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Particle Swarm Optimization (PSO) for Magnetotelluric (MT) 1D Inversion Modeling

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Abstract. Particle Swarm Optimization (PSO) is one of nature-inspired optimization algorithms that adopts swarm (insects, school of fish, flock of birds etc.) behaviour in search for food or common target in a collaborative manner. The particles (or agents) in the swarm learn from their neighbours as well as themselves regarding the promising area in the search space. The information is then used to update their position in order to reach the target. The search algorithm of a particle is dictated by the best position of that particle during the process (individual learning term) and the best particle in its surroundings (social learning term) at a particular iteration. In terms of optimization, the particles are models defined by their parameters, while the promising area in the model space is characterized by a low misfit associated with optimum models. Being a global search approach, PSO is suitable for non-linear inverse problem resolution. The algorithm was applied to a simple minimization problem for illustration purpose. The application of PSO in geophysical inverse problem is demonstrated by inversion of synthetic magnetotelluric (MT) data associated with simple 1D models with satisfactory results in terms of model recovery as well as data misfit.

1. Introduction

In geophysics, the inverse modeling is performed to the observed (field) data to obtain the model representing the physical property (e.g. density, resistivity, magnetization, seismic velocity etc.) distribution of the sub-surface. The problem is a function minimization, where the function to be minimized is the misfit or difference between the observed data and the theoretical response of a model. For a linear inverse modeling, i.e. the model is linearly related to the data, the solution is relatively straightforward and well-known. For non-linear inverse problems, the solution is seek by linearization of the misfit function around a starting model and update that model iteratively until a minimum misfit is obtained. Such localized approach suffers from two main drawbacks, i.e. sensitivity to the starting model and possibility to converge to a local minimum rather than to a global one [1,2].

In the global approach, no linearization and gradient-free optimization is performed by intensive exploration of the model space in the search for solutions. This approach can effectively overcome difficulties of local search or linearized approach of strongly non-linear inverse problems. The Monte-Carlo based algorithms such as Simulated Annealing (SA), Genetic Algorithm (GA), Markov Chain

Monte Carlo (MCMC) are among the most popular non-linear inverse modeling algorithms that have been applied to geophysical inverse problems [3,4,5]. One of global approach algorithms which is gaining interest for geophysical inverse modeling is the Particle Swarm Optimization (PSO) proposed by Kennedy and Eberhart [6,7]. The algorithm is based on the behavior of animals in swarm to achieve a common goal with a collaborative manner. Individuals (models) occupying positions in the model space are modified iteratively by considering the best position of each individual (cognitive learning term) and the group's best position (social learning term) to reach the optimum position or solution. The position of a model in the model space is qualified with its misfit.

In electromagnetic (EM) geophysics (e.g. magnetotellurics, transient EM, etc.) the forward and inverse modeling for 1-D structure has been considered as solved problems. However, non-linearity and ill-posedness of the problem remain to be an interesting subject for research, especially for the application of global search approach e.g. [8,9,10]. The PSO algorithm has been applied to non-linear inversion of magnetotelluric (MT) and Vertical Electrical Sounding (VES) data using 1-D model with satisfactory results [11,12]. In this paper, we follow the same approach with the emphasis for MT 1-D inversion of synthetic data. It is also our purpose to illustrate the application of a global search approach to invert geophysical data with a relatively simple and yet non-linear forward problem such as MT.

2. Basic of Particle Swarm Optimization (PSO)

The Particle Swarm Optimization (PSO) is a stochastic optimization algorithm that belongs to the larger family of evolutionary computation techniques, e.g. GAs. Although it shares many similarities with GAs, PSO does not use a variety of selection, recombination and mutation operators to maintain and generate new population of potential solutions iteratively. The PSO adopts the collaborative behavior of a group of animals (insects, birds, fish) in search for a common target, e.g. food. Each member of the swarm (particle) in the search space adapts its trajectory according to the best experiences of the swarm to search for the target. The particle represents a potential solution, i.e. a model with associated fitness value, in the solution space. The target location is in fact the global optimum of the optimization problem.

