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Central Loop Time Domain Electromagnetic Inversion Based on Born Approximation and Levenberg-Marquardt Algorithm

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Abstract. Time Domain Electromagnetic (TDEM) method uses electromagnetic wave to detect resistivity or conductivity difference of lithology in the subsurface, measured in the time domain. TDEM method has been developed in decades. There are forward modeling and inversion programs have been made. The purpose of making the forward modeling programs is to calculate TDEM response so that data acquisition parameters can be chosen correctly. The inversion program is for synthesizing geological model from measured TDEM data. Several TDEM programs have been made, but the procedures require heavy and complex computation, which need high computer specification and lot of calculation time. Nowadays program that used less complex computation and faster calculation is needed to match field data acquisition productivity. To achieve faster and more accurate process, we use Born Approximation of the apparent conductivity for the forward modeling program and Levenberg-Marquardt algorithm for the inversion program. We use many circumstances in testing our programs, from one layer to multi layers by varying the resistivity or thickness of lithology and compared with a validated program, EMUPLUS. At the end of the test, inversion of real data is taken as confirmation that the program can be used to process real TDEM data. Inversion results of both synthetic and real TDEM data show pleasant results. These test results indicate that our program can be used as daily forward modeling program for determining data field acquisition parameters and do the inversion procedure for synthetic and real TDEM data.

1. Introduction

Time Domain Electromagnetic (TDEM) method uses electromagnetic wave to detect resistivity or conductivity difference of lithology in the subsurface, measured in the time domain. Inversion program gives geological parameters from measured data of the Time Domain Electromagnetic. Previous forward modeling procedures require heavy and complex computation, which need high computer specification and lot of calculation time. To achieve a faster and more accurate process, we use approximation procedures in our program. The approximation procedure has been done by using Born Approximation [1,2]. This study will present modification of Levenberg-Marquardt algorithm that is suitable with the forward modeling algorithm. Forward modeling and inversion algorithms in this study will be used in our pyTEM1D program that is based on Python programming language.

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2. Time Domain Electromagnetic

Central Loop Time Domain Electromagnetic measured secondary vertical magnetic field (H_z) and the time derivative of vertical magnetic field $\left(\frac{\partial H_z}{\partial t}\right)$ [3]. The vertical magnetic derivative for homogeneous half-space can be expressed in the following equation.

$$\frac{\partial H_z}{\partial t} = -\frac{I}{\mu_0 \sigma a^3}\left[3erf(\theta a) - \frac{2}{\sqrt{\pi}}\theta a(3 + 2\theta^2 a^2)e^{-\theta^2 a^2}\right] \quad (1)$$

The transmitter radius is symbolized by a, transmitter current is I, and σ is conductivity of the homogenous earth. Generally, in TDEM measurement, apparent resistivity can be distinguished as early time and late time resistivity, although in geophysical prospecting, late time resistivity is used:

$$\rho_a \approx \frac{I^{2/3}\mu_0 a^{4/3}}{20^{2/3}\pi^{1/3}t^{5/3}}\left(\frac{-\partial H_z}{\partial t}\right)^{-2/3} \quad (2)$$

3. Forward Modeling

Forward modeling method in this research is Adaptive Born Forward Mapping [2]. The apparent resistivity can be calculated as.

$$\sigma_a(t_i) = \sum_{j=1}^{L} \sigma_j . \mathcal{F}_{ij} \quad (3)$$

$\mathcal{F}_{ij}$ is integral of Frechet kernel which is a function of depth (z). $\sigma_a(t_i)$ is apparent conductivity from subsurface layer in time t_i, i is i^{th} measurement time, and σ_j is synthetic conductivity models for j^{th} layer $(j = 1, \dots, L)$. The linear approximation of the integral of Frechet kernel can be expressed as:

$$\mathcal{F}\left(z_j, t_i, \sigma_a(t_i)\right) = \begin{cases} \frac{z_j}{D_i}\left(2 - \frac{z_j}{D_i}\right), & for\ z_j \le D_i \\ 1, & for\ z_j > D_i \end{cases} \quad (4)$$

With

$$D_i = \sqrt{\frac{c\,.t_i}{\mu_0 \sigma_a(t_i)}} \quad (5)$$

c is a constant that affect the sensitivity of Frechet kernel, which in this study used a value of 1.2. Later using the known apparent conductivity in a certain time, the time derivative of second magnetic field can be calculated using the analytic equation as in equation 1 and will give the time derivative of secondary magnetic field response in time domain.

