (-\kappa \nabla T) + Q \\ \rho = \rho_0 (p \beta_T - T \alpha_0) \end{cases} \quad (1)$$

where the dependent variables are density fluctuation ρ , particle velocity \mathbf{u} , temperature fluctuation T , acoustic pressure p , volume heat source Q and the constant quantities are steady-state density ρ_0 , ambient temperature T_0 , thermal conductivity κ , isobaric heat capacity C_p , the isobaric coefficient of thermal expansion α_0 , the isothermal compressibility β_T . Both sides of the governed equations are transformed into the frequency domain by Fourier transform:

$$\begin{cases} i\omega \rho + \rho_0 \nabla \cdot \mathbf{u} = 0 \\ i\omega \rho_0 \mathbf{u} = -\nabla p \\ i\omega \rho_0 C_p T - i\omega T_0 \alpha_0 p = -\nabla \cdot (-\kappa \nabla T) + Q' \\ \rho = \rho_0 (p \beta_T - T \alpha_0) \end{cases} \quad (2)$$

The finite element tool we employ is Comsol Multiphysics®. The geometry model can be seen in figure 1.

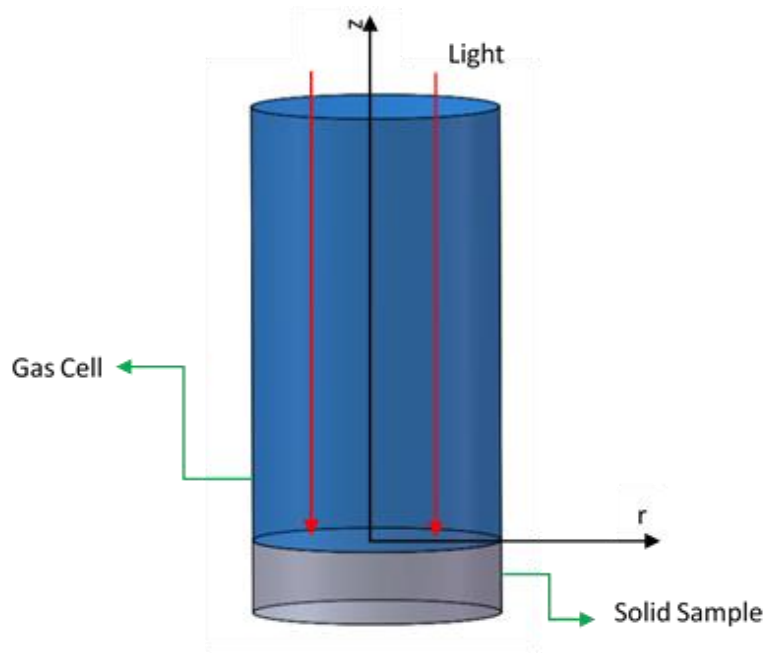


Figure 1. The geometry model of photoacoustic cell.

The upper gas cell is a cylinder with radius $r_a = 1.35\text{mm}$ and length $l_g = 6\text{mm}$. The lower solid sample is also a cylinder with $r_a = 1.35\text{mm}$ and thickness $l_s = 1\text{mm}$ and heated by an incident light. Material parameters are given: for Si, $\alpha_0 = 2.6 \times 10^{-6}/\text{K}$, $\beta_T = 0.98 \times 10^{-12}/\text{Pa}$, $\rho_0 = 2330\text{kg/m}^3$, $\kappa = 148\text{W}/(\text{m} \cdot \text{K})$, $C_p = 712\text{J}/(\text{kg} \cdot \text{K})$, $\beta = 3.9 \times 10^5/\text{m}$ and air is considered as ideal gas, $\rho = 1.2\text{kg/m}^3$, $\kappa = 0.0258\text{W}/(\text{m} \cdot \text{K})$, $C_p = 1000\text{J}/(\text{kg} \cdot \text{K})$. By using isothermal, slip boundary conditions and Frequency Domain Study, we can get the amplitude-frequency response in the photoacoustic cell with attenuated different Gaussian radii distribution light source ($Q' = \beta I / 2 \exp(\beta z) \exp(-r^2 / (2\sigma^2))$ at solid sample domain) in low frequency in figure 2.

