Monopulse joint parameter estimation of multiple unresolved targets within the radar beam

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Abstract. Aiming at the problem of the parameter estimation of multiple unresolved targets within the radar beam, using the joint bin processing model, a method of jointly estimating the number and the position of the targets is proposed based on reversible jump Markov Chain Monte Carlo (RJ-MCMC). Reasonable assumptions of the prior distributions and Bayesian theory are adopted to obtain the posterior probability density function of the estimated parameters from the conditional likelihood function of the observation, and then the acceptance ratios of the birth, death and update moves are given. During the update move, a hybrid Metropolis-Hastings (MH) sampling algorithm is used to make a better exploration of the parameter space. The simulation results show that this new method outperforms the method of ML-MLD [11] proposed by X.Zhang for similar estimation accuracy is achieved while fewer sub-pulses are needed.

1 Introduction

Monopulse processing is widely used in modern radar angle measurement system with sub-beam accuracy. However, the case that only one target falls in a radar resolution cell is the precondition of conventional monopulse processing to measure the target's angles accurately. Otherwise, the angle estimation using the conventional monopulse processing can wander far beyond the angular separation of the targets and these merged measurements often lead to high errors in data association and track filtering. Therefore, the parameters estimation of multiple unresolved targets within the radar beam has been a hot topic and a challenging problem.

There were many research literatures on multiple unresolved targets extraction and parameter estimation. Some [1-3] modified the antenna configuration to aid in the resolution process, while some took advantage of the array signal processing methods including high-resolution beam-forming or high-resolution direction finding [1,2]. Others still made use of the monopulse system, but extended the concept of the monopulse ratio. The complex monopulse processing using both the in-phase and quadrature parts (or real and imaginary parts) of the monopulse ratio, was firstly used to estimate the direction of arrivals (DOAs) of two fixed amplitude targets with known relative radar cross section (RRCS) [1], and then further improved to estimate DOAs of two unresolved Rayleigh targets under different conditions [2] and detection methods [3-5].

However, all of the above techniques can only pull out at most two unresolved targets for the limited information in an isolated matched filter sample, that is, targets are assumed to be located exactly where the matched filter is sampled, and there is no "spillover" of target energy to adjacent matched filter return. We can regard the above methods as ideal single bin processing. A more realistic case was proposed by X. Zhang, et al [1,2], in which sub-bin range and angular estimates of up to five targets between them can be detected by utilization of the ML extractor. At the same time, the number of targets present can also be detected by applying Rissanen's Minimum Description Length (MDL) criterion. However, the validity of MDL depends on the reliable procedures for ML parameter estimation for each possible model and the evaluation of the criteria, which lead to great computational burden and easily get into the local extremum. In this paper, we follow a Bayesian approach using the joint bin processing model to extract multiple unresolved targets within the radar beam. ML estimators are based on the likelihood function, while Bayesian estimators are based upon the posterior probabilities whereby the unknown parameters are regarded as random quantities with known prior distribution. Different from the case of knowing the number of targets existed in advance [1,2], the case treated here is more complex—the number of targets is unknown and assumed random during the derivation of the posterior probability distribution. To evaluate the joint posterior distribution of the number of targets and their position parameters, an efficient stochastic algorithm based on RJ-MCMC [1-3] is proposed.

2 Joint bin processing model

As mentioned in [11], we assume the output of the matched filter is triangular in the absence of noise, that is, the radar waveform has a rectangular envelope. And also we assume that the matched filter sampling rate is once per pulse-length T, meaning that a target can appear only in its contiguous matched filter samples. Figure 1 reveals the model of two targets located between two sampling points considering the spillover of target energy. ΔT is defined as the time offset of the target with respect to the first sampling point, representing the sub-bin range information. x represents the amplitude of the return peak after matched filter. Define $\alpha = \frac{\Delta T}{T}$, then the amplitudes of the return at the first and second sampling point are $(1-\alpha)x$ and αx, respectively.

