

# Elastoplastic stress analysis of tunnel considering different principal stress directions

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**Abstract.** Based on the unified strength theory, the elastic-plastic stress analysis is carried out under the premise that the first principal stress in two cases of radial stress and circumferential stress, and the formula of the radius of the surrounding rock and the plastic zone is derived. Tunnel surrounding rock has three states which including the elastic state, the elastoplastic state that the maximum principal stress is the radial stress and the elastoplastic state that the maximum principal stress is the circumferential stress. When the elastic-plastic analysis was carried out on the tunnel, the state of the surrounding rock should be judged and the correct formula is chosen to calculate. Analysis results show that the intermediate principal stress is benefit to the rock to give full play to the strength of the potential, so as to improve the stability of surrounding rock of the tunnel and the smaller intermediate principal stress coefficient, the higher sensitivity of the surrounding rock stability to the intermediate stress. When the inside pressure is less than the second critical stress and increasing the hole pressure is beneficial to improving the stability of surrounding rock, while the hole pressure is greater than the second critical stress, the stability of surrounding rock will reduce with the increase of the hole pressure.

## 1 Introduction

The engineering practice usually adopts Mohr-Coulomb strength criterion for tunnel elastic-plastic analysis, the deduced Fenner formula <sup>[1]</sup>, the modified Fenner formula and Kastner formula <sup>[2-4]</sup> is widely used in tunnel engineering. However, it can not reasonable to consider the influence of the intermediate principal stress on the analysis of surrounding rock by using the Mohr-Coulomb strength criterion, which leads to the analysis result is conservative.

The unified strength theory is not only reasonable considering the effect of intermediate principal stress, but also can degenerate into various corner model between Mohr-Coulomb strength criterion, Mises criterion, Tresca criterion and twin shear strength criterion and between Mohr-Coulomb and the twin shear strength criterion, witch have been widely applied in various engineering fields<sup>[5-6]</sup>. In this paper, based on the unified strength theory, the formula of plastic zone width under different principal stress directions is derived. The applicable conditions of different formulas are defined at the same time.



## 2 Calculation Model

The radius of circular tunnel is  $r_0$ , ground stress  $P$ , internal pressure is  $P_i$ , radius of plastic zone is  $r_p$ . In the tunnel of elastoplastic stress analysis, solve the problem of plane strain, the basic assumptions are as follows: (1) Tunnel section equivalent to circular, infinite length; (2) the lateral pressure coefficient is 1; (3) the homogeneous and isotropic rock and the incompressible materials, regardless of physical effects in the process of calculation. Mechanical calculation model shown in Figure 1

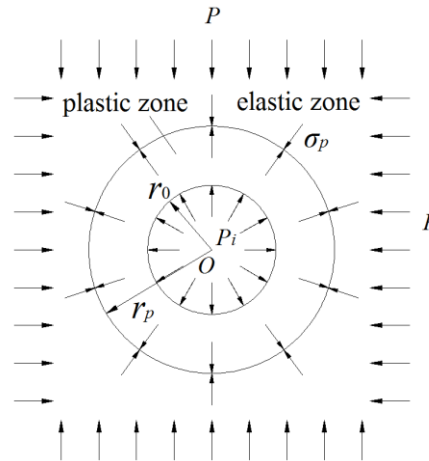


FIGURE 1. Mechanical analysis model

## 3 Elastic Plastic Stress Analysis

*Radial Stress is the First Principal Stress*

According to the unified strength theory expression:

$$F = \sigma_r - \frac{1 - \sin \varphi}{(1 + b)(1 + \sin \varphi)} (b\sigma_z + \sigma_\theta) = \frac{2c \cos \varphi}{1 + \sin \varphi}, \quad \sigma_z \leq \frac{\sigma_r + \sigma_\theta}{2} + \frac{\sin \varphi}{2} (\sigma_r - \sigma_\theta) \quad (1a)$$

$$F' = \frac{1}{1 + b} (b\sigma_z + \sigma_r) - \frac{1 - \sin \varphi}{1 + \sin \varphi} \sigma_\theta = \frac{2c \cos \varphi}{1 + \sin \varphi}, \quad \sigma_z \geq \frac{\sigma_r + \sigma_\theta}{2} + \frac{\sin \varphi}{2} (\sigma_r - \sigma_\theta) \quad (1b)$$

Formula:  $c$ 、 $\varphi$  is the cohesion and internal friction angle of the surrounding rock,  $b$  is the coefficient of intermediate principal stress.