In what follows, we describe only the basic PSO algorithm, while variations from the basic e.g. [13,14] are beyond the scope of this paper. The position and search trajectory of particles are defined in an N-dimensional model space. By adopting displacement formula in physics, the position $\mathbf{x} = [x_j]$; $j = 1, 2, \ldots N$ of the i-th particle at k-th iteration is expressed by,

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \mathbf{v}_i(k) + \tfrac{1}{2}\mathbf{a}_i(k) \tag{1}$$

where $\mathbf{v}$ and $\mathbf{a}$ represent velocity and acceleration respectively each define in the N-dimensional space as for $\mathbf{x}$. With a unitary time step, the terms t and t^2 in velocity and acceleration respectively are eliminated. During each iteration, each particle learns from its own experience (personal influence) and from other members' experiences (social influence) as well. These two influences affect the particle's total acceleration such that,

$$\mathbf{a}_i(k) = c_1 R_{1i}\ (\mathbf{p}_i(k) - \mathbf{x}_i(k)) + c_2 R_{2i}\ (\mathbf{g}(k) - \mathbf{x}_i(k)) \tag{2}$$

where c_1 and c_2 denote the acceleration coefficients, R_{1i} and R_{2i} replacing the number ½ in equation (1) are random numbers introduced to include stochastic behavior of the process. The personal best $\mathbf{p}_i$ and the global best $\mathbf{g}$ in equation (2) are respectively: the best solution of the i-th particle and the best solution amongst all particles so far up to the k-th iteration.

The velocity update is the sum of the current velocity and the total acceleration expressed by equation (2), i.e.

$$\mathbf{v}_i(k+1) = \omega\ \mathbf{v}_i(k) + \mathbf{a}_i(k) \tag{3}$$

where the velocity update is expressed for the (k+1)-th iteration and ω set less than 1 is the inertia weight such that only a fraction of the velocity is carried over to the next iteration to prevent velocities from growing out of control. The position update of the i-th particle as in equation (1) becomes,

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \mathbf{v}_i(k+1) \quad (4)$$

In practice, a particle's velocity is limited to a predefined range [$-\mathbf{v}_{max}$, $+\mathbf{v}_{max}$] to reduce the possibility of particles leaving the search space. The basic PSO algorithm is graphically illustrated in Figure 1. The term memory and cooperation denote personal and social influence respectively, while inertia is the previous velocity.

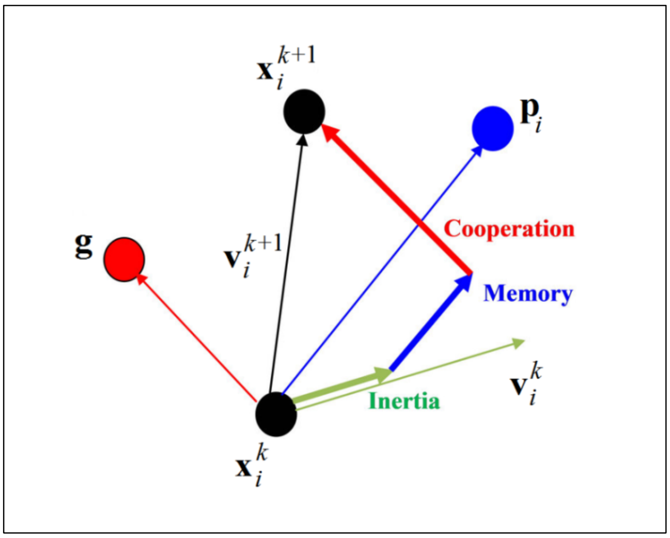

Figure 1. Graphical illustration of the basic PSO algorithm, in which particle $\mathbf{x}_i^k$ moves to $\mathbf{x}_i^{k+1}$ under the influence of its personal best $\mathbf{p}_i$ (memory) and the global best $\mathbf{g}$ (cooperation) while still having a part of velocity of the previous movement (inertia).