Assuming there are k targets present, we have the in-phase observations for k targets between two sampling points as follows.

$$\begin{cases} s_1(m) = \sum_j^k \left(1-\alpha_j\right) x_j\left(m\right) + n_{s1}\left(m\right) \\ s_2(m) = \sum_j^k \alpha_j x_j\left(m\right) + n_{s2}\left(m\right) \\ d_{a1}\left(m\right) = \sum_j^k \left(1-\alpha_j\right)\eta_{aj} x_j\left(m\right) + n_{da1}\left(m\right) \\ d_{a2}\left(m\right) = \sum_j^k \alpha_j \eta_{aj} x_j\left(m\right) + n_{a2}\left(m\right) \\ d_{e1}\left(m\right) = \sum_j^k \left(1-\alpha_j\right)\eta_{ej} x_j\left(m\right) + n_{e1}\left(m\right) \\ d_{e2}\left(m\right) = \sum_j^k \alpha_j \eta_{ej} x_j\left(m\right) + n_{e2}\left(m\right) \end{cases} \tag{1}$$

where the ranges $\left\{\alpha_j\right\}_{j=1}^k$, the azimuth $\left\{\eta_{aj}\right\}_{j=1}^k$ and the elevation $\left\{\eta_{ej}\right\}_{j=1}^k$ are the parameters to be estimated. Here η is not the true off- boresight angle but can be determined from the ratio of the antenna pattern gains in for the sum and difference channels [10]. The observations at the first matched filter sampling point are denoted as $\left\{s_1(m)\right\}_{m=1}^M$, $\left\{d_{a1}(m)\right\}_{m=1}^M$ and $\left\{d_{e1}(m)\right\}_{m=1}^M$, while $\left\{s_2(m)\right\}_{m=1}^M$ $\left\{d_{a2}(m)\right\}_{m=1}^M$ and $\left\{d_{e2}(m)\right\}_{m=1}^M$ are the observations at the second. $x_j(m)$ represents the return peak of jth target at mth sub-pulse. M is the number of sub-pulses. In this paper, we consider a Swerling II radar target model, where targets have pulse-to-pulse Rayleigh fluctuation, meaning $\left\{x_j(m)\right\}_{m=1}^M$ are independent and identically distributed (iid) which are Gaussian with zeros mean and variance σ_{tj}^2.

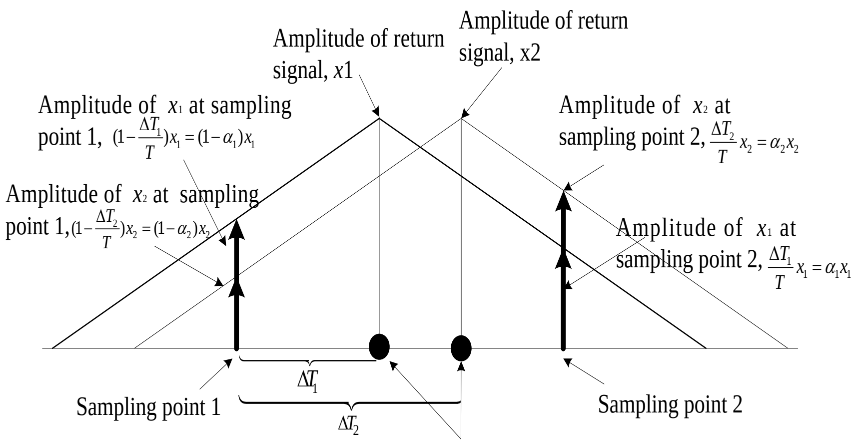

Figure 1: Signal model of two targets located between two sampling points considering the spillover of target energy [11]

The noise $\{n_s\}$, $\{n_{da}\}$ and $\{n_{de}\}$ are also independent, zero mean and Gaussian, with variance:

$$\begin{cases} E\left[n_{s1}^2\right]=E\left[n_{s2}^2\right]=\sigma_s^2 \\ E\left[n_{da1}^2\right]=E\left[n_{da2}^2\right]=E\left[n_{de1}^2\right]=E\left[n_{de2}^2\right]=\sigma_d^2 \end{cases} \tag{2}$$

According to (1), the observation model can be expressed in vector-matrix form as follows.