4. Inversion

Inversion method is used to find target models or parameters from data. In the inversion process, data and desired model can be simplified as a matrix [4]. Measured data with N number data can be defined as ($\boldsymbol{d}$). Further M number desired models can be stated as ($\boldsymbol{m}$) matrix.

One of the methods to solve non-linear problem is Levenberg-Marquardt algorithm. It is known that inverse matrix cannot be calculated if the matrix singular value is zero, while sometimes become unstable when the model change very small [5]. To overcome this problem, Levenberg-Marquardt equation is solved by using singular value decomposition as explained in [4].

$$\Delta \boldsymbol{m}_n = \boldsymbol{V}\left[diag\left(\frac{\mathrm{Q_i}}{\lambda + Q_i^2}\right)\right]\boldsymbol{U}^T(\boldsymbol{d} - \boldsymbol{g}(m_n)) \quad (6)$$

Where Jacobian $(\mathbf{J})$ matrix which is partial derivative of forward model function and models, is defined as singular value decomposition matrix

$$\mathbf{J} = \boldsymbol{UQV}^T \quad (7)$$

$\boldsymbol{U}$ is NxM matrix, $\boldsymbol{V^T}$ is MxM matrix and $\boldsymbol{Q}$ is diagonal matrix of singular value with MxM dimension. λ is damping factor, which at the beginning of the iteration the is big enough. Because of this, the equation can control the result so that the updated model is not overshoot. After getting a better model, the damping factor is reduced so that the equation can give the best result faster.

5. Forward Modeling Test

The forward modeling process is done by using approximation and analytic equation from previous sections. Here, three basic models and two three layers models that will be processed (Figure 1). Time domain electromagnetic (TDEM) central loop transmitter has 1 Ampere current, 25 m radius and sampling time 1 μs – 1 s, with 20 sampling numbers.

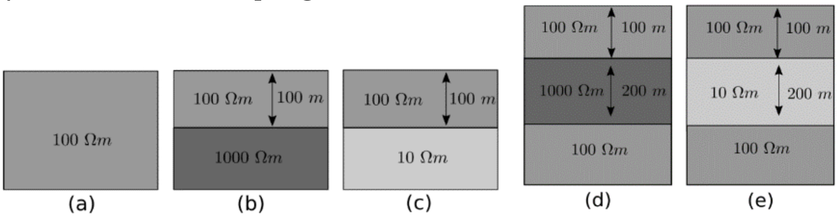

Figure 1. Synthetic models for forward modeling. Three basic models: **(a)** A, **(b)** B, and **(c)** C. Three layers models: **(d)** D and **(e)** E

The time derivative of magnetic field responses from pyTEM1D and EMUPLUS are similar as shown in Figure 2. The homogenous model has a constant trend of decay. Model with a more resistive and conductive second layer has the exact same value for earlier part of the curve, but in 0.1 ms the curve is bent to the lower value of time derivative of magnetic field for the model B and bigger value for the model C.

