

Research on Coupling Method of Watershed Initial Water Rights Allocation in Daling River

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Abstract. As a typical abnormal, nonlinear and multidimensional system decision-making problem, watershed initial water rights allocation involves various resource distribution, economic, social and environment objectives. Because of the large subjectivity of weight determination, different from the traditional methods, we adopt a new coupling method which is a dimension reduction method in this study to solve the watershed initial water rights allocation problem. The data allocated in Daling watershed can be calculated in optimum projection direction to gain the watershed initial water rights allocation scheme in low-dimensional space.

1. Introduction

Water is one of the basic resources for agricultural development and plays a strategic position in the agricultural production. As one of the increasingly scarce strategic resources, water resources have a global and long-term impact on food security and agricultural economic development. Nowadays, water resources scarcity is becoming much severe due to the increasing population, climate change and water pollution problems. Groundwater resources in many regions have been exploited to irrigate crops in dried years which result the drop of the water tables. Many issues lead to the severe damage to the environmental and social efficiency decreased. The contradiction between water supply and demand has become increasingly prominent, especially the excessive consumption and indiscriminate abuse of the severe problem of water supply and demand. The situation of the water shortage and water pollution is very grim for the sustainable use of water resources and water environment. How to achieve the rational use of water resources, improve the efficiency and equity of water resources utilization has become the focus of economic theorists and policy makers concerned.

Although the excessive use and waste of water resources are the main reasons leading the water shortage, structural defects of the existing system of water rights can be called the deep-seated reason of current water shortage. Due to the unclear water property, economic agents regard the water resources as the free "public resources" which lead to excess use and waste of water. If the current water rights system in China continues without fundamental change, only by increasing people's awareness of water conservation and expansion of the limited water supply is impossible to solve the current low water use efficiency fundamentally.

Some researchers have studied the hydrology, ecology and other subject models. Most of them focused on one factor but ignored the multi-factor analysis, it is still a challenge to directly solve the water rights allocation which is a multi-factor cluster problem. How to get a single-factor question



from the multi-factor one is the key issue. As a consequence, we adopt Projection Pursuit (PP) introduced by Friedman and Tukey (1974) to solve the above problem. Projection pursuit (PP) is an effective method based on different functions of projection indexes has been presented in this paper when standard deviation and partial density are used to structure the function of projection indexes, it deduces and acquires the empirical formula of calculation of exclusive parameter the density window breadth of projection pursuit cluster model. Fu (2003) also studied this model to solve the nonlinear questions. By transferring high-dimensional data into a low dimensional space (e.g., three-dimension, two-dimension or one-dimension), PP model can be directly driven by the low-dimensional data. Except the PP model, some researchers studied some derived models to analyze the high-dimensional data. Such as projection pursuit regression (PPR), projection pursuit density estimation (PPDE), projection pursuit cluster (PPC), projection pursuit learning network (PPLN), projection pursuit wavelet learning network (PPWLN), etc. (Friedman and Stuetzle. 1981, Friedman et al. 1984, Hall 1989, Hwang et al. 1994, Lin et al. 2003). The method of PP model, possessing advantages of good stability, strong anti-interference and high accuracy, has been widely used in many areas.

Coupling method has been used successfully in different areas to evaluate the multi-factor problems. But this method just provides the projected characteristic value remaining the major characteristics of data according to the projection index (Wang et al. 2002). In this paper, we propose a mathematical model by projection pursuit technique to solve the above mentioned problems, and find the optimal projection direction according to the Coupling optimization algorithm. Set up a new projection index to overcome the difficulties of weights determination. The method owns the advantages of complying with watershed initial water rights allocation mechanism and meeting the control requirements of water quantity, water quality and water utilization efficiency, which help to achieve effective allocation of water resource.