$$\begin{aligned}\mathbf{Z}(:,m)&=\left[s_1(m)\quad s_2(m)\quad d_{a1}(m)\quad d_{a2}(m)\quad d_{e1}(m)\quad d_{e2}(m)\right]^T \\ &=\mathbf{DX}(:,m)+\mathbf{W}(:,m)\end{aligned} \tag{3}$$

where

$$\mathbf{D}=\begin{bmatrix} 1-\alpha_1 & 1-\alpha_2 & \cdots & 1-\alpha_k \\ \alpha_1 & \alpha_2 & \cdots & \alpha_k \\ (1-\alpha_1)\eta_{a1} & (1-\alpha_2)\eta_{a2} & \cdots & (1-\alpha_k)\eta_{ak} \\ \alpha_1\eta_{a1} & \alpha_2\eta_{a2} & \cdots & \alpha_k\eta_{ak} \\ (1-\alpha_1)\eta_{e1} & (1-\alpha_2)\eta_{e2} & \cdots & (1-\alpha_k)\eta_{ek} \\ \alpha_1\eta_{e1} & \alpha_2\eta_{e2} & \cdots & \alpha_k\eta_{ek} \end{bmatrix} \tag{4}$$

$$\mathbf{X}=\begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(M) \\ x_2(1) & x_2(2) & \cdots & x_2(M) \\ \vdots & \vdots & \ddots & \vdots \\ x_k(1) & x_k(2) & \cdots & x_k(M) \end{bmatrix} \tag{5}$$

$$\mathbf{W}=\begin{bmatrix} n_{si}(1) & n_{si}(2) & \cdots & n_{s1}(M) \\ n_{s2}(1) & n_{s2}(2) & \cdots & n_{s2}(M) \\ n_{da1}(1) & n_{dai}(2) & \cdots & n_{da1}(M) \\ n_{da2}(1) & n_{da2}(2) & \cdots & n_{da2}(M) \\ n_{de1}(1) & n_{de1}(2) & \cdots & n_{de1}(M) \\ n_{de2}(1) & n_{de2}(2) & \cdots & n_{de2}(M) \end{bmatrix} \tag{6}$$

Combine the M observations, we obtain that

$$\mathbf{z}=\mathbf{Z}(:)=\mathbf{R}\left(\mathbf{s}_k\right)\mathbf{a}_k+\mathbf{v} \tag{7}$$

where

$$\mathbf{R}=diag\left(\underbrace{\mathbf{D}\quad\mathbf{D}\quad\cdots\quad\mathbf{D}}_{M个}\right)_{6M\times kM} \tag{8}$$

$\mathbf{s}_k\triangleq\left[\boldsymbol{\alpha}^T;\boldsymbol{\eta}_a^T;\boldsymbol{\eta}_e^T\right]$ is the unknown parameter set and

$$\begin{cases} \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_k \end{bmatrix}^T \\ \boldsymbol{\eta}_a = \begin{bmatrix} \eta_{a1} & \eta_{a2} & \cdots & \eta_{ak} \end{bmatrix}^T \\ \boldsymbol{\eta}_e = \begin{bmatrix} \eta_{e1} & \eta_{e2} & \cdots & \eta_{ek} \end{bmatrix}^T \end{cases} \tag{9}$$

$(\cdot)^T$ denotes transpose operation. $\mathbf{z} = \mathbf{Z}(:)$, $\mathbf{a}_k = \mathbf{X}(:)$ and $\mathbf{v} = \mathbf{W}(:)$ are the rearrangement of the return array $\mathbf{Z}$, the amplitude array $\mathbf{X}$ and the noise array $\mathbf{W}$ along with the column, respectively.

3 Bayesian model and computation

Define the unknown parameter set $\boldsymbol{\theta}_k \triangleq \{\mathbf{s}_k, \mathbf{a}_k, \sigma_k^2\}$, which include the position parameter matrix $\mathbf{s}_k$ of k targets, the amplitude vector of k targets and the noise variance (here we assume that $\sigma_s^2 = \sigma_d^2 = \sigma_k^2$ for convenience). Consider the unknown target number k jointly, we can get the overall parameter space $\Theta = \bigcup_{k=0}^{k_{\max}} \{k\} \times \Theta_k$ corresponding to the unknown parameters $\{k, \boldsymbol{\theta}_k\}$, where $\Theta_0 \triangleq \mathbb{R}^+$ and $\Theta_k \triangleq \mathbb{R}^{kM} \times \Phi_k \times \mathbb{R}^+$ for $k \in \{1, \cdots, k_{\max}\}$, and Φ_k represents the parameter space of $\mathbf{s}_k$.