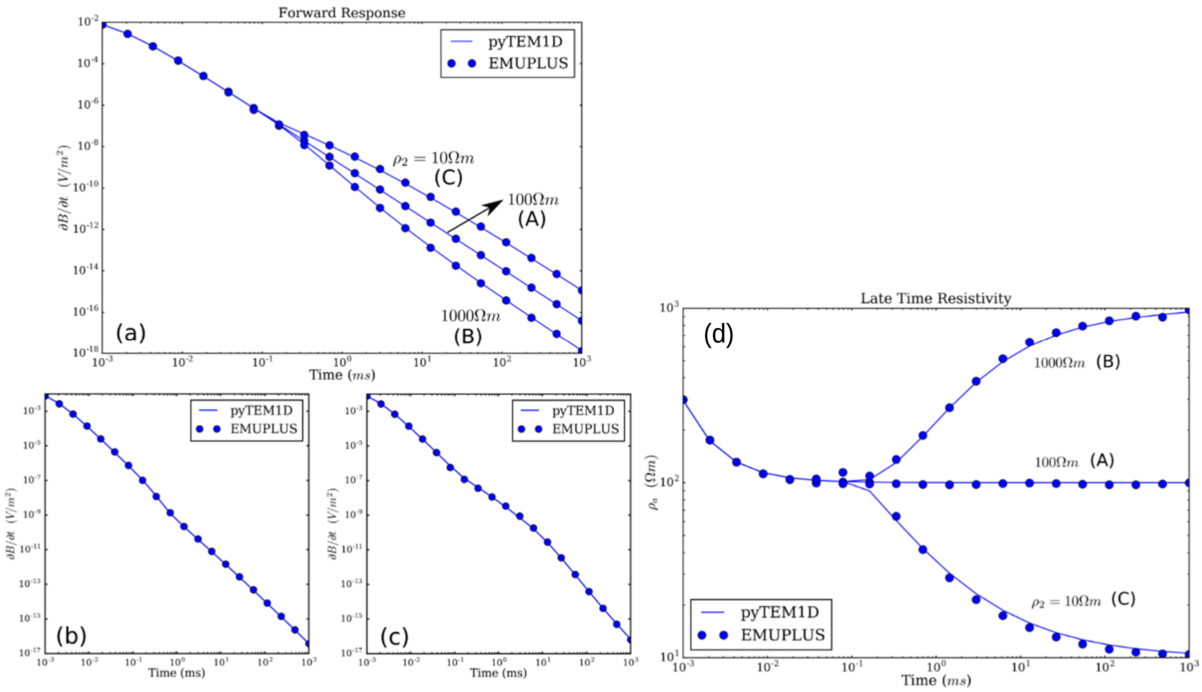

Figure 2. The time derivative of magnetic field of **(a)** three basic models: A, B, and C. Three layers models: **(b)** D and **(c)** E. **(d)** Late time apparent resistivity of three basic models

The time derivative of magnetic field from Figure 2 then transformed to become apparent resistivity at late time condition by using equation 2. The transformation process produces apparent resistivity for

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the late time in Figure 2. The late time apparent resistivity responses in time earlier than 0.1 ms give greater value than it has supposed to be (100 Ωm). This time range is usually called as early time condition [6].

6. Inversion Test

In this section, the inversion algorithm will be applied to the synthetic models from the forward modeling process, model B, C, D, and E (Figure 1). The inversion algorithm is Levenberg-Marquardt that has been explained. Starting models for this process are four starting models, 3-6 layers, 10 Ωm and thickness every layer is 50 m. Figure 3a is time derivative of the magnetic field response of the model E and its inversion results from four different starting models. In this figure, all inversion responses give similar to one another and match with the sytnetic respones of the model E. In determining the best result, a magnification is done to the Figure 3a (Figure 3b). In magnification figure, it appears that the curves of the inversion results are separated from each other.

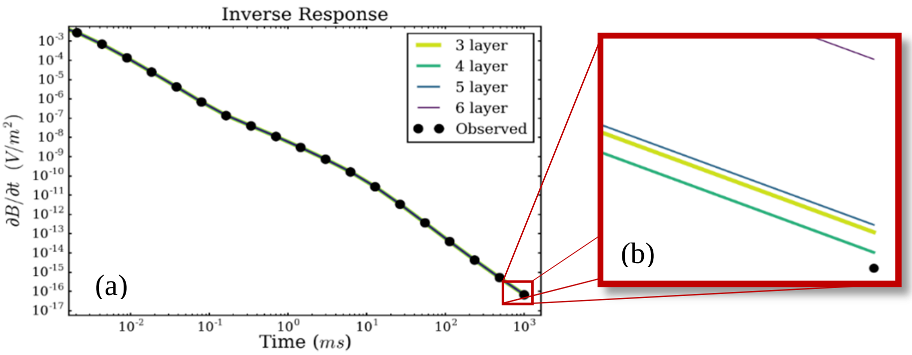

Figure 3. (a) The time derivative of magnetic field of the model E and its inversion results using four different starting models; **(b)** The magnification of the inversion results, shows separated curves for each starting model.