The equilibrium equation of axisymmetric plane strain mechanics model is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (2)$$

The calculation model can have boundary and elastic-plastic contact surface conditions as follows:

$$\left. \begin{aligned} (\sigma_r)_{r=r_0} &= -P_i \\ (\sigma_r^e)_{r \rightarrow \infty} &= -P \\ (\sigma_r^e)_{r=r_p} &= (\sigma_r^p)_{r=r_p} \\ (\sigma_\theta^e)_{r=r_p} &= (\sigma_\theta^p)_{r=r_p} \end{aligned} \right\} \quad (3)$$

Solution as

$$r_p = r_0 \left\{ \frac{[(2+b) - (2+3b)\sin\varphi](c\cot\varphi + P)}{(2+b-b\sin\varphi)(c\cot\varphi + P_i)} \right\}^{\frac{(2+b) - (2+3b)\sin\varphi}{4\sin\varphi(1+b)}} \quad (4)$$

#### *Hoop Stress is the First Principal Stress*

When the water pressure or support pressure is too large, the tunnel rock may also yield region. Because the radial compressive stress is larger, the hoop compressive stress is small, and even becomes the tensile stress, so the first principal stress should be  $\sigma_\theta$ , and the yield criterion expression of surrounding rock becomes

$$F' = \frac{1}{1+b} (b\sigma_z + \sigma_\theta) - \frac{1-\sin\varphi}{1+\sin\varphi} \sigma_r = \frac{2c\cos\varphi}{1+\sin\varphi} \quad (5)$$

Repeat the above analysis process, solution as

$$r_p = r_0 \left\{ \frac{(2+b)(1+\sin\varphi)(c\cot\varphi + P)}{(2+b-b\sin\varphi)(c\cot\varphi + P_i)} \right\}^{-\frac{(2+b)(1+\sin\varphi)}{4(1+b)\sin\varphi}} \quad (6)$$

#### *Applicable Conditions of the Formula*

Under the action of ground stress, there are three kinds of state of tunnel surrounding rock: elastic state, maximum principal stress under the condition of radial stress, elastic-plastic state and maximum principal stress under the condition of hoop stress. In the tunnel by elastic-plastic analysis before, should first determine the rock in the stress field under the conditions of whether there is plastic failure, and then determine the direction of the maximum principal stress, and then choose the correct formula of tunnel stress analysis.

The critical stress  $\sigma_p$  is introduced, which mean  $r_p/r_0 = 1$  when  $P_i = \sigma_p$ , the surrounding rock will appear plastic failure. Analytical formulas (4) and formula (6) show that the first critical stress

$$\sigma_{p1} = \frac{[(2+b) - (2+3b)\sin\varphi]P - 2c(1+b)\cos\varphi}{(2+b-b\sin\varphi)} \quad (7)$$

The second critical stress

$$\sigma_{p2} = \frac{(2+b)(1+\sin\varphi)P + 2c(1+b)\cos\varphi}{(2+b-b\sin\varphi)} \quad (8)$$

It can be concluded that the application of formula (4) and (5) to the elastic-plastic stress analysis of tunnel:

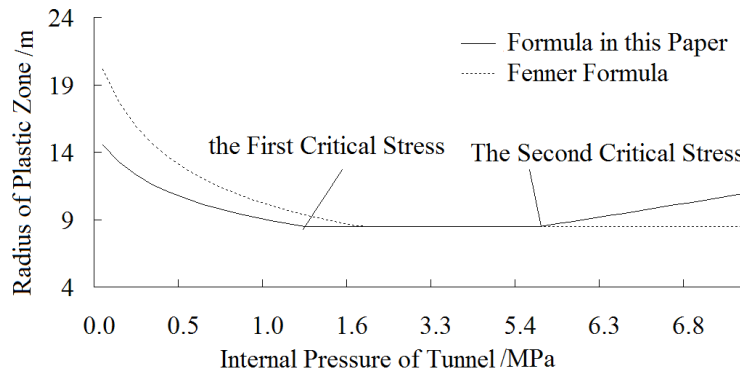
(1) when  $P_i < \sigma_{p1}$ , part of the surrounding rock is in a plastic state, and the first principal stress is radial stress, then the formula (4) is used for elastic-plastic analysis of the tunnel;

(2) When  $\sigma_{p1} \leq P_i \leq \sigma_{p2}$ , surrounding rock is in elastic condition completely and stress analysis can be carried out by elastic theory;

(3) when  $\sigma_{p2} < P_i$ , part of the surrounding rock is in a plastic state, and the first principal stress is hoop stress, then the formula (6) is used for elastic-plastic analysis of the tunnel.