3.1 Prior distribution assumption

According to the definition of sub-bin range α [1] and the relationship between DOA parameters η_a, η_e and angular deviation [1], it is reasonable to choose uniform distribution as prior distribution of α, η_a and η_e; that is

$$\begin{gathered} \alpha_j \sim U_{[0,1]}, \eta_{aj} \sim U_{[-1,1]}, \eta_{ej} \sim U_{[-1,1]} \\ (j = 1, 2, \cdots k) \end{gathered} \tag{10}$$

Assuming that targets are independent, then the parameter space Φ_k for $\mathbf{s}_k$ should be $\Phi_k \triangleq [0,1]^k \times [-1,1]^k \times [-1,1]^k$. Conditional on k, the distribution of the position parameters of all k targets should be

$$p(\mathbf{s}_k \mid k) = \prod_{j=1}^{k} \left(\frac{1}{1-0} \cdot \frac{1}{1-(-1)} \cdot \frac{1}{1-(-1)} \right) = \frac{1}{4^k} \tag{11}$$

The prior distribution of target number k is a truncated Poisson distribution; that is

$$p(k) \propto \frac{\Lambda^k}{k!} e^{-\Lambda},\ k \in [0, k_{\max}] \tag{12}$$

According to [11], k should not be larger than five; that is to say $k_{\max} = 5$. Λ can be interpreted as the expected number of targets and its influence can be removed by computing Bayes factors, namely, $p(\mathbf{z} \mid k_1) / p(\mathbf{z} \mid k_2)$ [1].

Conditional on $(k, \mathbf{s}_k)$ the amplitude parameter $\mathbf{a}_k$ is zeros mean Gaussian with covariance $\sigma_k^2 \boldsymbol{\Sigma}_k$, where $\boldsymbol{\Sigma}_k^{-1} = \delta^{-2} \mathbf{R}^T(\mathbf{s}_k) \mathbf{R}(\mathbf{s}_k)$ and δ^2 denotes the expected signal-to-noise ratio (SNR), following a conjugate inverse-Gamma prior distribution; that is

$$\delta^2 \sim Ig\left(\frac{\alpha_0}{2}, \frac{\beta_0}{2} \right) \tag{13}$$

Also, the noise variance is assumed to be distributed according to a conjugate inverse- Gamma prior distribution; that is

$$\sigma_k^2 \sim Ig\left(\frac{\upsilon_0}{2}, \frac{\gamma_0}{2} \right) \tag{14}$$

3.2 Posterior distribution formulation

According to Bayesian theory, the joint distribution of all variables can be derived as

$$\begin{aligned} p(k,\boldsymbol{\theta}_k \mid \mathbf{z}) &= p(k,\boldsymbol{\theta}_k,\mathbf{z}) / p(\mathbf{z}) \\ &\propto p(\mathbf{z} \mid k,\boldsymbol{\theta}_k)\, p(k,\boldsymbol{\theta}_k) \end{aligned} \quad (15)$$

where $p(\mathbf{z} \mid k,\boldsymbol{\theta}_k)$ is the conditional likelihood function which can be obtained from the model given by Eq.(7) as

$$p(\mathbf{z} \mid k,\boldsymbol{\theta}_k) = \left(2\pi\sigma_k^2\right)^{-\frac{6M}{2}} \exp\left[-\frac{\left\|\mathbf{z} - \mathbf{R}(\mathbf{s}_k)\mathbf{a}_k\right\|^2}{2\sigma_k^2}\right] (16)$$

The joint distribution of $\{k,\boldsymbol{\theta}_k\}$ can be expressed as follows using the hierarchical structure.

$$p(k,\boldsymbol{\theta}_k) = p(k,\mathbf{s}_k,\mathbf{a}_k,\sigma_k^2) = p(k,\mathbf{s}_k,\mathbf{a}_k \mid \sigma_k^2)\, p(\sigma_k^2) \quad (17)$$

Referring to the prior distribution given in Section 3.1, we introduce the prior distribution of $(k,\mathbf{s}_k,\mathbf{a}_k)$

$$p(k,\mathbf{s}_k,\mathbf{a}_k \mid \sigma_k^2) \propto \frac{\Lambda^k e^{-\Lambda}}{k!} \cdot \frac{1}{4^k} \cdot \frac{\exp\left(-\frac{\mathbf{a}_k^T \Sigma_k^{-1} \mathbf{a}_k}{2\sigma_k^2}\right)}{\left|2\pi\sigma_k^2 \Sigma_k\right|^{\frac{1}{2}}} \quad (18)$$