Figure 4 shows the inversion results of the model E using four starting models. Generally, the four inversion results show a model similar to one another. Resistivity value of each layer can be approached with pleasant results for all starting models. Starting model with more layers than the synthetic model can produce inversion models close to the synthetic model by combining thickness and resistivity of the layers.

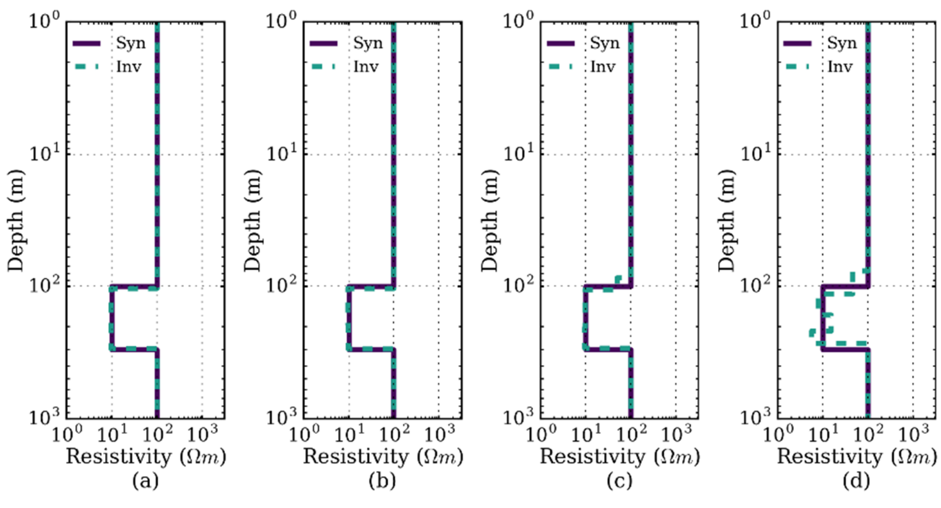

Figure 4. Inversion results of model E by using starting model **(a)** 3, **(b)** 4, **(c)** 5, and **(d)** 6 layers. Dash line is inversion results and full line is synthetic models.

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7. Real data inversion

This section will use real TEM data for testing the inversion and forward modeling algorithm. The TEM data were measured in Volvi basin. The Volvi basin, a neotectonic graben structure, is located northeast of the city of Thessaloniki in Northern Greece. The research was done by Widodo, with a combination of radio magnetotelluric and transient electromagnetic to analyze the structure. In this research, three data are chosen for further inversion processes, which are TEM 1, 2, and 3 [7].

The results of this inversion show that the models of three layers staring model have not been able to produce the appropriate model, this refers to the high RMS error (Figure 5). Inversion of four layers model give best RMS error.

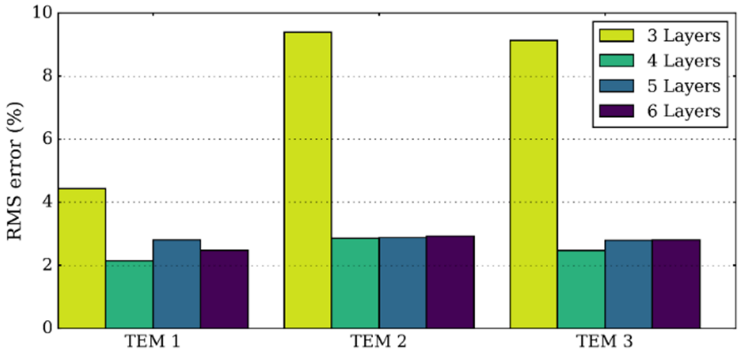

Figure 5. RMS error graph of inversion results for data TEM 1, 2 and 3 by using four synthetic models by using four different layers starting models.