3. Magnetotelluric 1D Forward Modeling

The magnetotelluric (MT) response of a given conductivity model is obtained from the solution of Maxwell's differential equations. For one-dimensional (1D) model, the response function is well-known and usually expressed as a recursive formula relating the impedance at the surface of two successive layers (Z_j and Z_{j+1}) as follows [8],

$$Z_j = Z_{0j} \frac{1 - R_j \exp(-2(Z_{0j}/\rho_j) h_j)}{1 + R_j \exp(-2(Z_{0j}/\rho_j) h_j)} \; ; \quad R_j = \frac{Z_{0j} - Z_{j+1}}{Z_{0j} + Z_{j+1}} \; ; \quad Z_{0j} = \sqrt{i\,\omega\mu_0\,\rho_j} \quad (5)$$

where Z_{0j} denotes the impedance of a homogeneous medium with a resistivity ρ_j and h_j is the thickness of the j-th layer, while $\omega = 2\pi/T$ is the angular frequency for a period T and μ_0 is the magnetic permeability of a free space. All units are in SI. At surface (layer 1) of a model consisting of N-layer (Figure 2), the MT response is usually expressed as apparent resistivity (ρ_a) and phase (ϕ),

$$\rho_a = \frac{1}{\omega\mu_0} |Z_1|^2 \qquad \phi = \tan^{-1}\left(\frac{\mathrm{Im}\, Z_1}{\mathrm{Re}\, Z_1}\right) \quad (6)$$

From equations (5) and (6) it is obvious that the parameter models are non-linearly related to the data.

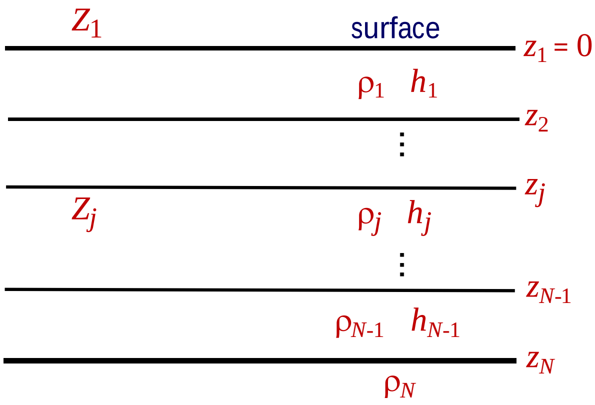

Figure 2. *N*-layered earth (1D) model, each layer with resistivity ρ_j, thickness h_j and impedance Z_j.

4. Inversion Results

The PSO algorithm was applied to invert synthetic MT data with 1D model. Four 3-layered synthetic models associated with H-, K-, Q-, and A-type sounding curves were used to generate the synthetic data (Table 1). Gaussian noise with 10% standard deviation was added to the theoretical response of the synthetic models. For the sake of brevity of the paper, only results with H- and K-type sounding curves (Model A and Model B) are presented. In general, models from both H- and K-type sounding curves are more difficult to resolve in the inversion.

Table 1. Synthetic model parameters used for generating the synthetic data.

	Model A (H-type)		Model B (K-type)		Model C (Q-type)		Model D (A-type)	
Layer	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)
1	1000	500	100	500	1000	500	10	500
2	10	1000	1000	1000	100	1000	100	1000
3	100	-	10	-	10	-	1000	-

We performed inversions with the correct (3-layer) and un-correct (5-layer) number of layers to test the robustness of the algorithm faced to different *a priori* information on the number of layers. The PSO algorithm was also tested with different number of models (or particles), i.e. 50 and 200 particles. Inversions were done systematically up to 200 iterations, where in general, convergence was obtained since 100-th iteration. Therefore, the results presented in Table 2 (Model A) and Table 3 (Model B) are averaged models at 200-th iteration, with negligible uncertainties.