Using the above assumptions and formulations, we can get

$$\begin{aligned} p(k,\boldsymbol{\theta}_k \mid \mathbf{z}) &\propto p(\mathbf{z} \mid k,\boldsymbol{\theta}_k) p(k,\mathbf{s}_k,\mathbf{a}_k \mid \sigma_k^2) p(\sigma_k^2) \\ &\propto \left(2\pi\sigma_k^2\right)^{-\frac{6M}{2}} \exp\left[-\frac{\left(\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}\right)}{2\sigma_k^2}\right] \left(\sigma_k^2\right)^{-\frac{\upsilon_0}{2}-1} \frac{\Lambda^k}{k!} \cdot \quad (19) \\ &\frac{1}{4^k} \cdot \left|2\pi\sigma_k^2 \Sigma_k\right|^{-\frac{1}{2}} \exp\left[-\frac{\left(\mathbf{a}_k - \mathbf{n}_k\right)^T \mathbf{N}_k^{-1} \left(\mathbf{a}_k - \mathbf{n}_k\right)}{2\sigma_k^2}\right] \end{aligned}$$

where

$$\mathbf{N}_k^{-1} = \boldsymbol{\Sigma}_k^{-1} + \mathbf{R}^T(\mathbf{s}_k)\mathbf{R}(\mathbf{s}_k) \quad (20)$$

$$\mathbf{n}_k = \mathbf{N}_k \mathbf{R}^T(\mathbf{s}_k)\mathbf{z} \quad (21)$$

$$\mathbf{P}_k = \mathbf{I} - \mathbf{R}(\mathbf{s}_k)\mathbf{N}_k \mathbf{R}^T(\mathbf{s}_k) \quad (22)$$

Integrating the so-called nuisance parameters $\mathbf{a}_k$ and then σ_k^2 yields the expression of $p(k,\mathbf{s}_k \mid \mathbf{z})$ up to a normalizing constant as follows.

$$p(k,\mathbf{s}_k \mid \mathbf{z}) \propto \left(\frac{\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}}{2}\right)^{-\frac{6M+\upsilon_0}{2}} \frac{\left[\Lambda / (1+\delta^2)^{\frac{M}{2}}\right]^k}{k! \cdot 4^k} \quad (23)$$

This posterior distribution is obviously highly nonlinear and the posterior distribution of ith target's position parameters vector $\mathbf{s}_{k,i}$ in $\mathbf{s}_k$ conditional on k $\mathbf{s}_{ki}$ and $\mathbf{z}$ can be denoted as

$$p(\mathbf{s}_{k,i} k,\mathbf{s}_{ki},\mathbf{z}) \propto \left(\frac{\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}}{2}\right)^{-\frac{6M+\upsilon_0}{2}} \quad (24)$$

where $\mathbf{s}_{ki}$ represents the remaining parameter matrix except $\mathbf{s}_{k,i}$ of parameter matrix $\mathbf{s}_k$. And $1 \le i \le k$.

3.3 Bayesian computation using RJ-MCMC

RJ-MCMC would be a good choice with the capability of jumping between subspaces of different dimensions. That is to say, RJ-MCMC can make sampler directly from different model orders based on the joint distribution on $(k,\boldsymbol{\theta}_k)$, meaning that the number of targets and the parameters of each target can be estimated at the same time.

The essence of RJ-MCMC method is a general state-space MH algorithm. For our problem, there are three states needed, including birth, death and update whose corresponding probabilities are b_k, d_k

and u_k respectively. For all $k \in [0, k_{\max}]$, $b_k + d_k + u_k = 1$ and

$$\begin{cases} b_k \triangleq c \cdot \min\{1, p(k+1)/p(k)\} \\ d_{k+1} \triangleq c \cdot \min\{1, p(k)/p(k+1)\} \end{cases} \quad (25)$$

where c is a tuning parameter controlling the update move to jump moves, and $p(k)$ is the prior probability of kth model. As explained in [15], $c = 0.5$ is also chosen used in this paper. The main steps of RJ-MCMC algorithm and the procedure of birth move, death move and update are described in detail in [16] (not explained here).