2. Study Area

As the largest single flow in the western of Liaoning province in China, Daling watershed stretches across the provinces of Liaoning, Inner Mongolia and Hebei. This region belongs to the typical continental monsoon climate characterized by hot and rainy summers, cold and dry winters, which results in uneven amount of precipitation of year and rainfall is concentrated in July and August. Meanwhile the average rainfall of years increases by degrees from north to south. The annual mean precipitation of this watershed is between 400mm and 600mm. The per capita possession of water resources of this watershed is merely 392m³, which accounts for 18% of the national level. In spite of the shortage and conflict of water resources, the frame work, such as the comprehensive scheme of social and economic development of this watershed, and the comprehensive scheme of water resources, are relatively complete. Hence, this article selects the initial water rights allocation scheme of Daling watershed as a case to study. Choose the fairness, efficiency and sustainability as three basic principles, show as table 1.

Table 1. The Index System of Water Rights Allocation

Principle layer	Index layer
Fairness(B ₁)	Rate of water shortage(C ₁)
	Per capita water resources(C ₂)
	Per capita water consumption(C ₃)
	Per capita cultivated land area(C ₄)
	Per capita effective irrigation area(C ₅)
	Population per unit area(C ₆)
	Livestock numbers per unit area(C ₇)
	Urbanization rate(C ₈)
	Per Capital GDP(C ₉)
Efficiency(B ₂)	Ratio of GDP in the whole watershed(C ₁₀)
	Ratio of industrial production in the whole watershed(C ₁₁)
	Ratio of agriculture production in the whole watershed(C ₁₂)
	Economic growth rate(C ₁₃)
	GDP per cubic water(C ₁₄)
	Agriculture production per cubic water(C ₁₅)
	Industrial production per cubic water(C ₁₆)
	Utilization rate of industrial water duplication(C ₁₇)
	Utilization rate of Irrigation water(C ₁₈)
Sustainability(B ₃)	Rate of water resources development(C ₁₉)
	Standard rate of water quality(C ₂₀)
	Per capita emissions of pollutants(C ₂₁)
	Unit GDP sewage discharges(C ₂₂)
	Rate of vegetation coverage(C ₂₃)
	Rate of population growth(C ₂₄)
	Per capita net income(C ₂₅)

3. Research methods

3.1. Coupling optimization algorithm

Coupling is a random phenomenon with seemingly rule in deterministic system, because of its randomness and regularity properties, chaotic motion could go through on its own rule within a certain range. Thus, by using chaotic variables, we can obtain globally optimal solution with high efficiency. The basic idea of coupling optimization algorithm is to linearly map chaotic variables to the value interval of optimized variables, and then search iteratively to get the optimal solution. Generally, the mathematical model of nonlinear programming can be expressed as:

$$\begin{cases} \min f(X) \\ g_i(X) \geq 0 \\ h_j(X) = 0 \end{cases} \quad (1)$$

Where $X \in E^n$, ($i=1,2,\dots,m$) and ($j=1,2,\dots,n$), $f(X)$ is the objective function. $g_i(X)$ And $h_j(X)$ are constraint functions of inequality and equality respectively, m and n are the number of inequality and equality constraint functions. The constraint condition can be expressed as:

$$S = \{X \mid g_i(X) \geq 0, i = 1, 2, \dots, m; h_j(X) = 0, j = 1, 2, \dots, n\} \quad (2)$$

S is called feasible set and the elements included are feasible points.

Logistic model is one of the most classical model of fuzzy research, chaotic variables generated by the model can be used to search the optimal solution, and the equation is:

$$X_{k+1,j} = \lambda \cdot X_{k,j} \cdot (1 - X_{k,j}) \quad (3)$$

Where λ is the control parameter, and it is selected in the range of 0 and 4, when $\lambda = 4$, the system is in the fuzzy state.

3.2. Coupling Model

Coupling model sets high dimensional data into a low dimensional data space, finds out structure or characteristics projection can reflect the original high dimensional data and studies the purpose of high dimensional data. The problem of structuring projection index function and its optimization is the key of applying method of projection pursuit successfully. This problem is very complex, traditional projection pursuit has considerable amount of calculation, it limits the method of thorough research and widely use to some extent. The concrete steps of the model are as follows:

Step 1: Structuring coupling model index function

In order to eliminate the effect of different ranges of values of cluster factors, the initial data are standardized before it is used in the PPDC model. Samples experience indexes set as $\{x^*(i, j) \mid i = 1, 2, \dots, n; j = 1, 2, \dots, p\}$, where $x^*(i, j)$ is the j th value of the i th sample, n, p are sample number (sample size) and index number respectively. To eliminate the dimensions of index value and range of unity each index, following formula can be adopted for extreme normalized. In case the index the bigger the better:

$$x(i, j) = \frac{x^*(i, j) - x_{\min}(j)}{x_{\max}(j) - x_{\min}(j)} \quad (4)$$