Based on one-dimensional photoacoustic theory [2], the magnitude has an $f^{-1/2}$ dependence on frequency. So logarithmic magnitude and frequency exhibits a linear relationship which is shown by theoretical log line in figure 2. In fact, the three-dimensional simulated log curve with attenuated

different Gaussian radii distribution light source are inconsistent with theoretical log line that indicates the influence of lateral magnitude variation of temperature fluctuation cannot be ignored. According to heat transfer theory [9], the magnitude of temperature fluctuation decreases exponentially with the distance r as

$$|T(r)| \propto e^{-\frac{r}{\mu}} \quad (3)$$

where $\mu = (\alpha/\pi f)^{1/2}$ characterizes the thermal penetration depth in gas. After three diffusion lengths the magnitude of temperature has fallen approximately 95% of its original value, so that the photoacoustic signal has no relation to r when $r_a \geq 3\mu$. So there is a threshold frequency below which the simulated results do not match the one-dimensional theory. In our model, $r_a = 3\mu = 1.35\text{mm}$, and the expected threshold frequency is about 35Hz which is consistent with the simulated results.

When the Gaussian spreading is changed, the threshold frequency is unchanged. That fact indicates that the Gaussian spreading has no effect on the threshold frequency. Furthermore, the shape of the light source determines the distribution of the heat deposited, so the heat deposited distribution has no influence on the threshold frequency when $r_a \gg \mu$.