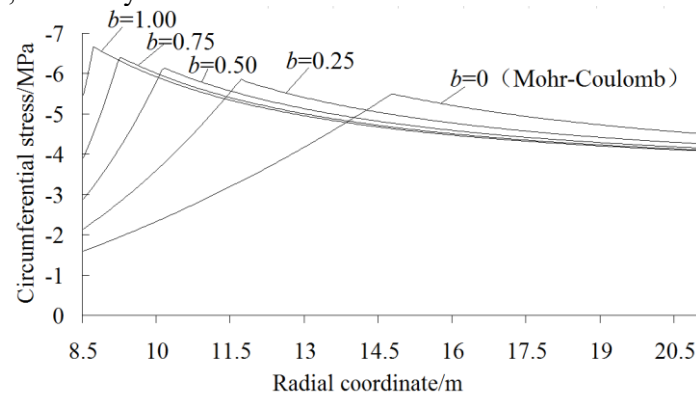
#### 4 Example Analysis

Radius of circular tunnel  $r_0 = 8.5$  m, buried depth is 180m, original rock stress  $P = 3.55$  MPa; surrounding rock cohesion  $c = 0.2$  MPa; internal friction angle  $\varphi = 30^\circ$ , internal pressure  $P_i = 0.3$  MPa; intermediate principal stress coefficient  $b = 0.25$ .



**FIGURE 2.** Radius change curve of plastic zone under different tunnel pressure  $P_i$

Figure 2 shows the variation curve of plastic zone radius under different pressure in tunnel. Using the formula (7) and (8) calculation to determine the first critical stress  $\sigma_{p1} = 1.26$  MPa, the second critical stress  $\sigma_{p2} = 5.84$  MPa. When  $P_i < 1.26$  MPa, Due to the stress release caused by tunnel excavation, the surrounding rock is in the elastic-plastic state, and the maximum principal stress is radial stress, so the radius of plastic zone should be calculated by formula (4). With  $P_i$  from 0 to  $1 \sigma_{p1}$ , the radius of plastic zone decreased from 20.16m to 8.50m. When  $1.26 \text{ MPa} \leq P_i \leq 5.84 \text{ MPa}$ , surrounding rock is in elastic condition completely and stress analysis can be carried out by elastic theory. When  $P_i > 5.84$  MPa, Because of the excessive plastic pressure in the tunnel and the maximum principal stress being hoop stress, the plastic zone radius of the surrounding rock should be calculated by formula (6). Analysis by using the Fenner formula, the first critical stress value is 1.68MPa, compared with the calculation results of (7) 0.42MPa is increased, and the same tunnel pressure under the condition of the radius of the plastic zone is larger, and there is no second critical stress value, which mean that no matter the tunnel pressure increases to any value, the surrounding rock will not appear plastic failure, obviously this conclusion is unreasonable.



**FIGURE 3.** Radial stress curves under different  $b$  values

Figure 3 show the distribution law of circumferential stress when the intermediate principal stress coefficient  $b$  is taken to be 0, 0.25, 0.5, 0.75 respectively. It can be seen from the figure, with the increase of  $b$  value, the plastic zone in the same position of the radial compressive stress and circumferential stress are increased, and the stress of the radial coordinate change rate is increased; and in the same place in the elastic region of circumferential stress and radial pressure with the increase of  $b$  stress decreased; with the increase of  $b$  value of the plastic zone is obviously reduced, and the elastic-plastic interface circumferential compressive stress increased from 5.49MPa to 6.63MPa, and the radial compressive stress is reduced from 1.60MPa to 0.48MPa, the elastic-plastic interface maximum shear stress increased from 1.95MPa to 3.07MPa, witch shows that rock in the larger shear stress remains stable and does not have the plastic damage.

## 5 Conclusion

(1) The stress solution of the tunnel and the corresponding radius of plastic zone under the condition of radial stress and hoop stress are derived based on unified strength theory. When  $b=0$  and only consider the radial stress as the first principal stress, the theory in this paper is reduced to the Mohr-Coulomb strength theory, and the plastic zone radius formula is reduced to Fenner formula.

(2) With the influence of intermediate principal stress coefficient  $b$  increases, the elastic-plastic interface of maximum principal stress and maximum shear stress are increased and the radius of plastic zone is decreased; the surrounding rock is easier to enter the fully elastic state with the maximum principal stress as the radial stress, and the larger the thickness of the surrounding rock is in the elastic state at the same time.

(3)  $P_i < \sigma_{p1}$ , part of the surrounding rock is in a plastic state, and the first principal stress is radial stress,  $\sigma_{p1} \leq P_i \leq \sigma_{p2}$ , surrounding rock is in elastic condition completely,  $\sigma_{p2} < P_i$ , part of the surrounding rock is in a plastic state, and the first principal stress is hoop stress. In the elastic-plastic analysis of surrounding rock, we should determine the state of the surrounding rock firstly, and then use the corresponding calculation formula for stress analysis.

(4) The above analysis shows that the intermediate principal stress is beneficial to fully exert the strength potential of the surrounding rock, thus improving the stability of tunnel surrounding rock, and the smaller the  $b$  is, the higher the sensitivity of the surrounding rock stability state to the intermediate stress is.

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