In case the index the smaller the better:

$$x(i, j) = \frac{x_{\max}(j) - x^*(i, j)}{x_{\max}(j) - x_{\min}(j)} \quad (5)$$

Where, $x_{\max}(j)$ and $x_{\min}(j)$ is the max and min value of the j th indicator respectively.

Step 2: Construct coupling model index function $Q(a)$.

Essentially, projection is used to observe data characteristic from all angles. The main purpose is to find the hidden structure from high-dimensional data sets by searching through all their low-dimensional projections (Cui 1997). Using projection pursuit technique, $\{x^*(i, j) \mid j = 1, 2, \dots, p\}$ can be projected into the direction $a = \{a(1), a(2), a(3) \dots a(p)\}$, that is

$$Z(i) = \sum_{j=1}^p a(j)x(i, j) \quad (i = 1, 2, \dots, n) \quad (6)$$

Projection index function is:

$$Q(a) = S_z D_z \quad (7)$$

$$S_z = \sqrt{\frac{\sum_{i=1}^n [Z(i) - E(Z)]^2}{n-1}} \quad (8)$$

$$D_z = \sum_{i=1}^n \sum_{j=1}^n (R - r(i, j)) \cdot u(R - r(i, j)) \quad (9)$$

Where S_z is the standard deviation of projection value $Z(i)$; D_z is the local density of $Z(i)$; $E(Z)$ is the mean of $\{Z(i)|i=1, 2, \dots, n\}$; R is the window radius of local density, normally is set to $0.1 S_z$; $r(i, j)$ is the distance between samples, $r(i, j) = |Z(i) - Z(j)|$; $u(t)$ is the unit step function, it equals to 1 when $t \geq 0$, otherwise 0.

Step 3: Optimize the coupling index function.

When the sample set of indicators is given, $Q(a)$ only depends on the projection direction a , the best projection direction is the one that exposures the characteristics of high-dimensional data structure of maximum possible, which can be obtained by solving the max value of projection index function:

$$\max Q(a) = S_z D_z \quad (10)$$

Constraint is:

$$\sum_{j=1}^p a^2(j) = 1 \quad (11)$$

The model is a complex nonlinear problem with variables $\{a(j)|j=1, 2, \dots, p\}$, and can be solved by Coupling optimization algorithm.

Step 4: Initialize parameters.

Assume X in (1) has p dimensions, $X = (x_1, x_2, \dots, x_p)$, $x_i = [a_i, b_i]$. Set control variable of iterations $k = 0$, generate p initial values $x_{i,k}$, $i = 1, 2, \dots, p$, $X_k = (x_{1,k}, x_{2,k}, \dots, x_{p,k})$, let

$$x'_{i,k} = a_i + (b_i - a_i)x_{i,k} \quad (12)$$

Where $X'_k = (x'_{1,k}, x'_{2,k}, \dots, x'_{p,k})$, when $X'_k \in S$, let $X^* = X'_k$ and $f^* = f(X^*)$. Otherwise, go to step (4) until X_k is found, which has the according X'_k that satisfies constraints of (1).

Step 5: Coarse search.

Set the max iteration step N , generate fuzzy sequence X_k , $k = 1, 2, \dots, N$, and then test its feasibility, if successful and $f(X'_k) < f(X^*)$, let $X^* = X'_k$, $f(X^*) = f(X'_k)$; otherwise test X'_{k+1} , repeat step (5), until $f(X'_k)$ satisfies N or precision requirement.

Step 6: Fine search.

Set the max iteration steps N' , let $Z_0 = X^*$, and $Z_k = Z_{k-1} + \alpha Z'_{k-1}$, $k=1, 2, \dots, N'$. Where Z'_{k-1} is the chaotic sequence generated by Logistic mapping, α is the given small real number. Test the feasibility of Z_k , $k=1, 2, \dots, N'$, if successful and $f(Z_k) < f(X^*)$, let $X^* = Z_k$, $f(X^*) = f(Z_k)$, otherwise test Z_{k+1} , repeat step (6) until $f(X^*)$ satisfies precision requirement. Finally, X^* is the optimal projection direction a^* of (10).