For the 3-layer model, the inverse models are practically similar and in very good agreement with the synthetic model, especially for the Model A (H-type) with both 50 and 200 particles. Similar observation is valid for Model B (K-type), except that the resistivity of the second layer is under-estimated. For the 5-layer model, the first 2 layers in the inverse models correspond relatively well with the same layers of the synthetic Model A. The last 3 layers of the inverse models are the equivalence layers of the third (last) layer of the Model A. This led to un-correct depth estimate of the last (moderately conductive) layer. The same difficulty in resolving the resistivity of the sandwiched resistive layer prevails for the Model B. However, the resistivity and depth of the last (conductive) layer were relatively well resolved. The comparisons of synthetic and inverse models along with the data fit for representative results are presented in Figure 3.

Table 2. Inverse model parameters from inversion of synthetic data (Model A).

	3-layer model				5-layer model			
	50 models (particles)		200 models (particles)		50 models (particles)		200 models (particles)	
Layer	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)
1	1001.37	516.00	1001.37	516.00	1007.45	517.60	1051.01	511.31
2	10.03	973.69	10.03	973.69	10.07	876.06	11.05	1120.71
3	96.65	-	96.65	-	43.76	149.57	49.72	608.49
4	-	-	-	-	5.85	44.48	128.89	10345.62
5	-	-	-	-	97.63	-	81.73	-

Table 3. Inverse model parameters from inversion of synthetic data (Model B).

	3-layer model				5-layer model			
	50 models		200 models		50 models		200 models	
Layer	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)	Resistivity (Ohm.m)	Thickness (m)
1	98.96	503.72	99.53	502.51	96.75	392.52	103.04	469.01
2	722.13	987.36	702.21	1004.68	286.19	457.89	411.41	738.15
3	9.48	-	9.75	-	248.94	582.21	200.46	310.16
4	-	-	-	-	22.16	198.02	37.94	53.86
5	-	-	-	-	9.78	-	9.91	-

5. Concluding Remarks

There are growing interests in computer intensive global optimization methods due to availability of computing power in recent years. The global and gradient-free approach avoids the use of linearization of highly non-linear inverse problems. Hence, it can overcome the fundamental limitations of linearized techniques. The Particle Swarm Optimization (PSO) which is one of nature-inspired global search algorithms has been presented with the application to invert MT data to obtain resistivity-depth variation of the subsurface represented by the 1D model. The results are encouraging in terms of model recovery as well as data misfit. The resolved inverse models are in accordance with the general characteristics of 1D MT modeling, i.e. difficulties in determining the resistivity of a resistive layer sandwiched between more conductive layers. It was also emphasized that a conductive layer at shallow depth may limit the ability of estimating parameters of the deeper layer, i.e. screening effects. The RMS misfits are in general around 18 to 19% which are consistent with the noise added in the synthetic data.

The PSO algorithm presented in this paper is relatively simple to implement with minimum inversion parameters to adjust, i.e. acceleration coefficients, inertia weight and number of particles or models. Those inversion paramaters have a relatively unsignificant influence on the overall inversion results. It is also possible to extend the basic PSO algorithm to solve more complex geophysical inverse problems. However, the application would be limited to a relatively small number of model parameters in order to be amenable with current computing capability. The possible applications include 1D inverse modeling of electromagnetic (EM) data, i.e. Transient EM (TEM), Vertical

Electrical Sounding (VES), Controlle-Source EM (CSEM), Nuclear Magnetic Resonance Sounding (NMRS) etc.