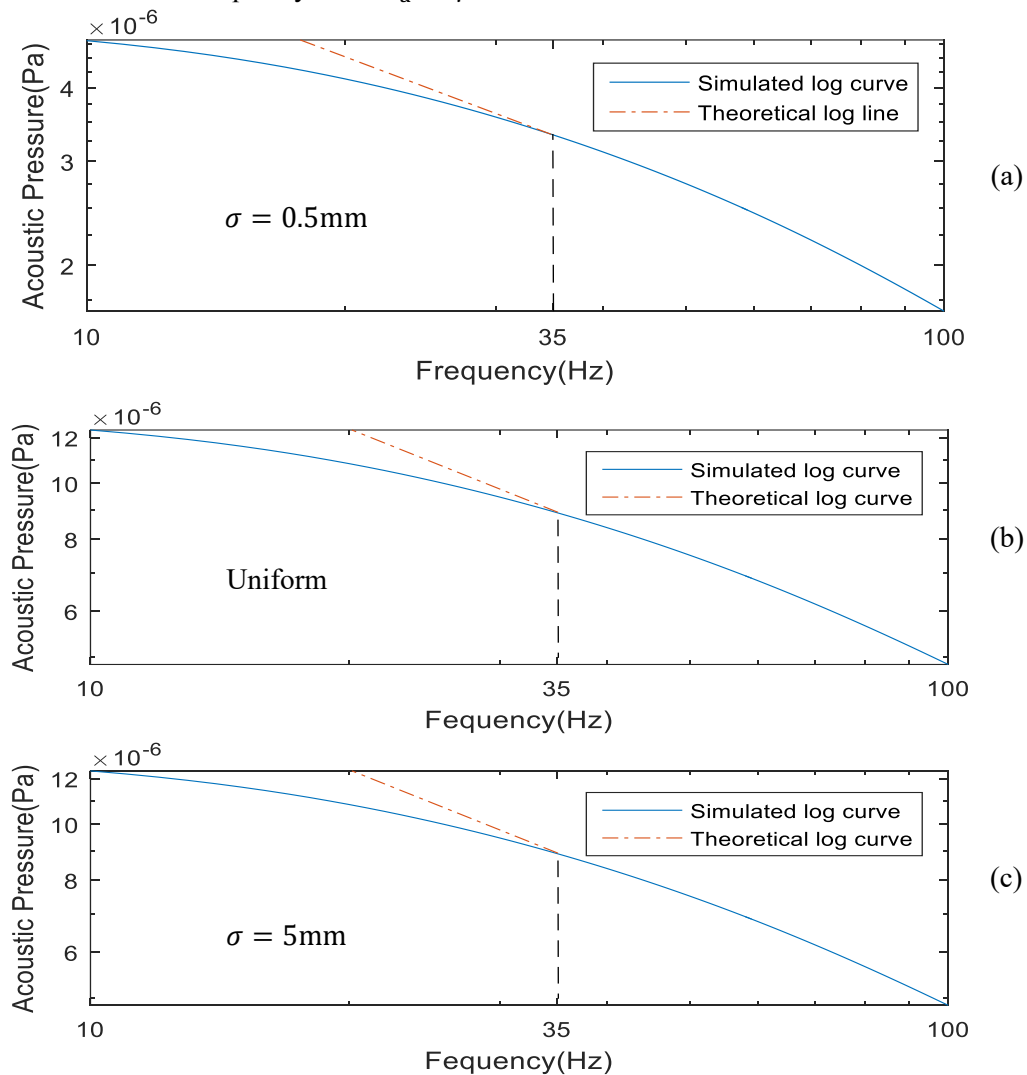


Figure 2. The simulated log curve and theoretical log line of acoustic pressure in low frequency.

In the intermediate frequency, we can get the amplitude-frequency response in the photoacoustic cell with the attenuated different Gaussian distribution light source in figure 3. According to vibration theory [10], resonance frequency satisfy the following relationship:

$$\operatorname{Re}(k) \cdot l_g = n\pi, n = 1, 2, 3, \dots \quad (4)$$

where $k = \omega/c_s$ stands for the wave number in acoustic mode by one-dimensional theory. So resonance frequency f is:

$$f = \frac{nc_s}{2l_g}, n = 1, 2, 3, \dots \quad (5)$$

Take the simulated data into the above expression, $f = 0.2858n \times 10^5 \text{ Hz}$, $n = 1, 2, 3, \dots$ coincided with the results. We can see the resonance frequency is proportional to isentropic sound velocity and inversely proportional to photoacoustic cell length.

It is the fact that the resonance frequency does not change with the Gaussian spreading, but the resonance peak decreases with the decrease of the Gaussian spreading when $\sigma < r_a$ and remain unchanged when $\sigma > r_a$ because of the amount of energy.

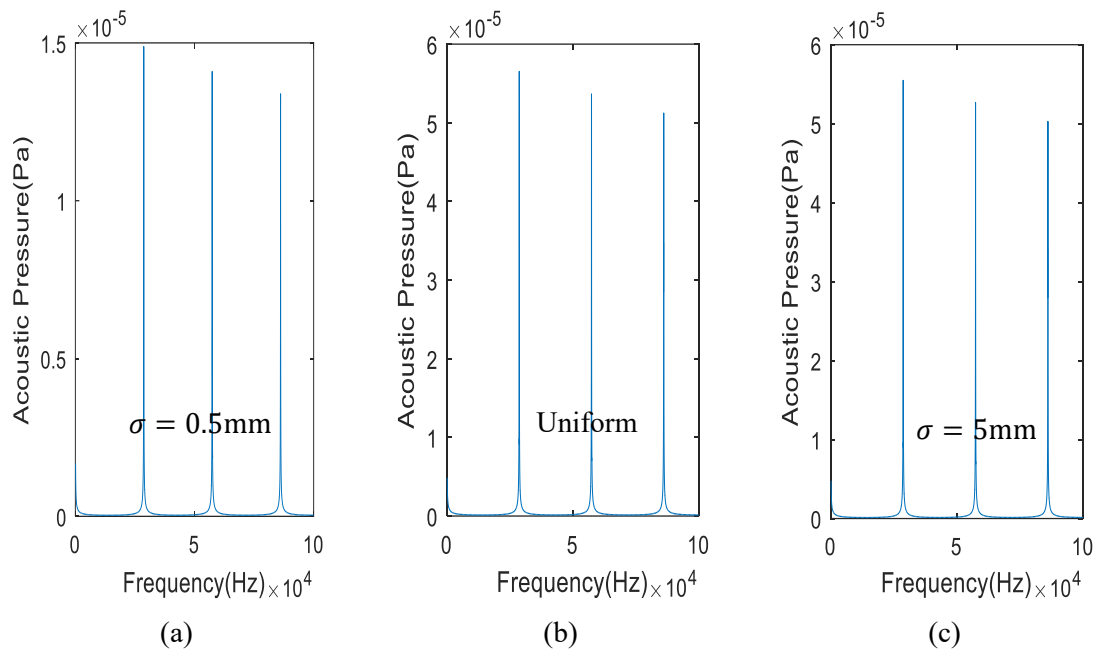


Figure 3. Amplitude-frequency response in the intermediate frequency.