For each state, the candidates $(k^*, s^*_{k,i})$ are chosen from corresponding proposal distributions. An acceptance ratio defined as (26) decides whether these candidates are accepted.

$$r \triangleq \begin{pmatrix} \text{posterior} \\ \text{distributions ratio} \end{pmatrix} \times (\text{proposal ratio}) \quad (26)$$

The acceptance ratio ensures reversibility and, hence, invariance of the Markov chain with respect to the posterior distribution.

The acceptance ratio of birth move, death move and update move can be derived as

$$\begin{aligned} r_{\text{birth}} &= \frac{p(k+1, \mathbf{s}_{k+1} \mid \mathbf{z})}{p(k, \mathbf{s}_k \mid \mathbf{z})} \cdot \frac{q(k, \mathbf{s}_k \mid k+1, \mathbf{s}_{k+1})}{q(k+1, \mathbf{s}_{k+1} \mid k, \mathbf{s}_k)} \\ &= \left(\frac{\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}}{\gamma_0 + \mathbf{z}^T \mathbf{P}_{k+1} \mathbf{z}} \right)^{\frac{6M+\nu_0}{2}} \cdot \frac{1}{(1+\delta^2)^{\frac{M}{2}}} \cdot \frac{1}{k+1} \end{aligned} \quad (27)$$

$$\begin{aligned} r_{\text{death}} &= \frac{p(k-1, \mathbf{s}_{k-1} \mid \mathbf{z})}{p(k, \mathbf{s}_k \mid \mathbf{z})} \cdot \frac{q(k, \mathbf{s}_k \mid k-1, \mathbf{s}_{k-1})}{q(k-1, \mathbf{s}_{k-1} \mid k, \mathbf{s}_k)} \\ &= \left(\frac{\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}}{\gamma_0 + \mathbf{z}^T \mathbf{P}_{k-1} \mathbf{z}} \right)^{\frac{6M+\nu_0}{2}} \cdot (1+\delta^2)^{\frac{M}{2}} \cdot k \end{aligned} \quad (28)$$

and

$$r_{\text{update}} = \left(\frac{\gamma_0 + \mathbf{z}^T \mathbf{P}_k \mathbf{z}}{\gamma_0 + \mathbf{z}^T \mathbf{P}^*_k \mathbf{z}} \right)^{\frac{6M+\nu_0}{2}} \frac{q_i(\mathbf{s}_{k,j} \mid \mathbf{s}^*_{k,j})}{q_i(\mathbf{s}^*_{k,j} \mid \mathbf{s}_{k,j})} (i = 1, 2) \quad (29)$$

Furthermore, during the update move, a hybrid MH sampling algorithm is used to make a better exploration of the parameter space. Illuminated by the importance sampling, we choose the proposal distribution $q_1(\mathbf{s}^*_{k,j} \mid \mathbf{s}_{k,j})$ as

$$q_1(\mathbf{s}^*_{k,j} \mid \mathbf{s}_{k,j}) \propto \frac{\exp\left[\rho_g I(\mathbf{s}^*_{k,j})\right]}{\int \exp\left[\rho_g I(\mathbf{s}^*_{k,j})\right] d\mathbf{s}^*_{k,j}} \quad (30)$$

where

$$I(\mathbf{s}^*_{k,j}) = \left| \sum_{m=1}^{M} \rho_j \mathbf{Z}^T(:, m) \mathbf{D}_j(\mathbf{s}^*_{k,j}) \right|^2 \quad (31)$$

and $\mathbf{D}_j(\mathbf{s}^*_{k,j})$ represents the jth column of matrix $\mathbf{D}$. ρ_j is a real value on $\left[\frac{2}{\sqrt{3}}, \sqrt{2}\right]$ and ρ_g is not sensitive to the algorithm performance in practice [1]. Compared with the uniform distribution as the proposal distribution, a better global exploration of the posterior distribution of target position can be got with (30).