Step 7: Determine the proportion of allocated water.

Substitute the optimal projection direction a^* in step (6) into (6), and projection value $Z^*(i)$ of every sample can be obtained. Normalizing $Z^*(i)$ can get the proportion of allocated water, multiplied by the total amount of water is the final allocated water.

4. Results and analysis

In addition, the result obtained through coupling optimization algorithm in this study is an optimal solution. This is because coupling optimization algorithm due to its ergodicity characteristic as well as enlarging the fuzzy sequences to the value interval of optimized variables to carry on iterative optimization. As long as the fuzzy sequences are long enough, we can obtain the globally optimal solution through the coarse search and the fine search. However, the solution of genetic algorithm is uncertain because many random numbers are used during calculation process, and it is easy to be trapped in the local optimal solution even divergent. Table 2 and table 3 are the curves between optimization iterations and projection index values of Coupling optimization algorithm and genetic algorithm.

As shown in the following tables, the optimization iterations of Coupling optimization algorithm and genetic algorithm are given. With this comparison, coupling optimization algorithm can obtain the optimal projection value and calculation speed is faster and the result is more stable, which proves the effectiveness of the model in this study fully.

Table 2. Normalized Index Values of Water Rights Allocation

Index	Region			Index	Region		
	Inner Mongolia	Liaoning	Hebei		Inner Mongolia	Liaoning	Hebei
C1	0.289	0.390	0.321	C14	0.185	0.192	0.623
C2	0.448	0.415	0.137	C15	0.248	0.224	0.528
C3	0.359	0.391	0.250	C16	0.191	0.198	0.611
C4	0.464	0.298	0.238	C17	0.311	0.315	0.374
C5	0.463	0.393	0.144	C18	0.325	0.307	0.362
C6	0.365	0.416	0.219	C19	0.310	0.362	0.328
C7	0.394	0.504	0.102	C20	0.410	0.312	0.278
C8	0.231	0.215	0.554	C21	0.356	0.365	0.279
C9	0.198	0.165	0.637	C22	0.405	0.392	0.203
C10	0.188	0.190	0.622	C23	0.385	0.330	0.285
C11	0.221	0.201	0.578	C24	0.198	0.195	0.607
C12	0.357	0.382	0.261	C25	0.199	0.143	0.658
C13	0.305	0.312	0.383				

Table 3. Water Rights Allocation Scheme of Daling Watershed

Region	Projection Value	Proportion of water allocation
Inner Mongolia	1.6356	35.2%
Liaoning	1.8965	39.3%
Hebei	1.3575	25.5%

Table 4. Comparative Analysis of the Results of Various Models

Region	Proportion of water use in history	Results of PP	Results of AHP
Inner Mongolia	34.9%	35.2%	34.1%
Liaoning	41.8%	39.3%	38.9%
Hebei	23.3%	25.5%	27.0%

5. Conclusion

With the rapid development of the economy of our country, the demand for water resources is increasing every day. The shortage of water resources has become an important restrictive factor for the sustainable development of the economy of our country, which causes water use conflicts between the upstream and downstream in the watershed or between different areas, so it's really important to allocate water rights fairly. This study proposed a new coupled fuzzy optimization projection pursuit model, taking advantage of projection pursuit to map high-dimensional data to one-dimensional space, using coupling optimization algorithm to find the optimal projection direction. This model is objective and fair in weights determination, which overcomes the difficulties of traditional methods that the objective functions and constraints must be continuous and differential and avoids being trapped in the local optimal solution during the process of optimization. Through an example analysis, the result of initial water rights allocation obtained through the model is reasonable and effective, simultaneously enriching and developing initial water rights allocation theory and method. However, the window radius of local density of projection pursuit in the model is determined by empirical formula, lack of theoretical basis and the optimal solution obtained through coupling optimization algorithm has great dependence on the length of chaotic sequences and the number of iterations, which need to be improved in the future.

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