At the same time, the distribution of acoustic pressure is shown in figure 4. It can be seen that there are plane waves in the photoacoustic cell, but there is a Gaussian distribution of acoustic pressure near the solid surface when $\sigma < r_a$. But the distribution is the same as that of uniform distribution light source when $\sigma > r_a$. This fact shows the temperature distribution within the gas layer adjacent to the solid surface is different which can be seen in figure 5.

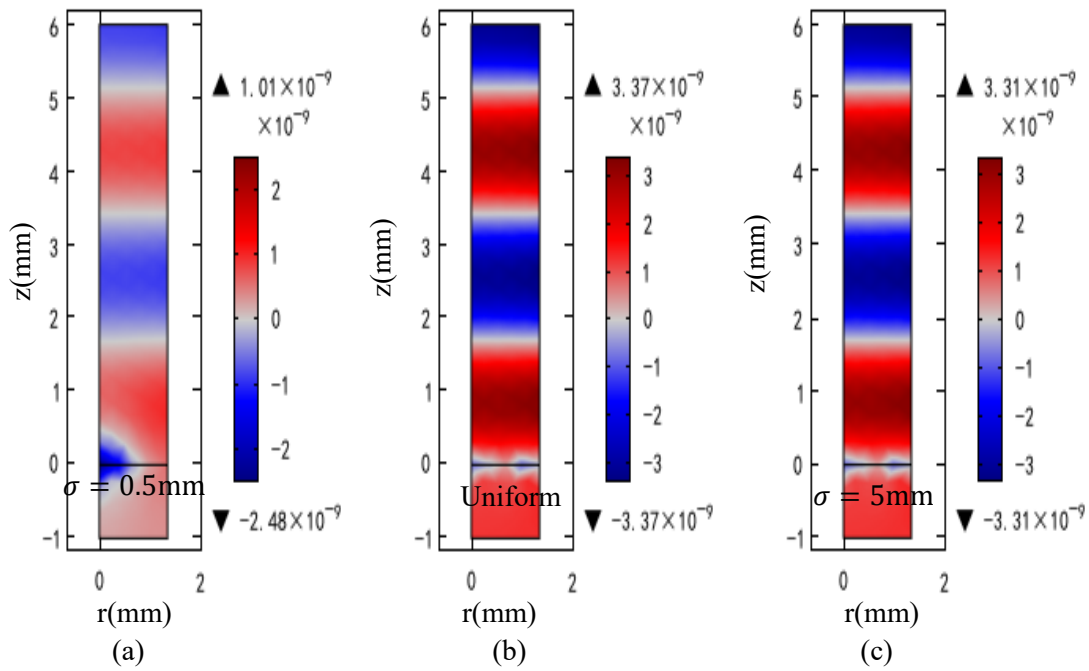


Figure 4. The distribution of acoustic pressure at 100kHz.