The proposal distribution $q_2(\mathbf{s}^*_{k,j} \mid \mathbf{s}_{k,j})$ yields a candidate $\mathbf{s}^*_{k,j}$ according to a random walk around $\mathbf{s}_{k,j}$; that is, $\mathbf{s}^*_{k,j} \sim N(\mathbf{s}_{k,j}, \mathbf{\Sigma}_{q_2})$. $\mathbf{\Sigma}_{q_2}$ is a diagonal covariance matrix whose diagonal elements corresponding to $(\alpha_j, \eta_{aj}, \eta_{ej})$, respectively, mean a perturbation of target position parameters.

We calculate the ergodic mean every 50 iterations and decide the convergence when the absolute values of three consecutive ergodic means difference are less than a assumed threshold value.

4 Simulation

Like [12], we assume that there are at most five targets between two sampling points, and their sub-bin range, azimuth and elevation are as specified in Table 1. The hyperparameters and other relative parameters are given as $\nu_0 = \gamma_0 = 0.1$, $\alpha_0 = 2$, $\beta_0 = 10$, $\varepsilon_1 = \varepsilon_2 = 10^{-4}$, $\lambda = 0.2$, and $\mathbf{\Sigma}_{q_2} = \frac{1}{30M}\mathbf{I}$ [1]. The simulation results below are all from 200 Monte Carlo runs.

Target index	Sub-bin range	Azimuth	Elevation
1	0.15	-0.8	-0.9
2	0.35	-0.6	0.7
3	0.55	0.1	-0.4
4	0.75	0.5	0.8
5	0.95	0.9	-0.8

Table 1 Five targets' positions between two sampling points

We make two numerical simulations to demonstrate the proposed method.

Example 1 Targets number estimation performance analysis
The qualities of the targets number estimates obtained under different assumed numbers of targets are compared in [12]. Same with [12], we assume that SNR of each target between two sampling points is also 25dB. P_c is defined as the probability of correct detection of the number of targets. Figure 2 shows the results of P_c of both methods.

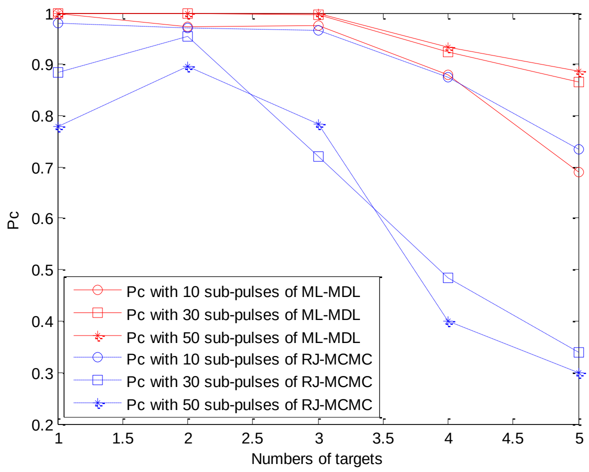

Fig 2 Probability of correct detection of the number of targets with same SNR (25dB)

From Figure 2, we can see that it becomes easier for both methods to make the correct decision with fewer targets. However, different with the method of ML-MDL [11,12], increasing the number of sub-pulses doesn't improve the situation for the method of RJ-MCMC. Furthermore, further calculations show that there is little difference for the root mean square errors (RMSEs) of the estimates (not given for the space reasons) no matter with 10, 30 or 50 sub-pulses. So in the following simulations we choose 10 as the number of sub-pulses. Owing to space reasons, we will make further analysis to the choice of number of sub-pulses in the follow-up work.

Example 2 Targets' position estimation performance analysis compared with ML

Expect the number of sub-pulses, other parameters are same with [12]. Figure 3-5 show the RMSEs of targets' position estimates (including the sub-bin range, azimuth and elevation) for target number one under different numbers of actual targets and different SNRs.

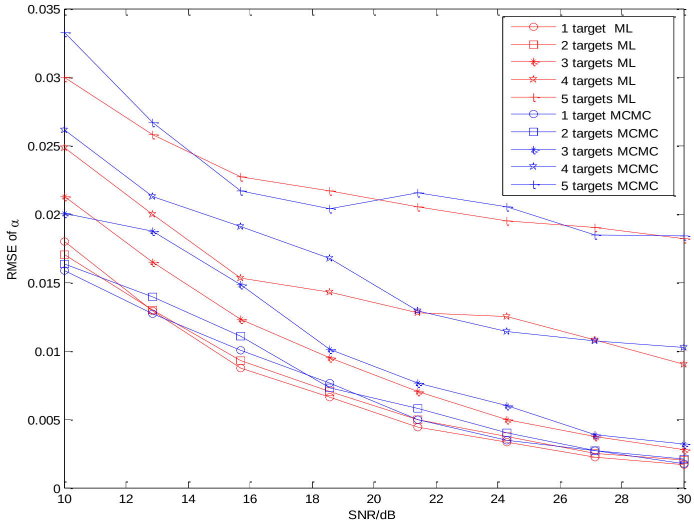

Fig. 3 RMSEs of the estimates of the sub-bin range α of targets 1 with different numbers of actual targets under different SNRs