Figure 6a is the time derivative of magnetic field response of TEM 1 and its inversion results from four different starting models. The synthetic and inversion responses are combined in order to compare it qualitatively. With the objective of determining the best results, magnification is done to the Figure 6a (Figure 6b). In the magnification figure (Figure 6b), it appears that the curves of the inversion results are separated from each other. Further inversion result from starting model with four layers is the closest to the synthetic response.

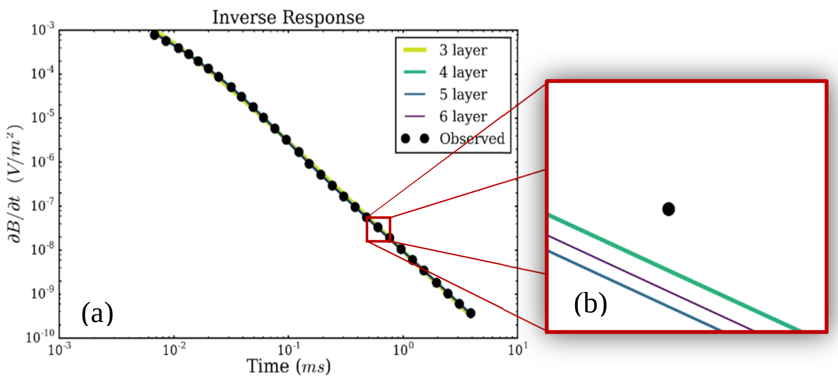

Figure 6. (a) The time derivative of magnetic field of data TEM 2 and its inversion results using four different starting models; **(b)** The magnification of the time derivative of magnetic field inversion results, shows separated curve for each starting model.

Through qualitative analysis using the RMS error (Figure 5) and qualitative analysis, it can be specify that the four layers of inversion result is the best reisitivity model for data TEM 1, TEM 2, and TEM 3 (Figure 7).

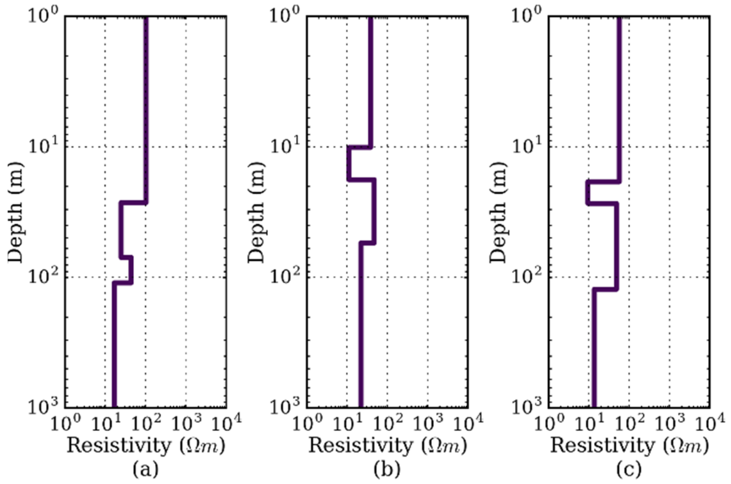

Figure 7. (a) The best inversion model for data TEM 1; **(b)** data TEM 2; and **(c)** data TEM 3.

8. Conclusion

Central loop time domain electromagnetic (TDEM) or transient electromagnetic (TEM) is a method to detect lithology layer by its conductivity properties. TDEM forward modeling program based on Born approximation is a simple method and very fast in the computation. This method is acceptable, as gives similar results with EMUPLUS's. For the inversion algorithm, Levenberg-Marquardt approach also gives pleasant results, though different layers, thickness and resistivity of starting models are used. So far, the inversion algorithm has stable inversion for solving synthetic data or real TEM data, but good starting model must be used in order to get good inversion results.

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