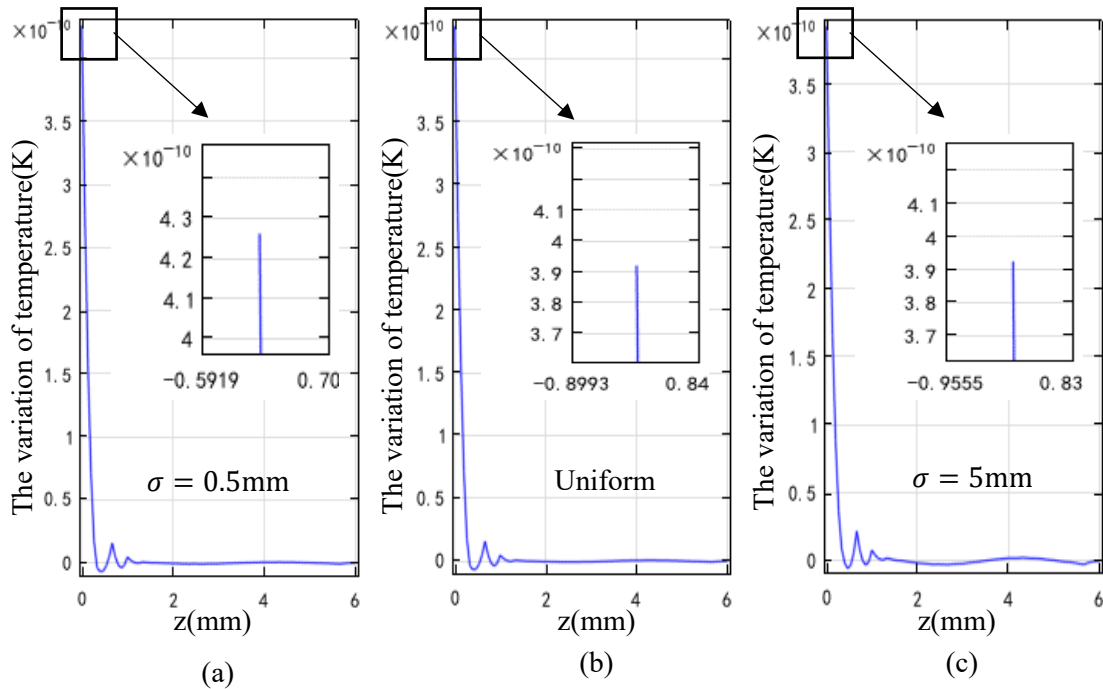


Figure 5. The variation of temperature along the z axis at the center of cylinder.

It can be obtained from figure 5 that the variation of temperature at the original point increases with the decrease of Gaussian spreading when $\sigma < r_a$ and remains unchanged when $\sigma > r_a$. In order to verify our conclusion we can plot the variation of temperature when $\sigma = 0.1\text{mm}$ in figure 6.

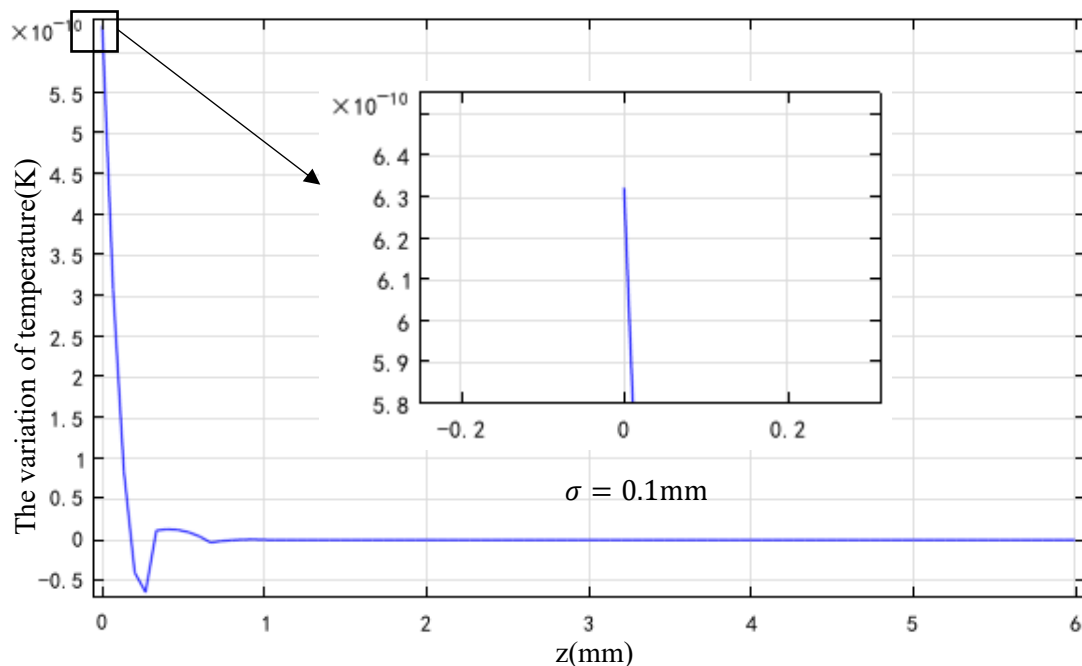


Figure 6. The variation of temperature when $\sigma = 0.1\text{mm}$.

3. Conclusion

It has been seen that there is a threshold frequency under which there is a large deviation between three-dimensional simulated and theoretical results in low frequency but remains unchanged with different Gaussian radii light source. So one-dimensional theoretical formula is not applicable for three-dimensional model under the threshold frequency which is about $f = 3\alpha/r_a^2$ without considering the size of the light spot. Therefore three-dimensional photoacoustic cell can use the one-dimensional theory in other frequency, but there is a Gaussian distribution of acoustic pressure near the solid surface and the variation of temperature at the original point increases with the decrease of Gaussian spreading when $\sigma < r_a$.

Acknowledgement

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