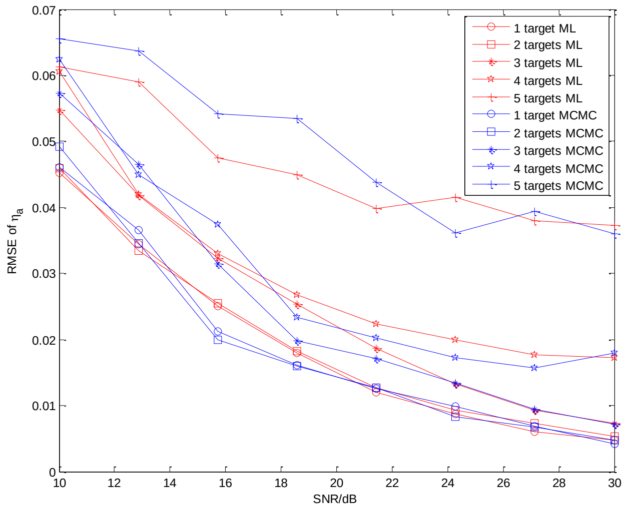

Fig. 4 RMSEs of the estimates of the azimuth η_a of targets 1 with different numbers of actual targets under different SNRs

Obviously from the figures, the RMSEs of sub-bin range, azimuth and elevation can keep within the range of less than 0.035, 0.07 and 0.14, respectively. And the RMSEs of estimates grow larger as the number of actual targets increase for both methods. When there are only three or fewer targets or the SNR is larger than 25dB, the RMSEs of the estimates even can reduce to less than 0.02, 0.02 and 0.06, respectively, which can satisfy many practical applications. Furthermore, we can also see that the method proposed in this paper can achieve similar estimation accuracy only with 10 sub-pulses compared with the method proposed in [11] and [12] with 50 sub-pulses, which means great importance in real-time processing.

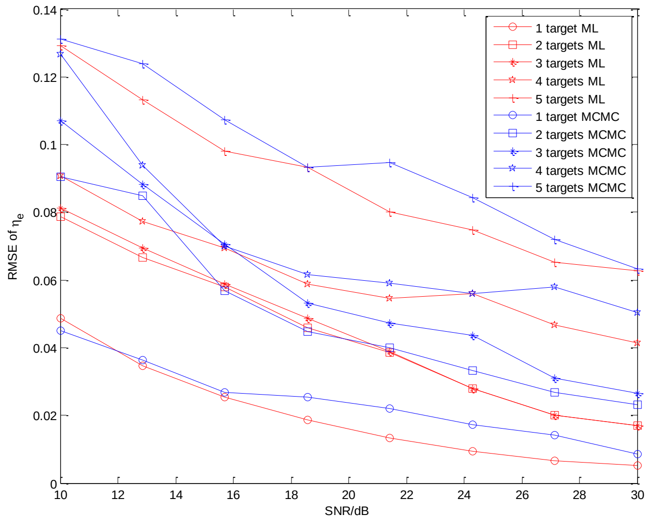

Fig. 5 RMSEs of the estimates of the elevation η_e of targets 1 with different numbers of actual targets under different SNRs

5 Conclusions

For the problem of the joint parameter estimation of multiple unresolved targets within the radar beam，motived by the joint bin processing model, we propose a method of jointly estimating the number and the position of the targets based on RJ-MCMC. Different from the conventional step-by-step estimation method, the proposed method based on RJ-MCMC can jointly estimate the number of targets, their position and other parameters. And simulations show that similar estimation accuracy can be achieved with only 10 sub-pulses, which would be of great importance in real-time processing. Next we will further analyze the estimation performance under different targets' signal power.

Acknowledgements

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