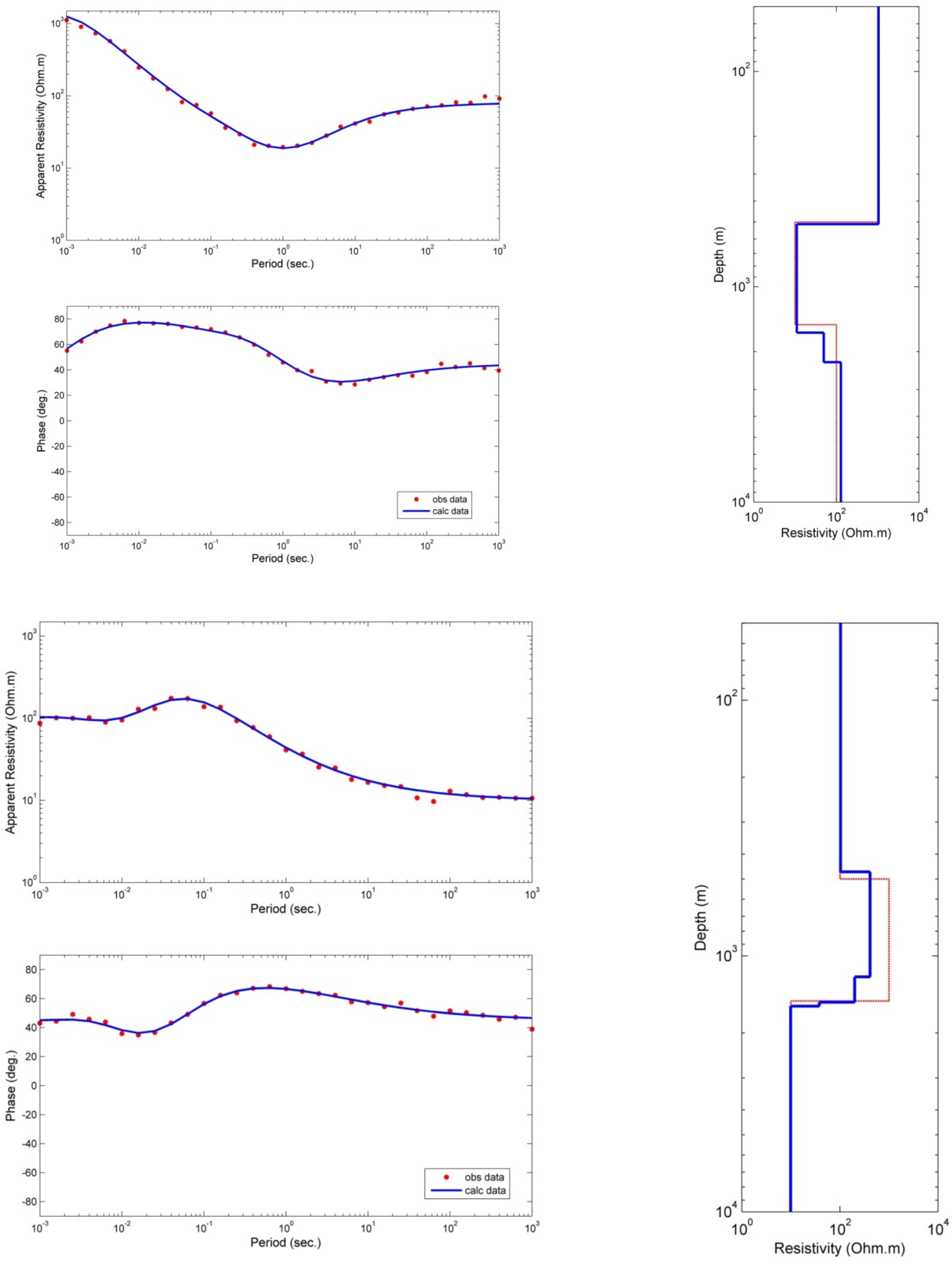

Figure 3. The fit between observed and calculated data along with the comparison of inverse (5 layers) model and synthetic model for Model A or H-type (top) and Model B or K-type sounding curve (bottom). In general, all results have RMS error around 18% to 19%.

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